

# Teachers' Characteristics: One Lesson — Two Learning Environments

Carmel Diezmann  
*Queensland University of Technology*  
<c.diezmann@qut.edu.au>

This study explored the thinking exhibited by two teachers as they implemented a mathematical activity designed to introduce young children to investigations. The video data revealed substantial differences in the teachers' distinguishing teaching characteristics. Differences in the quality of their specific teaching characteristics were also identified using a classroom observation instrument. These differences in teaching characteristics suggest that though the teachers ostensibly implemented the same lesson, there was considerable variation in the learning opportunities for each class.

## Background

Mathematical investigations have been advocated for children because they provide opportunities for them to develop thinking skills and content knowledge (Baroody & Coslick, 1998). However because the implementation of inquiry-based approaches results in a radically different curriculum (Taber, 1998), my colleagues and I have explored some of the key issues in implementing mathematical investigations with children of seven to eight years. These issues include: the types of tasks that promote investigation; how children learn from investigations and the difficulties they encounter; and how teachers can support or inhibit students' reasoning in investigations (Diezmann, Watters, & English, 2001a, 2001b, 2002). The finding that teachers can inhibit students' reasoning highlights the need to understand teachers' thinking about the implementation of investigations. Investigations require teachers to reconceptualise the nature of mathematics and to teach mathematics in new and different ways (Taber, 1998). Teachers' thinking about how to create learning opportunities for students can be explored by studying teaching characteristics (Doerr, 2002; Doerr & Lesh, 2002). Thus, the purpose of this paper is to explore the characteristics of teachers as they implement an investigatory activity with young children.

### *The Learning Potential of Investigations*

Mathematical investigations should be challenging and motivating (Greenes, 1996): "Investigations present curiosity provoking situations, problems, and questions that are intriguing and captivate students' interest and attention (p. 37)". Thus, investigations support the development of thinking in two ways. Firstly, they provide students with the opportunity to learn mathematics in context, which has cognitive and motivational advantages (Brown, Collins, & Duguid, 1989). Secondly, investigations acculturate students into mathematical practices, which empower them to discover, invent and use mathematics to understand the world (Lappan & Briars, 1995). Patterns and relationships can be explored through problem solving (Romberg, 1994), representations (Goldin, 1998), physical and thought experiments (Simon, 1996), and reasoning (Russell, 1999). As context and culture are significant factors in the teaching of thinking (Sternberg, 1994), investigations are ideal for nurturing thinking.

Teachers can support students' thinking through their selection and implementation of mathematically challenging tasks. A fundamental goal of an effective task is to "stretch" all children's thinking irrespective of their current capabilities (Diezmann, Thornton, & Watters, 2003). The challenge of a task is not fixed but can be moderated at any point in its "life" from its selection, through its presentation by the teacher, its implementation, and finally, to the products accepted by the teacher (Henningesen & Stein, 1997). Thus, the learning potential from a task is influenced by a teacher's ability to maintain adequate challenge throughout its life.

The implementation of investigations with young children needs to accommodate their limited content knowledge, and generally, their unfamiliarity with investigations. Children's content knowledge is typically supported through the use of stories to contextualise mathematical situations (Whitin, 1994) and manipulatives, which provide concrete referents for abstract mathematical ideas and relationships (Hartshorn & Boren, 1990). Although some guidance is provided in implementing investigations (e.g., Baroody & Coslick, 1998), there are no well-established ways to support children's learning from investigations. One of the difficulties that young children confront when beginning investigations is their lack of understanding of what the problem to be investigated is and how to explore that problem (Diezmann et al., 2001a). Thus, before students can learn from investigations, they need to learn how to investigate — just as students need to learn how to read before they can learn from reading (Brown & Campione, 1991). While children are learning how to investigate mathematical situations, their teachers are learning how to support them to investigate.

