Shaping Understanding: How Does Investigating in a Spreadsheet Environment Affect the Conversations of Initial Training Students Doing Mathematics?

Nigel Calder
University of Waikato
<n.calder@waikato.ac.nz>

How do we make sense of mathematical phenomena? This paper examines the varying discourses of groups of pre-service teachers as they investigated problems using two different approaches: pencil and paper (with equipment available), and spreadsheets. The way this discourse shaped their understanding and generalisation of number patterns, through the differing pedagogical media, is also discussed.

This paper examines how mathematical phenomena are shaped by the pedagogical medium through which they are encountered. It considers how the nature of pedagogical support influences the understanding of the mathematical ideas being targeted, and how attention to other pedagogical aspects of the learning environment is activated. A particular focus is on how alternative strategies promote different styles of social interaction and how this interaction contextualises and hence conditions the mathematical ideas. Baker and Beisel (2001) and Calder, (2001) found the spreadsheet's suitability for visualising both graphical and tabular forms enhances the conceptualisation of some mathematical processes. Yet perhaps there is also a cost attached to these supposed benefits. In using spreadsheets to more directly focus on certain mathematical features, what gets lost? From a social-constructivist perspective, if the mathematical conversations, and the negotiation of learning are different, then the learning experience will have different qualities and thus support the range of pedagogical objectives in a different way. Thus the specific task of this paper is to consider how the execution of an investigational activity manifests itself in two alternative learning environments; one involving spreadsheets on a computer, the other using pencil and paper methods. The particular focus will be on how mathematical ideas shape themselves within mathematical discussions between pre-service training students according to the context within which they are presented.

The prevalence of ICT media generally has begun to challenge the map of mathematical ideas encountered in schools. Access to many key elements of school mathematics has been altered as different software offers new ways in which certain constructs are created and understood. In geometry, for example, a circle is understood differently according to whether it is constructed using a pencil and compass, a template, Cabri or LOGO. Meanwhile spreadsheets have been found to offer an accessible medium for young children tackling numerical methods. As a result of the spreadsheet's apparent suitability in visualising both tabular and graphical forms it can enhance the conceptualisation of some numerical processes. There is also some evidence that this medium can offer more than an approach to visualisation. Other benefits that researchers such as Beare (1993), have identified include its interactive nature; its propensity for linking concepts (Funnell, Marsh and Thomas, 1995), its capacity to give immediate feedback, its ability to manipulate large amounts of data; and its suitability for an investigative approach. Ploger, Klinger and Rooney (1997), for example, alluded to this propensity to foster an investigative approach in developing algebraic thinking. They found, significantly, that children learn to pose problems and to create explanations of their own. Unencumbered with numerical computation with decimal or large numbers, and using formulas in meaningful ways, the children gained access to the predictive quality of algebraic thinking, allowing them to pose rich "What if..." questions. Manouchehri (1997) reported similar findings.

These aspects, coupled with the speed of response to inputted data, it seems give the learner the opportunity to develop as a risk taker; to make conjectures, and immediately test them in an informal, non-threatening, environment. This permits the learner opportunity to reshape or construct their conceptual understanding in a fresh manner. Drier (2000), Sandholtz, Ringstaff and Dwyer (1997), and Baker, Gearheart and Herman (1993) in their research into using ICT in mathematics, more generally, likewise reported improved high level reasoning and problem solving linked with this capability. It seems that the nature of mathematical conversations between children, when investigation takes place in a spreadsheet environment, appears to be different to that evident in a classroom setting. The conversations influence and are influenced by, the above factors, but can also be considered as a separate, unique aspect.

