Perception of Mental Computation Practice: Reports From Middle School Teachers and Students

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This report details the responses of middle school teachers and students on a series of survey questions regarding perception of mental computation practice. Teachers were asked to report how frequently they developed mental computation skills with particular number topics and related activities. The students were asked to report how often they used mental computation to help them in the same topics and activities detailed in the teacher survey. From the two sets of questions it was possible to compare and contrast the responses and also observe individual teachers with their classes.

> "Mental computation by high school is seen as something most kids should already have! As such its importance is downgraded in yrs 7-10." [Grade 7/8 teacher]

> "Automatic response type activities may dominate upper primary mental computation sessions and written computation may dominate in secondary school." [Grade 5 teacher]

> "[It] becomes the 'flavour of the month' and slips by the way when teachers take on new ideas. However, I believe that mental computation must have a high priority in the maths curriculum." [Grade 5/6 teacher]

The above quotes by three middle school teachers motivate a consideration of what is actually happening in relation to mental computation in the late primary and early secondary years of schooling. McIntosh (2002) advocates that it is a matter of urgency to "find effective ways to ensure that well developed approaches to number find their way particularly into the majority of middle school classrooms" (p. 463). The middle years are arguably a critical time in addressing the essential needs of numeracy (Watson & Callingham, 2001). For students there is an explosion of key mathematical ideas to grasp and the challenges for teachers in guiding the development of their students at this time abound. This report sets out to learn what teachers are doing in the classroom and what students perceive they are using in the way of mental computation.

Background

The majority of mental computation research has focused on the early primary years (Caney, 2002) and as such developing strategies for working with whole numbers have dominated this field (Heirdsfield, Cooper, Mulligan, & Irons, 1999; Threlfall, 2002). For teachers the issue of how to teach strategies has arisen with concerns that mental computation is at risk of being treated simply as drill-and-practice routines in the classroom (Maclellan, 2001). This relates not only to answer-driven mental testing but also to the development and rehearsal of computational strategies.

The emphasis on mental computation is notably part of curriculum and standards documents both in Australia and worldwide, yet from a research perspective the views of teachers seem strangely absent. Reports on teachers' practices in the classroom, including their beliefs and attitudes, are valuable for other teachers, and at a system level in terms of being able to inform professional development programs for teachers. Particularly in times of educational reform, for example the introduction of the *Essential Learnings Framework* in Tasmania (2001), knowing what teachers are doing in certain areas of the curriculum, seems imperative. In terms of considering teachers and students, research studies have for the most part combined only the views of teachers and/or their classroom practices with student achievement, for example in *The Third International Mathematics and Science Study* (Lokan, Ford, & Greenwood, 1996).

The Study

In extending mental computation research to incorporate the middle years of schooling, this report is part of a wider research project investigating what mental computation might entail at this level, with contributions from both teachers and students. This report describes a series of questions, which are part of a teacher survey on mental computation. The purpose of the survey is to provide an overview of current mental computation teaching practices across Grades 5-8 and to ascertain what practises are perceived by teachers to be most effective. The student survey provides similar information on middle school students' beliefs and attitudes to mental computation and examines the influence of the primary/secondary transition on these beliefs. Given that many of the questions on the two surveys are related, it is possible to explore the impact of approaches to mental computation by linking the beliefs and practices of teachers with their students. The following research questions are addressed in this report:

> How frequently do middle school teachers work to develop mental computation skills when working with particular types of numbers and related activities?
>
> How frequently do middle school students use mental computation skills when working with particular types of numbers and related activities?
>
> Are there any associations between the responses of the teachers and the students in general and in individual classes?

Method

Participants

Thirty-four middle school teachers (Grades 5-8) participated by completing the mental computation survey. This sample included 16 primary teachers (Grades 5-6) and 18 secondary teachers (Grades 7-8). The teachers were from primary schools (Grades K-6), secondary schools (Grades 7-10), and district schools (Grades K-12) in the Tasmanian state government system. Schools were identified on the basis of having an interest in developing numeracy across the school. From this sample of teachers, four primary teachers and four secondary teachers were asked to participate further by involving their classes in the project. Consequently 172 middle schools students (Grades 5-8) completed a survey: 83 primary students in Grades 5 or 6, and 89 secondary students in Grades 7 or 8.

