

Can the Notion of Affordances be of Use in the Design of a Technology Enriched Mathematics Curriculum?

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The term affordances is rising in prominence in scholarly literature in mathematics education generally and in technology in mathematics education in particular. A proliferation of different uses and meanings is evident. The roots and use of the term and some of its applications are explored in order to clarify its many meanings. Its potential usefulness for developing a framework for a new research project which aims to enhance mathematics achievement and engagement at the secondary level by using technology to support real world problem solving and lessons of high cognitive demand is investigated.

Mathematical technologies have the potential to bring real world applications to life in the mathematics classroom. Some of the issues the incorporation of such an approach into curriculum design must address will be investigated in the RITEMATHS project, funded from 2004-2006 by the Australian Research Council Linkage Scheme, involving the Universities of Melbourne and Ballarat, Texas Instruments (TI), and a number of industry-partner schools from both Ballarat and Melbourne.¹ The project aims to enhance mathematics achievement and engagement by using technology to support real world problem solving and lessons of high cognitive demand in secondary mathematics classrooms. Working collaboratively with the researchers, industry-partner teachers will design, evaluate in classrooms, and iteratively refine technology supported tasks that sustain higher order thinking and deep engagement with context. For this project three research themes which especially impact on the use of real world applications have been identified, namely context, algebra, and *affordances* of technology. The third of these potentially provides an overarching framework for the project. "Research examining the concept of affordances is critical if we are to build ... a more flexible design orientation to the practices of education" (Pea, 1993, p. 52). An operational definition of affordances that serves the purpose of the research project is needed. This paper outlines the use and definition of the term affordances in scholarly literature particularly highlighting its use in mathematics education in technologically enhanced teaching and learning environments.

Affordances

A Potted History

The word affordances, often referred to without definition or source in very recent technology in mathematics education literature (e.g., Doerr, 2001; Kaput, 1998), was first coined in 1966 by the perceptual psychologist J. J. Gibson (Reed, 1988, p. 231) who later claimed,

the verb to afford is found in the dictionary, but the noun affordance is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment As an affordance

of support for a species of animal ... they have to be measured relative to the animal. They are unique for that animal. They are not just abstract physical properties. (1979, p. 127)

Gibson's main interest was an ecological approach to perception. He believed that animals directly perceive "the affordances of objects, places, events and persons" in their environment and from this he developed the "first truly alternative explanation of cognition" (Reed, 1988, p. 298). For a person, water, for example, affords drinking; not breathing but drowning (Gibson, 1977); but for a rainbow trout it affords breathing.

Mainly inspired by *The Psychology of Everyday Things* (Norman, 1988), the term was taken up in the area of human-computer interaction which focuses on human designed environments. Pea (1993) has suggested that the notion of affordances be used in exploring the interrelationship between tools such as computers as artefacts of distributed intelligence and educational practice. Although he acknowledges Gibson, Pea extends Norman's idea of the "psychopathology of everyday things" to artefacts used in teaching and mathematical representations. By this he means how "the affordances of objects deeply and often unnecessarily restrict their accessibility to the ordinary human" (p. 52). Affordances and constraints were discussed at length by Greeno (1994, 1998) within a *situative perspective* of educational practice. The term affordances is becoming increasingly more prominent in literature in technology related aspects of mathematics education (e.g., Guin & Trouche, 1999; Kennewell, 2000), general mathematics education (e.g., Watson, 2003), and the epistemology of mathematics (e.g., Van Kerkhove & Myin, in press). However, a diversity of explicit definitions and implied meanings exist.