### *Studying Classroom Teaching*

The assessment of teaching characteristics within a lesson is problematic. Brown et al. (2001) developed a classroom observation instrument based on previous work by Saxe (1991) and their own work in a large five-year Numeracy study. This instrument focuses on four key teaching characteristics of effective mathematics lessons, namely Tasks, Talk, Tools, and Relationships and Norms. Each of these characteristics is comprised of various components. For example, Tasks comprises (1) mathematical challenge, (2) the integrity and significance of the mathematical tasks, and (3) children's interest in the task (See Table 1). Brown et al.'s instrument describes four levels for evaluating the quality of these components. However, they concluded that this instrument had shortcomings due to its low predictive power for student attainment scores and argued that the instrument failed to account for the human factor — the teacher-class relationship. However, a further factor that seems likely to impact on predictive power is the validity of test scores as a measure of the learning that occurs when teachers emphasise Tasks, Talk, Tools, and Relationships and Norms. This issue is beyond the scope of this paper. However, Tasks, Talk, Tools, and Relationships and Norms are variously argued to be important in creating rich and supportive learning environments for investigations (e.g., Baroody & Coslick, 1998; Greenes, 1996). Thus, evaluation of these specific teaching characteristics should provide some insight into the "learning potential" of an environment, which aims to promote learning through mathematical investigations.

Doerr (2002) has also highlighted the importance of the human factor in the understanding of teaching characteristics. She argued that the identification and understanding of the distinguishing teaching characteristics of an individual implementing a particular lesson with a specific class within a realistic context could provide insight into that teacher's thinking. Thus, Doerr's (2002) approach can accommodate individual

(teacher), contextual (class/school), relational (teacher-class), and situational (lesson) variability. Brown et al.'s (2001) instrument provides the means to examine the situational variability of teaching in greater depth.

Table 1

*Specific Teaching Characteristics and Their Components*

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Tasks

1. Mathematical challenge for all pupils
2. Integrity and significance of the mathematical tasks in the lesson
3. Engagement of interest in the mathematics of the lesson

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Talk

1. Teacher talk that focuses on mathematical meanings and co-construction of knowledge
2. Teacher-pupil talk about mathematics
3. Pupil talk that focuses on reasoning and mathematical understanding
4. Management of talk to encourage pupils to talk about mathematics

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Tools

1. Range of modes of expression including oral, visual, and kinaesthetic
2. Types of models used to represent mathematics ideas

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Relationships and Norms

1. Community of learners comprising teacher and pupils
2. Empathy towards pupils' responses

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(summarised from Brown et al., 2001, p. 14)

The analysis of an individual's teaching characteristics is problematic due to "cultural blindness" to these characteristics. The term "cultural blindness" is used here to describe the situation noted by Hiebert and Stigler (2000) where aspects of teaching are so common that they are "invisible" to members of the culture including teachers themselves. Doerr (2002) accounted for this blindness to some extent through her comparison of the characteristics of more than one teacher. As this blindness results from high familiarity with the tasks of teaching, novel tasks should make the teaching characteristics of an individual more visible both to that individual and to observers. Thus, teaching characteristics in the implementation of an investigatory activity should be most apparent when more than one teacher is implementing a novel activity. Additionally, the use of Brown et al.'s instrument should provide particular insight into the quality of specific teaching characteristics that are important in investigations.

### Design and Methods

This research is part of a case study of teachers' implementation of mathematical investigations with young children (See Diezmann et al., 2001a, 2001b, 2002). The study reported here explores teachers' characteristics in implementing a story-reading activity, which was used as an introduction to a ten-week program of mathematical investigations. This particular activity was chosen for study because (1) teaching characteristics are overt in a story-reading activity, and (2) children's literature is promoted as a means of engaging students in meaningful mathematical thinking (Whitin, 1994). The participants were two

teachers, who taught in the same large outer metropolitan school. Ms I and Ms U each had in excess of ten years teaching experience. They taught comparative mixed-ability classes of 25 and 26 students, who ranged from seven to eight years old. Each week the teachers implemented a 90-minute session of mathematical investigations. The teachers met regularly with a colleague and I to discuss, plan, and debrief the investigations program. Thus, these sessions were consistent in many ways with “lesson study” (Hiebert & Stigler, 2000). The data reported here are observational lesson data, which were collected by three strategically focused video cameras that monitored the teacher, the whole class, and key classroom events. These data were analysed for emergent themes of distinguishing teaching characteristics (Doerr, 2002), and subjected to an evaluation of specific teacher characteristics (Brown et al., 2001). Only limited data are presented here due to space limitations. However, teacher interviews, and teacher and researcher notes supported the interpretation of video data.

