Alternative Learning Trajectories

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Lesson planning usually involves the generation of a hypothetical learning trajectory. This paper illustrates a teaching strategy that is one focus of a major research project. Alternative learning trajectories with different entry level prompts were used to enable students to access the concepts and procedures necessary for their joining the main learning trajectory. The strategy is being trialled in primary classrooms that have a large proportion of lower SES students, with the aim of maximising success in mathematics for all students.

Defining Learning Trajectories

Wood, Bruner, and Ross (1976) and Bruner (1986) developed the concept of leading children's learning forward through "scaffolding". This involves the teacher providing a pedagogical trajectory to support children's movement into new territories. In articulating Bruner's notion of guided participation, Rogoff (1991) argued that the teacher's main role is to "build bridges from children's current understanding to reach new understanding through processes inherent in communication" (p. 351). Later, Bruner (1996) drew on Vygotsky's notion of the "zone of proximal development" (Vygotsky, 1978, p. 86) when he further defined scaffolding as a logical structuring of ideas to be understood in an order that leads children to develop further and faster than they would on their own.

A variety of images for teachers' roles in scaffolding learning have been presented. In describing quality teaching, Wood (1991, p. 109) used the term "leading by following", noting that the most effective scaffolding draws on the interests and understandings of the child. Cobb and McClain (1999) described an instructional sequence that follows a conjectured learning trajectory that "culminates with the mathematical ideas that constitute our overall instructional intent" (p. 24). Hiebert et al. (1997) used the term "residue" to describe the knowledge that children gain from teaching that may be used as a basis for further planning of sequences of tasks aimed at the development of further particular residues over time. Scardamalia, Bereiter, McLean, Swallow, and Woodruff (1989) portrayed learning trajectories as social phenomena, with teachers employing scaffolding to create more general pathways of potential development of mathematical concepts and procedures. Lerman (1998) discussed the teachers' roles in setting up loci of development—social interactions with mutual appropriation by teachers and students.

Simon (1995) demonstrated how the continually changing knowledge of the teacher creates change in expectations of how students might learn a specific idea.

> A hypothetical learning trajectory provides the teacher with a rationale for choosing a particular instructional design; thus, I (as a teacher) make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions … as well as the spontaneous decisions that I make in response to students' thinking. (pp. 135-136)

Simon used the word "hypothetical" to suggest that all three parts of the trajectory are likely to be somewhat flexible, with teachers changing the learning goals and adapting

aspects of planned activities in response to (a) their perceptions of students' levels of understanding and (b) their on-going evaluations of students' performance of classroom tasks. Thus actual learning trajectories cannot be known in advance. Further, Simon noted that such a trajectory is made up of three components: the learning goal that determines the desired direction of teaching and learning, the activities to be undertaken by the teacher and students, and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136).

In discussion of Simon's paper, Steffe and Ambrosio (1995) described teachers' working hypotheses of what students could learn as being

> determined by the teacher as she interprets the schemes and operations available to the student's actions in solving different tasks in the context of interactive mathematical communication. The anticipation is based on the teacher's knowledge of other students' ways of operating, on the teacher's knowledge of the particular mathematics of that student, and on results of the teacher's interactions with that student. (p. 154)

In this discussion, Steffe and Ambrosio raised an important question—one that is at the heart of this paper: "How does a teacher modify a task that fails to activate certain schemes?" (p. 155). This is a question that we return to below.

Throughout these varied discussions about scaffolding of learning via the creation of specific learning trajectories, the general picture is one of a teacher planning to create a context in which the class will follow one learning trajectory, as shown in Figure 1.

Figure 1. A model of the common perception of a hypothetical learning trajectory.

The concepts of "remedial work" and "extension work", as well as teachers' everyday experience of some students being more successful than others, suggest that actual learning trajectories are likely to take a shape more like Figure 2.
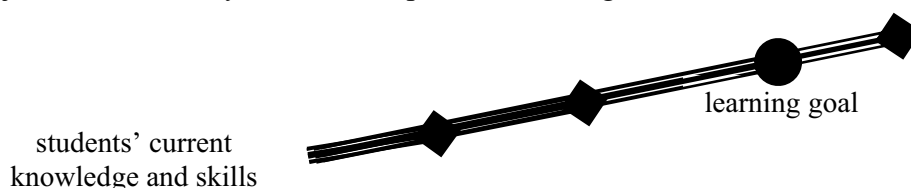

Figure 2. A model of the common perception of actual learning trajectories.