Gibson's Affordances

Origin of Gibson's affordances. The concept of affordance has its roots in Gestalt psychology. Koffka (1935) talks about *demand character*, "each thing says what it is ... a fruit says 'Eat me'" (p. 7). Koffka argued that "the value of something was assumed to change as the need of the observer changed" (Gibson, 1977, p. 78). According to Gibson, the phrase *Auffordungscharakter*, translated as valence, was used by Kurt Lewin (Brown, 1929). "The valence of an object was bestowed upon it in experience, and bestowed by a need of the observer" (Gibson, p. 78). Although related to demand character and valence, the concept of affordance has a crucial difference as affordances do not change as an observer's needs change. "Whether or not the affordance is perceived or attended to will change as the need of the observer changes, but being invariant, it is always there to be perceived" (p. 78). Gibson's initial interest was in understanding what motivates human behaviour (Reed, 1988). Before one can act, action must be *perceived* as possible, that is, the perception of an affordance motivates the doing of an action.

Affordances as relationships. Gibson (1977) considered affordances to be relationships between objects and actors involved in interactive *activity*. They are what the environment offers to a particular sub-class of animal. The affordance of a particular seat, for example, will differ depending on the size of the human. A highchair may afford sit-on-ability for a young child, however it is unlikely to offer this to readers of this paper. The object has certain *attributes* and the actor particular *abilities*. Gibson sees affordances as a precondition for activity defining allowable actions between the object and actor; however, the existence of an affordance does not necessarily imply that activity will occur. Van Kerkhove and Myin (in press) in proposing "to consider mathematical entities in terms of the sets of practices they 'afford'" use the example of numbers affording such operations as adding or multiplying and these mathematical entities being able to be "directly perceived" only by "someone possessing particular number-related skills".

Invariance of affordances. Affordances exist within the *constraints* of an environment and are invariant. To understand invariance of affordances one needs to consider the environment of one individual as opposed to the environment of all individuals (Albrechtsen, Anderson, Bødker, & Pejtersen, 2001). Consider a rectangular table viewed by an observer from a particular position. The observer's view of the table changes as the observer alters position. The table does not change although the observer's perception of it does. "Invariant structure ... specifies the environment of all observers, what any observer would see on any path" (Reed, 1988, p. 290) around the table. The invariants that are perceived by an observer looking at a table are the information specifying its affordances.

Affordances in Human-Computer Interaction (HCI)

Gaver (1991) was the first to use the term in the context of HCI. Gaver tried to produce a framework for separating affordances from design—"the information that specifies the affordance" (McGrenere & Ho, 2000, p. 5). Considerable ambiguity in the use of the term in HCI followed but mainly because of the influence of Norman (1988) who considered affordances to be perceived properties which may not be real but suggested how to use the object. Albrechtsen et al. (2001) suggest this definition confuses affordances with invariants. Norman (1988) argued that in an ideal situation a designer would take full advantage of the affordances of an object making it obvious at a glance to a user how the object was to be used. This idea has been of especial interest to designers of computer interfaces. Authors such as McGrenere and Ho (2000) have tried to unravel the diversity of definitions in this area. As a major focus in the area of design is on the visible interface, ideas developed in this area are not easily transferable to the more complex area of teaching and learning and hence will not be considered further.

Affordances in General Mathematics Educational Literature

As noted earlier, Pea (1993) sees research into the concept of affordances as critical. He points out that many of the touted goals of education are based on the presupposition of

the success of the social constructability of affordances — that one can get a learner to attend to the pertinent properties of the environment, or the designed object, or the inscriptional notations, such that the learner can join in to contribute to distributed intelligence in activity. (pp. 51-52)

He disputes Gibson's notion that affordances are directly perceived, suggesting that the "meeting of intentionality and artefact is not simply the direct perceptual pickup of the affordance structure of the object" (p. 52). Pea expands this by suggesting that culture and context also contribute.

Gibson's view that affordances are interactions is supported by Greeno (1994). Greeno (1994) sees any agent-system interaction involving conditions arising from properties of both the agent and the system. He differs from Gibson in that he does not see "direct perception as a defining characteristic" (p. 341) of affordances but suggests recognition plays a role. To Greeno an affordance is a property of whatever a person interacts with in the environment but this property must interact with a property of the person so as to support an activity. Abilities are "whatever it is about the agent that contributes to the kind of interaction that occurs" (Greeno, 1994, p. 338). Greeno (1994, 1998) relates these ideas to use of affordances and constraints in situation theory where "attunements to constraints and affordances" (1998, p. 8) are used to provide a broader explanation of activity in terms of agent-system interactions than the narrow analyses of the "structures of the informational contents of activity" provided by cognitive science (1998, p. 6).