Survey Questions

The focus questions for this report provide information on one aspect of teaching practice, namely the frequency with which teachers develop mental computation skills with whole number operations, rational numbers, and related activities involving estimation and calculators (Figure 1). These questions asked teachers to rate each component based on a five point Likert scale – Always (1), Frequently (2), Sometimes (3), Rarely (4), and Never (5). The questions were influenced by the survey instrument used in the *Leverhulme Numeracy Research Programme* (e.g., Askew, Denvir, Rhodes, & Brown, 2000). Students were asked a similar set of questions relating to their perception of their own practice (also in Figure 1). In adapting the questions for the student survey some alterations were made involving the language used; for example, mental computation was referred to as “mental maths” in the student instrument.

<table>
<tr><td colspan="2">Teacher survey questions
When you’re working on the following topics, how often do you try to develop mental computation skills? (Please rate each)</td></tr>
<tr><td>a) Basic number facts (addition & subtraction)
b) Basic number facts (multiplication & division)
c) Multi-digit addition & subtraction
d) Multi-digit multiplication & division</td><td>e) Fractions
f) Decimals
g) Percents
h) Estimation activities
i) Calculator activities</td></tr>
<tr><td colspan="2">Student survey questions
How often do you think you use mental maths to help you…? (Tick each)</td></tr>
<tr><td>a) Add & subtract numbers
b) Multiply & divide numbers
c) Work out percents
d) Work out fractions</td><td>e) Work out decimals
f) Check a calculator answer
g) Work out tables you can’t remember
h) Estimate an answer</td></tr>
</table>

Figure 1. Teacher and student survey questions.

Results

Whole Number Mental Computation

Overall, the majority of both primary and secondary teachers indicated that developing mental computation skills “always” (42.4%) or “frequently” (45.5%) occurred when working with basic number facts. Responses were identical for the two groups of operations, addition and subtraction, and multiplication and division. Those teachers who marked “sometimes” and “rarely” were from the secondary level.

In developing mental computation skills with addition and subtraction of multi-digit numbers more than half of the teachers indicated this occurred “frequently” in class (54.5%) (see Figure 2). More teachers marked “sometimes” (24.2%) than for basic number facts and this increased slightly for multi-digit numbers with multiplication and division operations (30.3%). A greater number of teachers indicated that with multiplication and division this “rarely” occurred (15.2%) (see Figure 3). The response patterns of both the primary and the secondary teachers were similar.

Over half of the students (50.3%) indicated that they “frequently” used mental computation to help them “add and subtract numbers” with a further 20.5% indicating that they “always” did (also in Figure 2). For the students the whole number questions were not separated into basic number facts and multi-digit numbers. The frequency decreased, however, for multiplication and division. Here, 32.7% of students indicated that they “frequently” used mental computation to “multiply and divide numbers” and 12.3% indicated that they “always” did (also in Figure 3). The percentage of students who marked “rarely” also increased to 17.0%. Student responses to the question regarding using mental computation to “work out tables you can’t remember” were very similar to reports on working with multi-digit numbers (multiplication and division), with 32.0% of students responding “frequently” and 34.9% indicating “sometimes.” There were no differences in responses between the primary and the secondary students when considering adding and subtracting numbers. Secondary students, however, more frequently marked the “rarely” category for multiplying and dividing numbers and this was similar for the question on working out tables.

In comparing the two sets of results, the teachers and the students, the distribution of student responses agreed more closely with the teachers' reports on multi-digit numbers than for basic number facts. This is particularly so for the operations of addition and subtraction (Figure 2).

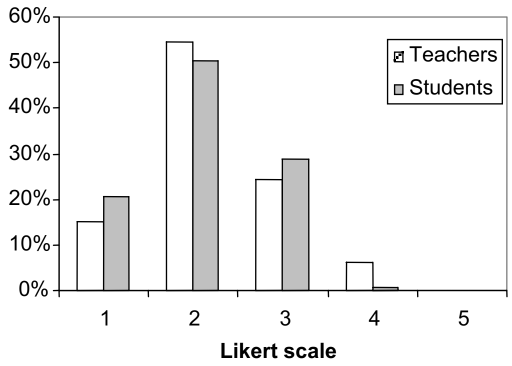

Figure 2. Responses to addition and subtraction survey questions.

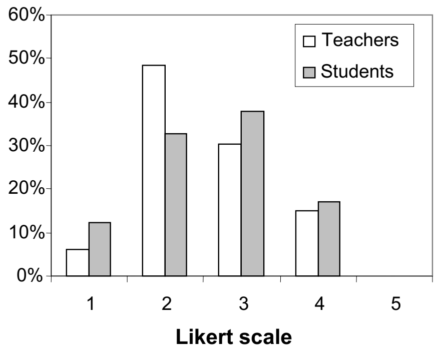

Figure 3. Responses to multiplication and division survey questions.