"Ability" grouping, setting or streaming present a further model, with teachers aiming to lead groups or whole classes of students to different learning goals (see Figure 3). However, the literature on the negative effects of such grouping in primary and lower secondary schools is extensive (see for example Boaler, 1997; Gamoran, 1992; Ireson, Hallam, Hack, Clark, & Plewis, 2002; Mousley, 1998; Zevenbergen, 2003).
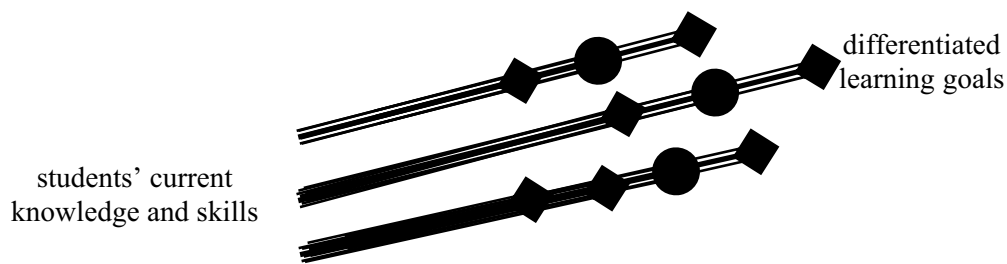
Figure 3. A model of the common perception of differentiated learning trajectories.

Negative effects of differentiated learning expectations can include lowering for some students of teachers' expectations, self- and peer-expectations, self concepts, opportunities for positive modelling and mentoring, and student motivation. In fact, this solution to diversity has the potential to exacerbate disadvantage due to self-fulfilling prophesy effects (Brophy, 1983). In our experience, teachers who use groups in this way are aware of these potential effects but also want to set achievable goals for all students. They realise that most classes have pupils with sufficiently divergent needs that any one task may not be appropriate for all. Clearly there is a need to research forms of pedagogy that may help teachers to adapt classroom tasks to the needs of the range of individual pupils in their classes. Thus it is important to research how teacher may modify tasks that fail to enable some students to meet specific learning goals.

The Research Project

This paper reports on a 3-year project that is exploring ways that can assist students with different starting points to reach the same learning objectives by making the mathematics involved more explicit and hence more accessible. The main focus of the research is on enhancing numeracy learning for students most frequently under performing in school: Indigenous and low socio-economic status (SES) background students.

During earlier stages of the project *Overcoming structural barriers to mathematics learning*[1], we identified and described aspects of classroom teaching that may act as barriers to mathematics learning for some students. Strategies for overcoming such barriers were developed (see Sullivan, Zevenbergen, & Mousley, 2002). We articulated some problematic aspects of the pedagogy associated with context-based, open-ended approaches to teaching mathematics, then focused on making explicit those aspects that may not have been clear to every student. This met with some success, engaging a range of students well. Yet in every class observed as part of this investigation, the teacher expected all students to complete the same task, with the only differences being in their expectations for performance levels. Such an approach failed to recognise the needs of individual learners.

In the current 3-year project, entitled *Maximising Success in Mathematics for Disadvantaged Students*[2], we are exploring "task differentiation" as a means of engaging all students successfully. We propose that teachers invite students experiencing difficulty to solve a similar problem with reduced cognitive demand. This scaffolding step may enable all students to join the hypothetical learning trajectory and hence to complete the original task successfully. For other students who complete the original task quickly, the teacher might pose additional applications of the mathematics. With such an approach, the whole class may complete the same basic task, with possibilities for further application; so all pupils should be able to participate in discussion and class reviews. Most importantly,

all students should be prepared to move together on to the next stage of the learning. We propose that this will allow teachers to maintain a sense of the class as a coherent learning community, even though the tasks posed would have been adapted for individual students.