Prior to the first lesson, both teachers participated in a planning session and decided to use “The Doorbell Rang” (Hutchins, 1986) to introduce their students to investigations. Both teachers agreed that this story provided an ideal context for an introductory investigation and was mathematically relevant for their classes. This story commences with a small number of children sharing out a batch of 12 cookies. The doorbell then rings announcing the arrival of more children and a subsequent need to re-share the cookies. This story line repeats until there are 12 children present to share 12 cookies. The climax of the story occurs when the doorbell rings again but this time it is Grandma with another batch of 12 cookies. The mathematics in this story includes division, multiples of 12, doubling, and inverse relationships (children and cookies). At the culmination of the planning session, both teachers had decided to implement this activity similarly with children acting out the story using real cookies and stating their intent to capitalise on the mathematical situations during the role-play.

## Results and Discussion

This section presents a brief description of how teachers implemented the activity and the three most distinguishing characteristics of their implementation. Only three characteristics are discussed due to space limitations. The evaluation of the participants’ specific teaching characteristics is then presented.

### *Teachers’ Implementations and Distinguishing Characteristics*

Ms I selected children for the characters in the story and directed them to a small group of tables where the cookies were to be shared. The remainder of the class was seated nearby on the carpet. Ms I provided an overview of the story and directed “mother”, to collect the tray of cookies. She paused each time a new group of children “arrived” in the story and waited until the nominated children acted out their arrival and shared the cookies. The actors had a good view of events, however, the children who were seated on the floor had difficulty seeing. This activity culminated with Ms I asking whether all children in the class had a share of the cookies, instructing some children to share their cookies with those who did not have one, and encouraging the class to eat their cookies. Ms I also advised children that they had participated in a mathematical investigation.

The three most distinguishing characteristics of Ms I’s implementation of the activity follow:

*A. The story enactment had a dramatic rather than mathematical focus.* Ms I's focus from commencement to conclusion was to faithfully act out the story. For example, she provided the child chosen as "mother" with a broom for sweeping. After the story concluded, Ms I checked whether all children had cookies and instructed some children to share their cookies with those who had none.

Ms M: Have we all shared the cookies?"

Class: Yeeeeees.

Ms M: We haven't shared yet! ... Lily doesn't have one and Ed doesn't have one.

*B. The mathematics was assumed to be self-evident.* During the planning session prior to this activity, Ms I endorsed the selection of this book for its wealth of relevant mathematics for her class. However, she failed to capitalise on the substantive mathematics in the storyline. The only mathematics in which students engaged were simple counts of the number of children, and one-to-one correspondence between children and cookies.

*C. Investigations were straightforward and fun.* Ms I assumed the role of the director and narrator in the acting out of this story. She organised the props, selected the actors, and cued them when to enter. Ms I's scripted approach implied that investigations were straightforward rather than ill defined and complex. The eating of cookies at the end of the story created a sense of fun. However, this physical enjoyment was unlike the mental satisfaction that results from the successful completion of a challenging task.

Ms U's implementation of the lesson differed substantially from Ms I's. She organised the class to sit in a large circle, asked for volunteers to act out the story, and directed the actors to a nearby location from which to enter at the appropriate times. She questioned the remaining students about "how many" people were seated at the table in the story and laid a square tablecloth inside the circle of children. She then read parts of the story frequently pausing to ask a range of questions. These questions included general recall — "What did Ma say?" — and prediction — "What do you think will happen next?" Ms U also incorporated many mathematically oriented questions throughout the story reading that ranged from simple questions — "Have they got the same amount?" to more complex questions — "How did that (when there were four children to share the cookies) compare to when Zeb and Cia were by themselves?"

In Ms U's class, children had opportunities to engage in thought experiments and physical experiments by predicting the outcomes of the sharing and by acting out the story. Prior to the conclusion of the story, there was a groan when "the doorbell rang" yet again and the 12 children each only had one cookie remaining. Ms U concluded the activity with a discussion about the mathematical situations throughout the story. She then posed the question of whether what they had done was a mathematical investigation. Children recorded their responses, which were discussed later. At lunch, the children ate the cookies.

The three most distinguishing characteristics of Ms U's activity implementation follow:

*A. The role of the teacher was to stimulate and support children's thinking.* Ms U cued the children to think about the mathematical situations. For example, her mathematical questions built on from each other — "How many do you each have?", "How did they share this time?", and "What did they do because two extra people came?".