Tanner and Jones (2002) develop Gibson's ideas further within an educational setting. They define an affordance as "a potential for action, the capacity of an environment or object to enable the intentions of the student within a particular problem situation" (p. 78). This idea of affordances as potentiality has become its more common usage. They build on the work of both Gibson and Greeno as they suggest constraints "limit possible actions" (p. 78) and that "students' *abilities* are determined by their potential to act within the particular environment. If *abilities* change, then learning has occurred" (p. 78).

Watson (2003) discusses how perception of the mathematics classroom is linked to opportunities to learn and how each classroom task affords engagement with mathematics in particular ways. Building on Gibson's ideas and Greeno's articulation of these in a social setting, Watson describes classrooms as having potential for developing learners' identities through activities and within these classrooms "a mathematical topic provides an arena which affords learning, [and] constraints [of the setting and the task] limit the variation which can be perceived" (p. 37). She suggests the analytical framework offered by the "complex interplay between what *could be* possible, what *is* possible, and what *is seen as possible*" (p. 37) can serve to analyse a broad range of educational settings and also the nature of the tasks and the activities they result in.

Constraints and Negative Affordances

"Analysing the affordances in a situation may need to be more than merely listing available items" (Kennewell, 2000, p. 57). Kennewell used a design experiment within a theoretical framework based on affordances, constraints, and abilities to explore the effect of computer modelling tasks on the learning of algebra. He describes affordances as "attributes of the supporting features" (2001, p. 106) or "the potential for action inherent in the features of the setting" (2000, p. 55). He acknowledges "that the setting imposes *constraints*" (2000, p. 55). Constraints are complementary: They "are not the opposite of affordances" (2001, p. 108). Kennewell claims that both affordances and constraints must be considered as the former describes potential for action; the latter the structure for action and it is often "the structure imposed by the setting [that] may facilitate task progress" (2000, p. 55). Within an educational setting novice learners are deliberately constrained in order to facilitate desired action with the teacher altering the available affordances and constraints so the gap between these and the learners' abilities allows intended learning to occur (2001, p. 107). Kennewell illustrates his ideas in a discussion of multiple representations, suggesting the mere provision of such an environment is insufficient. "The alternative representation must afford activity which engages the pupil with the ideas to be learned and the constraints must reflect the structure of those ideas" (2000, p. 70).

Interactions between learners and technological devices necessarily involve both the ability of the learner and the affordance of the technology. These combine to determine the potential of the interaction in any given situation. Some constraints are artefacts of the technological environment. For instance, in a graphing calculator learning environment one is constrained in terms of representational control to directly altering only the algebraic or numerical representations of a function. The graphical representation cannot be so controlled. Other constraints may be imposed by the teacher, the student, or derive from the general learning environment.

Kaput and others often imply that affordances are positive, with Kaput (2001), for example, stating that affordances of technology are the ways in which it provides successful solutions to dilemmas in teaching and exploits learners' strengths. Other writers including Gibson discuss the idea of negative affordances. For example, a reluctant

student, Mary say, presents her ideas to the class convincing them that she has correctly identified a particular graphical view of a function on her graphing calculator as linear. Taking a socio-constructivist approach, the class teacher would sustain the interaction and ask Mary to explain her certainty that the graph is linear. If the teacher simply told Mary that the graphical view observed was actually a zoomed in view of a cubic function, this “interaction” would become a “negative affordance” (Akhras & Self, 2002, p. 25) from a socio-constructivist view point, as the student was not able to construct her own learning, nor was the community of inquiry that constituted the classroom so allowed. For research the idea of negative affordances allows questions such as: “How do particular kinds of affordances of an environment allow or preclude a student to learn from engaging in particular kinds of interactions at particular times?” (Akhras & Self, 2002, p. 25).