The alternative pathways are shaped by "prompt" activities that provided stepping stones to the main trajectory leading to the learning goal. The idea of these alternative pathways is illustrated in Figure 4.
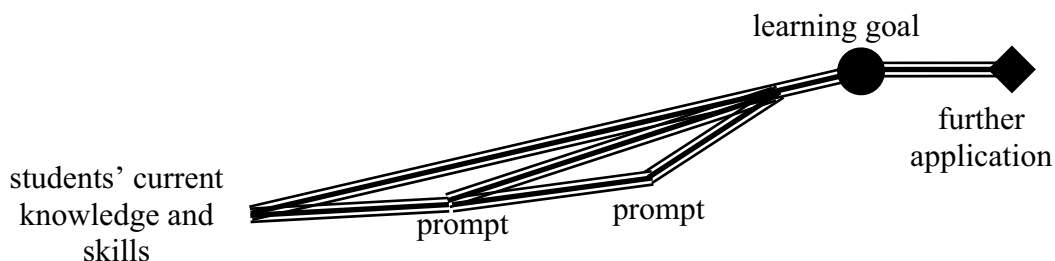

Figure 4. Alternative trajectories to reach the same learning goal.

Our research questions include ways that this form of task differentiation can be implemented in classrooms, whether teachers find it easy to use, what additional demands it may place on teachers, what they must know and be able to do to effect it, and ways in which it changes the learning experience of pupils. In particular, we wish to identify ways that such differentiation in pedagogy may improve outcomes for Indigenous pupils and those from lower SES backgrounds.

Our focus is on upper primary and junior secondary levels because of their potential for the results to be most easily transferred to other levels. We are using teachers in nine schools where there is a mix of pupil backgrounds, with most schools involved having significant numbers of Indigenous students.

Focus groups and classroom-based research are being used, but this paper reports only on the latter. The data collection and analysis is being guided by a framework, developed from Clark and Peterson (1986), that has teacher beliefs and understandings interacting with opportunities, constraints, intentions and actions. The research approach is a combination of (a) interpretive analysis of teaching and teacher development, and (b) broader quantitative data collection. Mixing quantitative and interpretive methods is now common, and has been defended elsewhere (e.g., Knobel, 1999; Patton, 1990).

Currently, teaching experiments using the processes established by Sullivan, Bourke, and Scott (1997) are being carried out, involving modelling and coaching (see Clarke, 1994, for a rationale for this approach). In each classroom, a researcher is working with the teacher, with collaborations following a reiterative approach similar to the action research cycle. Each researcher and teacher meet periodically to discuss the implementation, and a record of these discussions is transcribed. In each classroom, naturalistic observations of the implementation are being recorded, using a format developed as part of the investigation. The data below were drawn from a lesson observation and interviews with one teacher who was experimenting with the use of varied learning trajectories that were thought to be appropriate for particular children in her mixed SES classroom.

The Area-as-counting-squares Lesson

The goal of the model lesson that we call "Areas as counting squares" is for students to use squared paper to gain a sense of area as covering, and in particular to have them count units that can assist in calculating areas of shapes. To introduce the idea of area being

measured as squares, the lesson first has the children draw, on squared paper, letters of the alphabet using exactly 10 whole squares. As a first prompt for students experiencing difficulty, squared paper with one letter already drawn is available. Some square counters that children still experiencing difficulty can use to make a letter are also available. Students who finished the task quickly are given the challenge of making their name using letters of area 10. Some responses are shown to the class.

The students are then asked to draw, again on squared paper, other letters of the alphabet using a total of 10 squares, but using some half squares (triangles) to make the letters easier to read. As a first prompt for students experiencing difficulty squared paper with a sample letter drawn on it is available, and again there are whole- and half-square counters available. Students who finished quickly are asked to choose the hardest letter they can think of and draw it in three different ways, then to make some words with all letters the same height and each letter having an area of 10 squares.

A worksheet is then completed. The first worksheet task involves working out the area of some given solid-coloured rectangles that are drawn onto squared paper. As a first prompt, students experiencing difficulty are given a worksheet with all the lines in the first rectangle (only) shown, so that the unit squares can be counted. The second task is to work out the area of some solid-coloured triangles. The final challenge for the students is to draw many different triangles with an area of 12 square units. The model lesson then asks that some students come to the front of the room to explain their drawings and reasoning.

Virginnia's Lesson

We report here what happened when Virginnia, a Year 5-6 teacher, taught "Area as counting squares". Prior to the lesson, Virginnia had said that most of her students would need all of the prompt sheets. "Most won't be able to do that" (find the area of a rectangle), she said. When the children were asked to draw a letter using ten squares, none asked for help and all were successful. Most created several recognisable letters and some spontaneously wrote their names. They also all coped well with the activity where they could use squares and half squares, and without instruction some went on to decorate their work with shapes, such as yachts, that had a total area of ten squares. Virginnia circulated, challenging but not helping, "Fabulous. Have a go at drawing a z again, but a different way".