The emergence of social constructivist learning theory, over the last ten years, has seemingly resulted in a greater emphasis on inquiry methods comprising investigation, and problem solving with more explicit facilitation of social interaction between pupils. Teachers have increasingly encouraged students to link both the content and the processes of mathematical learning. With an emphasis on working in groups and verbalising interpretations of mathematical situations, negotiation of understanding was encouraged. As Neyland (1995) has indicated, the discussion that occurs within the context of mathematical activity facilitates the learning, with the teacher, the agent of enculturation, playing a key role in support of this. This sort of perspective activates interplay between the task of the individual learner and the way in which that is understood as an engagement with a more social frame. Cobb (1994) has highlighted the pedagogical tension between the perspectives of mathematics education being perceived as a notion of enculturation, as compared with one of individual construction and the theories that have been invoked in support of these.

Meanwhile Brown has sought to soften the individual/social divide with a phenomenological formulation with an emphasis "on the individual's experience of grappling with social notation within his or her physical or social situation" (Brown.1996, p.118). While the introduction of a social frame is inevitable, this will vary according to how the activity is constructed and the perceived environment within which this takes place. The mathematical activity is inseparable from the pedagogical device as it were, derived as it is from a particular understanding of social organisation, and hence the mathematical ideas developed will inevitably be a function of this device. Brown (2001) has also argued that such pedagogical devices should be regarded as worthy objects of mathematical learning insofar as school mathematical learning is largely carried out to be in support of the student's later engagement in mathematically oriented social activity.

Approach

The investigative work undertaken within this study considered the spreadsheet as a possible tool to facilitate discussion as part of the process of understanding. This was considered alongside an alternative approach based around pencil and paper methods. The specific aspects to be considered here are: the amount of conversation, the quality of articulated reasoning, the nature of the technical language used, and the way in which conversations, in the different environments, filtered the understanding of, and the approach to, the investigation.

Three groups of three first year, primary, pre-service students worked in a typical classroom setting with counters, calculators and pen and paper available, and three groups, from the same class, worked independently, in an ICT laboratory, doing the same investigation using spreadsheets. Their discussions were audio recorded and transcribed; each group was interviewed after they had completed their investigation; and their written recordings were collected. This data, together with informal observation and discussions, formed the initial basis for the research. Five weeks after the first data was gathered, a similar approach for data collection was used, with the students using the same medium, but a different investigation. Analysing both tasks provided greater depth to the data as the participants had undertaken more investigative work in the interim, and the data collection should feel less intrusive the second time.

The investigations were chosen to be suitable for exploring in a spreadsheet environment and this may have constrained the nature of the tasks, and hence guided thinking and conversations in a particular way.

For example the second activity:

Investigate the pattern formed by the 101 times table by:

- Predicting what the answer will be when you multiply numbers by 101
- What if you try some 2 and 3 digit numbers? Are you still able to predict?
- Make some rules that help you predict when you have a 1, 2, or 3 digit number. Do they work?
- What if we used decimals?
  How did you solve the problem?

Nevertheless, it was felt that this still would shed some valuable light on how students responded differentially to the alternative presentations of what might be viewed as the same activity.

The transcripts from both tasks were then systematically analysed for patterns in the dialogue, within the settings, and comparisons between the two settings. The students were labeled serially; students one to eight were in the classroom setting and students nine to fourteen in the spreadsheet one.

The Conversations

A social frame did inevitably emerge in both situations.

The classroom setting:

The conversations in the classroom situation began with a group member initiating the negotiation of the meaning and requirements of the activity. This initial negotiated sense making started with a single discrete numerical example. The students used this to not only begin the process of solving, but also to help determine the nature of the task; what it was asking them. For example:

> Student one: So if we had twenty three times a hundred you would have twenty three hundred…Lets say we do twenty three times a hundred and one, we would get twenty three hundred plus twenty three ones
>
> Student two: Does it look right?
>
> Student one: Yes that is what I would guess it to be. Like if it was eleven times a hundred and one it would be eleven hundreds and eleven ones.

While this clearly is the preamble to the process of generalising, they needed to then verify these and other examples before using more recognisable language of generalisation.

> Student one (later): Basically if you times your number by a hundred and then by one you would add them together and get your answer.

Group two likewise went initially to an example although they took a more sequential approach.

> Student five: What if we went one, two, three, four, five, six and multiply it by one hundred and one?