Applications of the Concepts of Affordances and Constraints within Technologically Rich Mathematics Learning Environments

Examples of affordances and constraints in various technologically rich mathematics learning environments to be explored in the RITEMATHS project are now presented. These include applications to environments involving graphing calculators, the computer application Mathworlds from the SimCalc project, and computer algebra systems (CAS).

Graphing calculators. Doerr and Zangor (2000) in discussing research findings relating to teaching with graphing calculators suggest some “teachers did not change their methods or approaches” (p. 80). This highlights the importance of teachers being aware of the affordances of a technological environment. Drijvers (2003, p. 78) noted that affordances of the graphing calculator (in the sense of “perceived opportunities” for the integration of technology into mathematics teaching) have been identified by many writers as including “the use of realistic contexts, an exploration approach to problem situations, visualisation and the integration of different representations, the experience of dynamics within a problem situation, [and] a flexible way of doing mathematics” (p. 78). However, he points out that for the affordances of any technological tool, to be realised in the classroom not only does this depend “on the affordances of the technological tool, but [also] on the exploitation of these affordances embedded in the educational context and managed by the teacher” (p. 78). Furthermore, in a study by Doerr and Zangor (2000) exploring how “the students’ learning and the teacher’s pedagogical practice” (p. 149) were enabled or constrained, it was found that “the graphing calculator emerged as a constraint” when used either as a ‘black box’ or as a personal private device which did not have to be shared with others in group work. Their discussion points to teacher’s knowledge of the constraints of technology being crucial if actions taking advantage of affordances are to be enacted.

Mathworlds. The computer application SimCalc Mathworlds (Kaput, 1998) has “hot, bidirectional links” between the algebraic, graphical and numerical representations of the movement of toy cars or simulated lifts or clowns. Changing any of the representations, or the physical or simulated situations, changes all the others. For example, the user can move the simulated lift, drag the graphical representation of its position or velocity, or change the algebraic representation, and the other representations adjust. Kaput describes these capabilities as affordances of the SimCalc system, seeing them as examples of more general affordances offered by technology. “These kinds of affordances turn a fundamental representational relationship between mathematics and experience from one-way to bi-directional. This in turn supports a much tighter and more rapid interaction on which to base learning” (p. 274). However, it is not only affordances that have educational potential.

The SimCalc recommended lessons take learners through sequenced experiences with progressively fewer constraints. In the early stages, only a few capabilities operate and the user is constrained to work from particular representations to achieve desired effects.

Symbolic Math Guide. Symbolic Math Guide (SMG) is a free Flash application for TI CAS calculators. The affordances of this application are its facilities to take users through the step-by-step algebraic solution of problems such as the solution of equations and simplification of expressions involving algebraic fractions. At each step SMG prompts the user to choose from a finite list of items dedicated to the particular form of the algebra at that point in the solution. When simplifying $\frac{x^3 - 2x + x}{x - 1}$, for example, the choices available are as shown in Figure 1. The calculator undertakes the selected action, and then presents the user with another set of choices. If the user chooses option 4: combine like terms, the calculator shows the interim step of the distributive law application, that is, $\frac{x^3 + (-2+1)x}{x - 1}$, followed by the simplification of that expression. Another selection would result in an equally, mathematically correct execution of that step but some selections lead to less than helpful expressions. The user has the option to select CLEAR at any time, returning to the initial problem where the solution steps can begin again. A negative affordance offered by the CLEAR feature of SMG is that the user must always begin at the initial step, rather than delete lines step by step to reconsider the problem at any point in the solution process. The affordances of SMG include simplification of linear equations, hints to aid this process, the use of a balanced method of solving equations, and explicit application of the distributive law in this process. However, users are also constrained to continue to follow the step by step solution long after interim steps would be subsumed in written solutions.

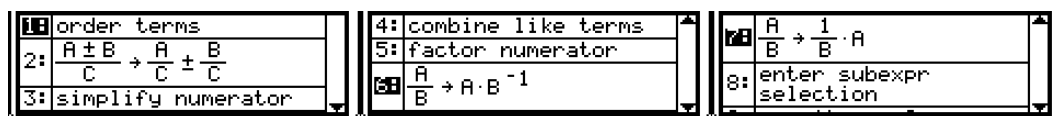


Figure 1. SMG simplification choices available at a particular point in the solution.