When introducing the "area of rectangles" worksheet, Virginnia pointed out that there were some "help sheets" available, and she left them in a pile on the carpet. Four children chose to use the prompt sheet and helped themselves—one before returning to her seat, then three after thinking about the task. The prompts provided enough of a stepping stone for them to complete the original task independently.

The children's methods of finding the areas of the rectangles were varied, but all successful. Many drew on the black rectangles with pencil and then counted the squares. Jack used his calculator to multiply the dimensions.

> *Researcher:* How are you working it out, Jack?
> *Jack:* It's 18 rows with 7 so it's times.
> Frank multiplied in his head (including 18 x 7).
> *Frank:* I know seven nines, then doubled it.

The most challenging part of the worksheet was to work out the area of a right-angled triangle (see Figure 5). Frank was the only child who did this easily.
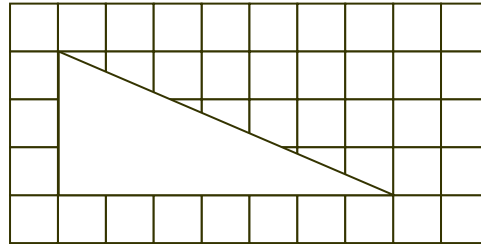

Figure 5. The most difficult triangle task.

Jack had counted the whole squares, and then counted parts of squares as halves.

Virginnia:	Are they exact halves? They seem to be different shapes, not halves. Some are more like quarters. Can you work out exactly what the total is?
Jack:	(pointing to coloured bits in adjacent squares along the hypotenuse) That bit looks like it goes with that one. Them two—that one and that one".
Virginnia:	You're still guessing.
Jack:	Yeah.
Virginnia:	There can't be any guessing involved. (To class) Hands up if you are up to the last triangle. No guessing. You have to try and work out exactly how many squares the area is. I'll give you a clue. Try and use what you have been doing with the rectangles to help you.
Dorothy:	Ohh!

After a few minutes, Virginnia sat down on the floor and children came out to show her their work. Within a few minutes, many children were saying, "It's ten and a half". You make it into a rectangle and then take half". Most had drawn the full rectangle, just as Dorothy had done (see Figure 6).
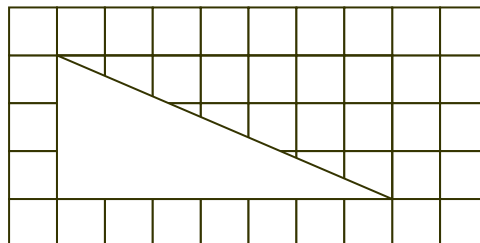

Figure 6. Dorothy's solution.

Researcher:	Did you work that out for yourself?
Cherrie:	No, Dorothy told us.

Most of the children finished the sheet quickly. Virginnia asked each child to explain why they had drawn the rectangle. Most were able to explain it and had gone ahead with drawing other rectangles. Interestingly, once Jack had heard the "half a rectangle" clue he was able to explain which sections of the triangle joined to make one square unit.

Jack:	That point is the same, so that goes there. It's those two, and those two, and those. And that one is the half. So it's (he counted 1–10) ten and a half.
Virginnia	It's like Dorothy's way, isn't it? Half the rectangle.

Jack: Yes. You turn the black triangle around to make the rectangle. The black triangle and the white triangle are the same, and the bits fit together. It's half, so ten and a half."

Virginnia: (to class.) Okay, most of you have worked out the method. Now use that method to draw some triangles and work out their area.

During the post-lesson interview, Virginnia claimed that no students had needed the prompt sheets. As we looked at the children's work she was surprised that four had used them. She felt that the careful structuring of the tasks had enabled "a surprising number of children to do the whole lesson on their own".

Researcher: What did you see as the goal of the lesson?

Virginnia: With the rectangles, to realise that they could multiply the dimensions, or at least count by sevens or fives or the length of the rows. Then to see the triangles as being half the area.

Researcher: Did they all get to those points?

Virginnia: I think so. Most of them drew at least one other triangle, and they talked about it confidently.

Of interest to us was the effect of the short verbal prompt that Virginnia gave: "Try and use what you have been doing with the rectangles to help you". This was followed by Dorothy's very public "Ohh!" The children had been asked to work independently, but several girls had then watched Dorothy draw her solution and word had soon spread. That had the potential to prevent further mathematical thinking, but the open-ended nature of the final creative task and the class discussion where children explained the thinking that underpinned their drawings required personal engagement with the mathematical idea.