*B. Highly structured representations supported mathematical understanding.* Ms U used the square tablecloth on the carpet to position the actors and share the cookies. She sat the first two children opposite each other with their share of six cookies on a plate. When the next two children arrived, they sat on the vacant sides of the tablecloth. Each of the first two children then shared their cookies with a newcomer. This arrangement made the act of giving the newcomers half their cookies very explicit. Ms U later explained that she had organised this visual layout to emphasise the mathematics.

*C. The community orientation was supportive and focused students on a shared goal.* Ms U fostered a supportive community orientation in many ways including ensuring that everyone could easily see the story being acted out. This support seemed related to the common goal of understanding the various mathematical situations throughout the story. For example, towards the end of the story when the actors only had one cookie left, Ms U's response indicates an appreciation of the children's concerns when the doorbell rang.

Ms U: As the doorbell rang (Story text).

Actors: (Muffled raised voices). I don't want to share.

Ms U: Oh, now we've got a bit of concern, what are we concerned about? (The actors started putting their hands over their cookies showing concern and groaning.) I wonder why? (A rhetorical question)

Ms I and Ms U's teaching characteristics varied considerably. This difference, for example, was reflected in how the cookies were used in the story. Ms U capitalised on the cookies as manipulatives to support thinking, whereas Ms I mainly used the cookies as story props. In summary, while Ms U seemed highly focused on mathematics learning, Ms I seemed to lack a similar focus.

### *Specific Teaching Characteristics of Instruction*

Figure 1 presents an analysis of the teachers' implementations of this activity using Brown et al.'s (2001) classroom observation instrument for the evaluation of mathematics lessons. The numbers on Figure 1 correspond to the codes for the components of these teaching characteristics (See Table 1). For example, in the Task section, "1" refers to "mathematical challenge". No hierarchical relationship should be implied from these numbers. Brown et al.'s instrument has four levels, which describe the quality of components of specific teaching characteristics. "Nil" here indicates that there was no evidence of that component or that it was ineffective. "Low", "medium" and "high" indicate increasing levels of quality.

The teachers' implementations of the lesson differed in three ways (See Figure 1). Firstly, each component of the specific teaching characteristics was observed in Ms U's implementation to at least a low level, whereas there was no evidence of components related to effective use of Tools in Ms I's implementation. Secondly, there was a variety in the components of Ms U's teaching characteristics; whereas only one component of each of Ms I's demonstrated teaching characteristics was evident. Finally, the majority of the components in Ms U's implementation were coded at the highest level. In contrast, those components demonstrated by Ms I were coded as ineffective or non-existent. Thus, overall, there were substantial differences. In the quality and extent of demonstrated specific teaching characteristics between Ms I and Ms U.

Levels	Tasks			Talk				Tools		Relationships & Norms	
	1	2	3	1	2	3	4	1	2	1	2
High	U	U	U	U			U		U	U	
Medium					U	U					U
Low			I				I				I
Nil	I	I		I	I	I		I	U	I	I

Note: I = Ms I; U= Ms U

Figure 1. Levels of Effective teaching characteristics.

## Conclusion

This paper reports on the exploration of the mathematical learning potential of a story-reading activity, designed to introduce young children to mathematical investigations. The two analyses of this activity revealed substantial but consistent differences between the teachers' implementation of the activities. Ms U's implementation reflected the principles of effective mathematics instruction (e.g., Brown et al., 2001) and investigatory approaches (e.g., Baroody & Coslick, 1998), and consequently, had high learning potential for children. In contrast, Ms I's implementation was inconsistent with these instructional principles, and hence, had low learning potential. This study also revealed three potentially fruitful avenues for further exploration. Firstly, why would an experienced teacher fail to capitalise on the mathematics in the story despite participating in the joint planning of this activity? Secondly, to what extent might story-reading activities result in limited mathematical learning opportunities? Story reading may cue off-task teacher behaviour through "seductive detail" — the interesting but unimportant information in text (e.g., Alexander, Kulikowich, & Jetton, 1994). Finally, what other typical practices may camouflage a lack of mathematical learning opportunities? For example, manipulatives can sometimes be an impediment to reasoning rather than an aid (Marojam, 1992). Thus, this exploration of teacher characteristics has provided considerable insight into teachers' thinking about how young children learn mathematics and the roles they adopt in supporting children learning about investigations.

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