Fractions, Decimals, and Percents

Approximately half of the teachers indicated that they would "sometimes" work to develop mental computation skills with fractions (48.5%) (see Figure 4). Although many of the teachers indicated this occurred "frequently" (27.3%) or "always" (12.1%), 12.1% of teachers indicated that they "rarely" developed mental computation with fractions; this included both primary and secondary teachers. The pattern of response changed, however, when working with decimal numbers. Here more teachers indicated that they "frequently" developed mental computation skills (45.5%) (see Figure 5). The two teachers who marked "always" were primary teachers with the responses of the secondary teachers concentrated around "frequently" and "sometimes." Both primary and secondary teachers marked "rarely" (15.2%) and like fractions one secondary teacher indicating mental computation skills were "never" developed. Although percent was a topic where most teachers appeared to develop mental computation skills "frequently" or "sometimes" (36.4% each) (see Figure 6), when compared to fractions and decimals, percent had a slightly higher number of teachers who marked "rarely" (18.2%) and one teacher who marked "never."

In considering how often students used mental computation to work out fractions, decimals, and percents, the responses were relatively consistent across all three topics, in particular the number of students marking "sometimes" was almost identical (also in Figures 4, 5, and 6). Percents seemed to be the area where more students indicated they "rarely" used mental computation (34.5%). There was a small number of students who indicated they "never" use mental computation across these topics. There were differences in the response patterns between the two groups of students for these topics, particularly with the spread of responses. In working with percents, for example, secondary responses were concentrated around "sometimes" and "rarely." Primary responses, however, were spread more evenly over "frequently" to "rarely." Responses for fractions were similar. In working with decimals the responses were spread for both the primary and secondary students with more secondary students indicating that they used mental computation here.

The response patterns for teachers and students for questions relating to fractions, differed in that many students indicated it was rare for them to use mental computation even though this was an area many teachers frequently worked to develop the skills: a small number of students marked "never" (Figure 4). This was different from the teachers who indicated they more frequently developed mental computation skills with fractions. The response patterns changed somewhat with decimals, with the modal response for the teachers being "frequently" dropping then to "sometimes," and for the students "sometimes" dropping to "rarely" (Figure 5). Similarly with percents, the response patterns for teachers and students differed (Figure 6). Students reported using mental computation less often with percents, though the majority of teachers indicated it was a topic they "frequently" or at least "sometimes" developed mental computation skills.

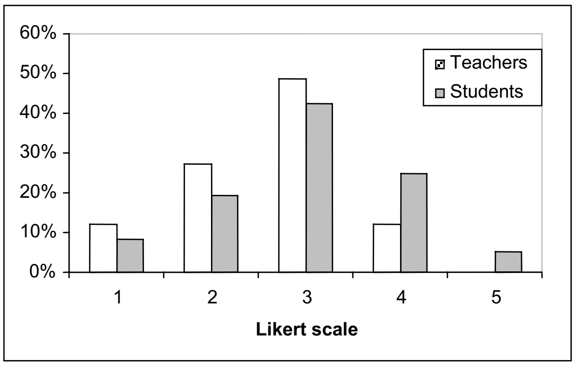

Figure 4. Responses to fraction survey questions.

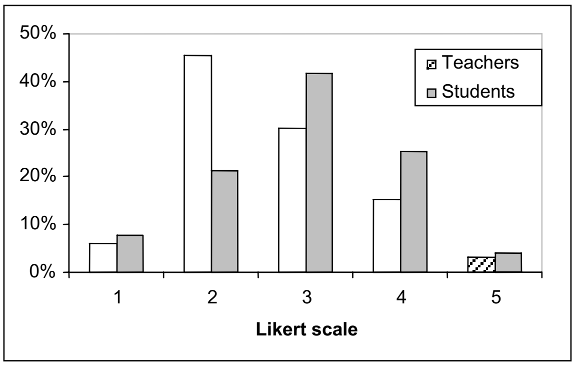

Figure 5. Responses to decimal survey questions.