This approach was evident in the first task too. For example:

> Student one: Shouldn't we work through each one?
>
> Student two: Two hundred by twenty equals four thousand. That was easy.

The students again looked to evaluate individual numerical examples, to build up a numerical picture, usually in a written tangible form, before trying at a later stage to order it, analyse and look for generalisations.

> Student two (later): We did them individually. We broke down twenty weeks and worked through the entire process of doubling one cent each week.

Likewise in their recorded thoughts after the investigation, this approach was highlighted.

> Student one: We went through one at a time and solved them. We solved them on paper and we solved them with a calculator.
>
> And Student seven: We didn't really predict from the beginning. We just jumped straight in and did it that way. [Student 8] found that she likes to see it, have each individual one to see the pattern.

The spreadsheet setting:
In contrast, those groups working in the spreadsheet environment, tended to initially perceive that the bigger picture was most easily accessed through entering a sequential, formulaic structure into the spreadsheet and then visually analysing for patterns. For example:

> Student 10: I haven't predicted. I was just going to put in A1 times 101 and drag it down (does it).
>
> Student 9: So we're investigating the pattern of 1 to 16 times 101.

This appears a more direct path to the patterning approach, and several comments later this group had recognised a pattern, and explored further based on the rule for their pattern.

> Student 9: If you did a huge number like five hundred times 101 it would be 500500 wouldn't it?
>
> Student 10: Lets have a look. It's 50500.
>
> Student 9: Let's try a hundred times 101.
>
> Student 10: 10100. If you put in 800 it would be 80800.

The discussion seems to focus more on the pattern through a visual lens rather than an operational one. That is, the pattern of the digits in the outcome rather than how the numerical operation affects the structure of the outcome. Another spreadsheet group highlighted this aspect of visualising the whole pattern to scrutinise for general qualities.

> Student 14: 101,202,303,404, and 505 onwards, because it is one times the number. It's straightforward in terms of doing the spreadsheet. It should continue to show that pattern throughout. Drag it down and I think it will probably pick it up.

This also introduces a difference in terms of the technical language utilised. Did this alter the way the students made sense of the situation or proceeded to analyse it? "Drag it down" is functioning language rather than mathematical, but the inference is clearly that there is a pattern, which might possibly lead to a generalisation; and that the spreadsheet by nature will enable users to quickly access that pattern.

The user needs to be enculturated into this schema though, and initially there is negotiation of these aspects also.

> Student 12: Equals sum? Is it A2?
>
> Student 11: A1? Maybe that's wrong.
>
> Student 12: Can't we just do it down the column? It should be the top one. A1 multiplied by 101 and then drag down.

Whether this negotiation of procedures, and the different style of social interactions initiated, changes the approach to the mathematical conversation is difficult to ascertain, but considered in conjunction with other aspects, it certainly seems to lead to a different contextualisation of the mathematical ideas.

The structure of the conversations evolved with several comparable aspects: the number of interactions between students was similar for both the classroom situation (mean=46.5) and the spreadsheet one (mean=48.6); the number of questions the investigations generated in the on task conversations was also equivalent: 15.2 on average for the classroom and 16 for the spreadsheet situation. These tallies of the questions discounted any that were the reading of activity prompts, but included those of a technical nature. A technical question was one such as: "How much would you know to drag it down by?" which called for clarification of an operational aspect of the equipment available, compared with one such as: "What if we tried some three digit numbers?" This type of question engaged the group in further mathematical thinking; it fostered the sense making of the context the problem was situated in, or of possible solutions.

Only a spreadsheet group had any technical questions (14) indicating less fluency with the tools they employed. The groups in the classroom setting articulated their mathematical reasoning more frequently; 7.75 times on average, compared to 5 for the spreadsheet group. In the interviews, they typically had noticeably more expansive responses both in explaining the process they used, and the explanation of their solutions. For example, one person in the classroom situation explained their approach:

> We completed each question and basically added up the totals of each problem. We went through each question first. I couldn't look at them and say that was the answer. We had to go through each and discover the answer. As a group, we did it individually because none of us believed number two was the best one. We had to prove to ourselves that it was the best answer.