Computer Algebra Systems. The notions of affordances and constraints have been useful also in the study of the educational use of computer algebra systems. A detailed analysis by Guin and Trouche (1999) describes specific features of the artefact (e.g., a CAS implemented on a calculator) as being either constraints or affordances. They view constraints as shaping the potential of an artefact “in terms of types of actions and their management” (p. 203), categorising them as internal, command and organisational constraints. Internal constraints are caused by the inevitable limitations of the artefact. They include the many consequences of the discrete pixilation of the screen on graph presentation, such as jumps in lines that should be straight and joining points across discontinuities. Another example is that a CAS may be able to recognise and use the exact value of $\cos \frac{\pi}{8}$ but not $\cos \frac{\pi}{16}$ even though the latter can be derived easily from the former using a half angle formula, which the CAS calculator “knows”. Command constraints relate to the need to learn and use syntax strictly and to appreciate the precise effect of commands. For example, the *solve* feature for equations may not “solve” an inequality; one needs to specify which variables are to be solved for explicitly; and *solve* works differently when the calculator is in exact or approximate mode (so that a cubic can be solved in either

mode, but most quintics can only be solved in approximate mode). Organisational constraints relate to the architecture of the artefact: what is “at the fingertips” on one brand of CAS may require following a sequence of steps on another. Guin and Trouche point out that the many constraints make learning to use a CAS skillfully a major undertaking for students. However, internal and command constraints can be important “training features”. In learning how the machine works, the students also learn about mathematics from a new viewpoint. Learning how to use a complicated instrument such as a CAS calculator requires discovering its affordances (these are not obvious) and learning to operate within its constraints. Thus the technology user has to learn to think about problems and mathematical operations in a similar way to the software designers. A challenge for research in mathematics education is to study how this process can be most productive.

Discussion and Conclusion

The ideas presented in this paper open a small window on the diversity of understandings of the term affordances, their underlying ideas, and companion terms such as constraints. Although in some research areas authors appear to believe this term needs no explanation or definition, its variety of meanings in scholarly literature point to this lack of definition as being potentially problematic as there certainly are not shared and agreed understandings even in specific research communities. The aim of this paper was to determine an operational definition for the purposes of research in a particular area of mathematics education, however many questions have arisen with regard to affordances. These include:

- Is it that something— an object, tool, artefact, or instrument— affords users to do things in particular ways by constraining them to think or act in a specific way?
- If you ask me to solve a problem algebraically, for example, is that constraining my thinking to the algebraic representation but nevertheless affording me to learn, consolidate, or demonstrate my understanding in an algebraic representation?
- Are affordances positive and constraints negative or are constraints a way of articulating more specifically the affordances that exist in a given situation?
- Do affordances describe possible specific interactions between the technology and the learner or are they something the technology offers?
- What does the term affordances and associated ideas provide in terms of a framework for analysis in the RITEMATHS project that other frameworks do not?

In the HCI area obvious affordances are of interest for very good reasons, as writers using the term affordances are focussing on the design aspects of technology. In designing mathematics curriculum, where the focus is the learning process, problems are more intransigent than Norman’s notion of ideal design involving obvious affordances. Obvious affordances are unlikely to occur in other than simple educational applications. They will not occur in environments involving major mathematical analysis tools nor would we expect them to as the learning process is a complex one. The learning process is much more complicated than directly perceiving an object by its affordances. We need to perceive the affordances of technological learning environments but we cannot do this merely by looking. However, the characterisation of constraints as “learning features” shows merit. The use of affordances and constraints as a framework for analysis of classroom activity as students are immersed in a mathematically and technologically rich learning environment appears to be worth pursuing. By manipulating the affordances and constraints of particular technologically enriched learning environments it may be possible to determine learning pathways that enhance mathematics achievement and engagement.

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