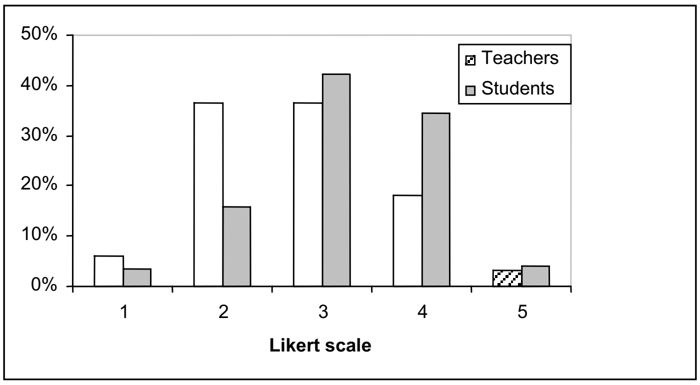

Figure 6. Responses to percent survey questions.

Estimation and Calculator Activities

The majority of teachers indicated that developing mental computation skills "always" (36.4%) or "frequently" (39.4%) occurred with estimation activities (see Figure 7). Responses for these two categories dropped, however, for calculator activities ("always" (12.1%) and "frequently" (24.2%)) and a greater number of teachers indicated mental computation skills were "sometimes" (42.4%) or "rarely" (18.2%) developed here (see Figure 8).

Primary and secondary students most commonly indicated that they used mental computation "sometimes" (43.5%) in estimation activities, with other categories "always" and "frequently" accounting for the majority of students overall (also in Figure 7). Students were asked about using mental computation to "check a calculator answer" and the majority indicated this occurred "sometimes" (41.8%) (also in Figure 8). More secondary students than primary students indicated that this happened "frequently."

In comparing the teachers and the students' responses to the estimation questions, the teachers' reported more frequently working to develop mental computation skills than students reported using them (Figure 7). Whereas the majority of teachers marked “always” or “frequently,” the modal student response was “sometimes” and responses were spread over all categories. The two sets of responses were more similar for students and teachers when considering calculator activities (Figure 8).

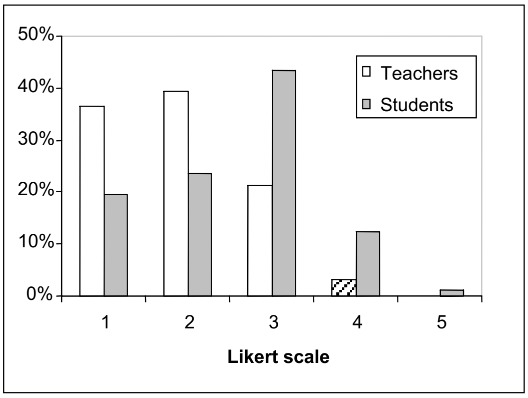

Figure 7. Responses to survey questions on estimation activities.

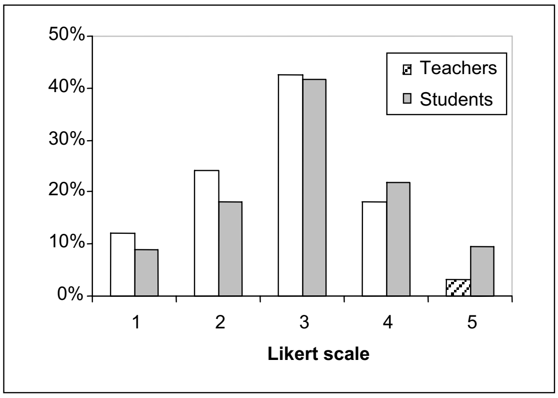

Figure 8. Responses to survey questions on calculator activities.

The responses of a primary teacher and her Grade 5/6 class (*n*=17) and a secondary teacher and her Grade 8 class (*n*=22) were contrasted regarding the agreement for the survey questions. The primary teacher indicated that she “always” attempted to develop mental computation skills in all nine situations. Her students, however, did not have the same perception of their own practice in using mental computation (see Figure 9). With percents, for example, none of the students marked “always” with responses being distributed across “frequently” to “rarely” with one student indicating that mental computation was “never” used in working out percents. In checking a calculator answer many of this class indicated this was rare for them. The second teacher responded in a way that was similar to the modal response pattern for teachers overall. This teacher indicated that she “always” developed mental computation skills with basic number facts, “frequently” with multi-digit addition and subtraction and “sometimes” with multiplication and division. She marked “sometimes” for fractions, decimals, and percents, along with “frequently” for both calculator and estimation activities. In this case the modal responses for students appear more closely associated with the teacher for multi-digit addition and subtraction, and decimals (see Figure 10). Generally, students appeared to use mental computation one level below the teacher’s emphasis: for example, although the teacher marked “sometimes” for percents and fractions, the majority of students indicated they “rarely” used mental computation to help them in these areas.