This compares with the explanation of someone who used a spreadsheet:

> You only have to know the formula for getting the answer.

This could be due to the environment fostering deeper mathematical thinking or just greater articulation of that thinking. It is an aspect that requires analysis.

The Approaches to the Investigations

Likewise, the approaches used to investigate the problems had similarities and contrasting features. The groups in both settings used a reflective, cyclical approach involving making sense of the problem, prediction, verification, reflection, and generalisation. Further iterations of this cycle occurred in varying degrees, followed by communication of the solution in terms of the problem's context.

The initial sense making of the problem, and exploration happened in distinct ways however. Those working in the classroom setting discussed the problem, while trying one or two explorations. As they made further sense of what the problem was about, they began to predict, verify and reflect in a discrete numerical manner, before looking for a methodical recording means so as to make generalisations. This tended to be tabular. The groups using the spreadsheet, after an initial perusal of the problem, looked immediately for formulae to generate tables of values. They predicted, then verified, within this tabular structure, ensuring that it was appropriate for exploring the problem, then moved more directly to the generalisation phase. This initiated the predict, verify, then reflect cycle.

While superficially these approaches have equivalent features, there is a contrasting approach to the initial exploration and making sense of the problem, that not only affected the way the best solution was attained, but the type of conversation that occurred and subsequently, possibly the understanding of the mathematics.

Discussion

Are the social discourses different in the two pedagogical settings? If so, to what extent; what is the nature of the differing conversations, and how might this contextualise the mathematical ideas, and thus condition the student's understanding?

As evident in the above section, discourse in each situation demonstrates a contrast in the initial approach to engaging in the mathematics. In the spreadsheet setting, the discussion told a story of using the spreadsheet to get a broad picture; using the formulas and copy down functions to create a numerical table that could then be analysed for any pattern. The language included technical questions and statements, primarily regarding spreadsheet operation. This may have indicated a variance in familiarity with the medium used in the two approaches, rather than an inherent pedagogical difference however.

The spreadsheet approach, perhaps due to the actual technical structure of the medium, led more directly to an algebraic process, with the language interactions containing both algebraic and technical terminology. This is consistent with Battista and Van Auken Borrow's (1998) observation that students developing spreadsheets to solve problems used greater accuracy in applying procedural structures. It seemed, in fact, that the spreadsheet setting, by its very nature, evoked a more algebraic response. The participants in these groups were straight away looking to generalise a formula that they could enter and fill down. Their language reflected this, but the interactions also contained more language of generalisation, and it took them generally less interactions to start a more formal generalisation process. This was possibly because there was more discussion of individual examples with the classroom setting groups, before they had enough data to make comparisons.

Those working in the classroom setting used a discrete numerical example to engage in the problem; to make sense of the requirements of the problem as well as initiating the process of solving. They tended to try, confirm with discussion as well as another method eg the calculator, then move more gradually into the generalisation stage as their set of discrete examples increased enough for comparison. Their initial dialogue seems more cautious, and contains comments requiring a degree of affirmation amongst group members before moving into a more formal approach to generalisation. As a consequence the descriptions of the process undertaken and the mathematical thinking were more fulsome. This may be evidence of more fulsome understanding too.

The groups in the classroom setting tended to have greater articulation of mathematical reasoning; both in the frequency of occurrence and the depth of explanation. Interestingly the explanations were more descriptive of step-by-step procedures rather than the comparative broad statements (see the last two quotes in the *conversation* section). This could be indicative of greater understanding of the process, or it might be just the language describing, rather than shaping, the process. The social interaction and the process certainly shape each other to some degree, but the extent of symbiotic relationship can't be ascertained here.

The spreadsheet groups used more algebraic language eg formula, while the pencil and paper groups had more numerical reference. The differences in language probably reflected the differences in approach the two settings engendered, rather than the differences in language evoking distinctive approaches. As well, the spreadsheet groups progressed more quickly into exploring larger numbers and decimals. This appears to indicate a greater propensity for exploration and risk taking engendered by the spreadsheet environment. Yet, although that is consistent with other findings (Beare, 1993; Sandholtz et al, 1997; Calder, 2001), certainly no correlation between using spreadsheets and greater mathematical risk taking can be drawn from this study.

Most significantly, the social interactions appear to shape the analysis of the patterns in distinct ways. Given that the path to, and manifestation of, the patterns differs, the conversations indicate a different approach once the patterns are viewed. Those using the spreadsheet used a more visual approach. They were observing and discussing visual aspects eg the situation of digits or zeros. For example:

> You take the zero out. What about when you get to the three digits? Was that 22? So the middle number is still a double? Okay, so when you've got three digits, you get two, two, five, three, three.

Those using pencil and paper were more concerned with the operation aspects that generated the patterns. For example:

> Basically if you times your number by a hundred and then by one you would add them together and get your answer.

To generalise a pattern in terms of the sequence of digits is significantly different to generalising in terms of an operation. In this aspect, the different settings have certainly filtered the conversation and approach, and by inference the understanding. As Brown (1996) argued, the mathematical understanding is a function of the social frame within which it is immersed, and the social frame evolves uniquely in each environment. This study demonstrated that the different pedagogical media provided a distinct lens to contextualise the mathematical ideas, frame the mathematical exploration, and condition the negotiation of the mathematical understanding.

References

Baker, J. D., & Biesel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics, 101*, 23-31.

Baker, E.L., Gearheart, M., & Herman, J.L. (1993) *The Apple classrooms of tomorrow: The UCLA evaluation studies.* CST Technical Report 353. Los Angeles, CA: Centre for the study of Evaluation/Centre for Technological Assessment, University of California.

Battista, M. T., & Van Auken Borrow, C. (1998). Using spreadsheets to promote algebraic thinking. *Teaching Children Mathematics, 4*(6), 470-478.

Beare, R. (1993). How spreadsheets can aid a variety of mathematical learning activities from primary to tertiary level. In B. Jaworski (Ed.), *Technology in Mathematics Teaching: A bridge between teaching and learning*. Birmingham.

Brown,T. (1996) The phenomenology of the mathematics classroom. *Educational Studies in Mathematics, 31*, 115-150.

Brown, T. (2001) *Mathematics Education and Language:Interpreting Hermeneutics and Post-Structuralism,* Dordrecht: Kluwer Academic Publishers.

Calder, N.S.(2001) *Does the use of spreadsheets in year 4 number programmes enhance children's numeracy development?* Tauranga:University of Waikato.

Cobb,P. (1994)'Where is the mind? Constructivist and socio-cultural perspectives on mathematical development', *Educational Researcher, 23*(7)*,* 13-19.

Drier, H. S. (2000). Investigating mathematics as a community of learners. *Teaching Children Mathematics, 6*(6), 358-362.

Funnell, L., Marsh, T., & Thomas, M. (1995). Strategies for integrating computers into mathematics lessons: emphasising spreadsheets - SAME papers. Hamilton: University of Waikato, Centre for Science, Mathematics and Technology Education Research, (pp. 223-238).

Manouchehri, A. (1997). Exploring number structures with spreadsheets. *Learning and Leading with Technology*, *24*(8), 32-36.

Neyland,J. (1995). Eight approaches to teaching mathematics. In J. Neyland(Ed.),*Mathematics Education: A Handbook for Teachers.* (Vol. 2). Wellington:Wellington College of Education, (pp.34-48).

Ploger, D., Klinger, L. & Rooney, M. (1997). Spreadsheets, patterns and algebraic thinking. *Teaching Children Mathematics, 3*(6), *330-335.*

Sandholtz, J.H., Ringstaff, C., & Dwyer, D.C. (1997). *Teaching with technology: Creating a student centred classroom.* New York: Teachers' College Press.