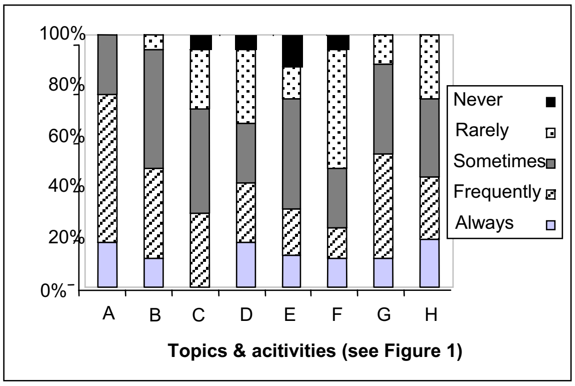

Figure 9. Grade 5/6 class responses to all survey questions.

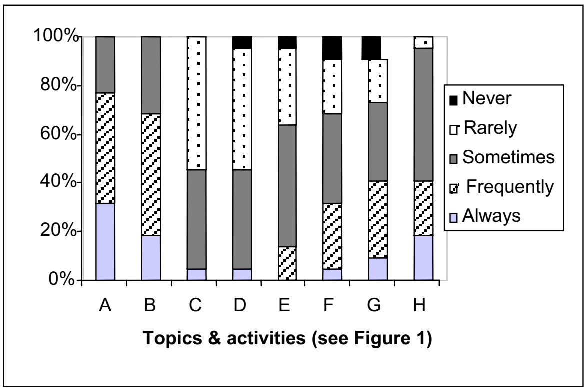

Figure 10. Grade 8 class responses to all survey questions.

Discussion and Implications

On the whole, the students in this study reported that they used mental computation less frequently than the teachers reported working to develop the skills. Possibly teachers need to be more explicit in emphasising the mental element(s) of the number work they do in the classroom with the students. Although the Grade 5/6 teacher reported that she "always" worked to develop mental computation skills in all the topics and related areas on the survey, she was not having the impact on her class that she might desire. Exploring the nature of interactions between teachers and their students and how teachers influence the decisions students make in using mental computation is one direction further research in this area could take. This is particularly so if mental computation is to be regarded "as a first resort" for students in situations that require a calculation (Australian Education Council (AEC), 1991, p. 109). One teacher in this report made the comment that in class "we rarely talk 'about' mental maths." Maybe emphasising mental computation might simply be a matter of helping students develop more of an awareness of the decisions they make in using mental computation (or not).

The responses of teachers and students agreed closely for the multi-digit whole number questions that involved addition and subtraction. For the questions involving multiplication and division, there was slightly less agreement. It might be that students prefer other methods of calculating, such as written computation or using a calculator, especially if these operations are perceived as "harder." Generally these areas are more traditionally associated with mental computation and as such students might be more likely to consider working and calculating mentally.

Developing mental computation skills with fractions and percents appears to be emphasised less frequently by teachers than with whole numbers. Teachers, however, indicated that they did develop skills with decimals more frequently: the link with whole number place value might account for this. Callingham and McIntosh (2002) note that in this area curriculum documents offer little guidance for teachers in terms of developing mental computation skills, for example, " Estimates and calculates mentally with whole and fractional numbers, including finding frequently used fractions and percentages of amounts" (AEC, 1994, p. 104). There is also inadequate information about the strategies students use to solve fraction, decimal, and percent problems mentally (Caney & Watson, (2003). Students' responses varied in using mental computation to work out fractions, decimals, and percents. Possibly students do not feel as competent in solving these types of problems mentally and do not have a range of strategies available. The questions involving estimation and calculators were slightly different in nature in that they concerned general mental activity, not specific types of numbers or operations. For the teachers mental computation was a frequent component of estimation activities but the students did not report using mental computation as frequently during these activities. Again, the fundamental link between mental computation and estimation may need emphasising by the teachers. Students may only count calculations with exact answers as "mental computation" and not regard estimating in the same manner.

Overall, there was little difference between the responses of the primary and secondary teachers who participated in the survey. Although completing the survey was voluntary, possibly only those teachers with a particular interest in numeracy responded. This might account for the lack of differentiation between the two groups of teachers. There was also little that differentiated the primary and secondary students even though the sample was larger. On one hand, it seems plausible to expect that the changes in moving from primary to secondary school might produce differences between the two groups of students. Yet, a "plateau" or "dip" in achievement is often associated with this transition (Callingham & McIntosh, 2002); perhaps this can also be linked with the students' beliefs and attitudes. More information in respect to this issue is likely to arise as part of the larger project.

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