Conclusion

In this paper we have aimed to explain one form of task differentiation: the creation of alternative learning trajectories that are aimed at a common learning goal through provision of resources that step children into the main task. The lesson described illustrates how (a) carefully sequenced activities; and (b) prompts that enable students to join the expected learning trajectory, can be used to enable children all children to succeed. This form of task differentiation has proved easy for teachers to use, given the model lessons. It remains to be seen whether they can apply the same principles to their own lessons. Other forms of task differentiation also need to be researched in the available classrooms. In the meantime, though, we have evidence from the lesson reported above, as well as other lessons observed, that this form of task differentiation does change the learning experience of pupils—especially those who are at risk of not being able to follow learning trajectories that are set with the whole class in mind.

Notes

References

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting.* Buckingham: Open University Press.

Brophy, J. (1983). Conceptualising student motivation. *Educational Psychologist, 18,* 200-215.

Bruner, J. S. (1986). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.

Bruner, J. S. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 255–296). New York: Macmillan.

Clarke, D. M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Croxford (Eds.), *Professional development for teachers of mathematics*. *1994 Yearbook of the National Council of Teachers of Mathematics* (pp. 37-48). Reston, VA: NCTM.

Cobb, P., & McClain, K. (1999). Supporting teachers' learning in social and institutional context. In Fou-Lai Lin (Ed.), *Proceedings of the 1999 International Conference on Mathematics Teacher Education* (pp. 18–28). Taipei: National Taiwan Normal University.

Gamoran, A. (1992). Is ability grouping equitable? *Educational Leadership, 50*(2), 11–17.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K, C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Ireson, J., Hallam, S., Hack, S., Clark, H., & Plewis, I. (2002). Ability grouping in English secondary school: Effects on attainment in English, Mathematics, and Science. *Educational Research and Evaluation: An International Journal on Theory and Practice*, *8*(3), 299–318.

Knobel, M. (1999). *Everyday literacies: Students discourse and social practice.* New York: Peter Lang.

Lerman, S. (1998). A moment in the zoom of a lens: Towards a discursive psychology of mathematics teaching and learning. In A. Olivier & K. Newstead (Eds.), *Proceedings of 22nd Conference of the International Group for the Psychology of Education* (Vol. 1, pp. 66–81). Stellenbosch, South Africa: International Group for the Psychology of Mathematics Education.

Mousley, J. A. (1998). Ability grouping: Some implications for building mathematical understanding. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, Gold Coast, pp. 397–403). Sydney: MERGA.

Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage.

Rogoff, B. (1991). Social interaction as apprenticeship in thinking: Guided participation in spatial planning. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives in socially shared cognition* (pp. 349–364). Washington, CD: American Psychological Association.

Scardamalia, K., Bereiter, C., McLean, R. S., Swallow, J., & Woodruff, E. (1989). Computer-supported intentional learning environments. *Journal of Educational Computing Research, 45,* 51–68.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145.

Steffe, L. P., & D'Ambrosio, B. (1995). Toward a working model of constructivist teaching. *Journal for Research in Mathematics Education, 26*(2), 146–159.

Sullivan, P., Bourke, D., & Scott, A. (1997). Learning mathematics through exploration of open-ended tasks: Describing the activity of classroom participants. In E. Pekhonnen (Ed.), *Use of open-ended problems in mathematics classrooms* (pp. 88–106). Helsinki: University of Helsinki.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2002). Contexts in mathematics teaching: Snakes or ladders? In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.). *Mathematics education in the South* Pacific (Proceedings of the 25th Annual Conference of The Mathematics Education Research Group of Australasia, Auckland, pp. 169–177). Sydney: MERGA.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (from the unpublished notebooks of L. S. Vygotsky, edited and translated by M. Cole, V. John-Steiner, S. Scriber, & E. Souberman). Cambridge, MA: Harvard University Press.

Wood, D. (1991). Aspects of teaching and learning. In P. Light, S. Sheldon, & M. Woodhead (Eds.), *Learning to think* (pp. 97–107). London: Routledge.

Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry, 17*, 89–100.

Zevenbergen, R. (2003). Streaming in mathematics classrooms: A Bourdieuian analysis. *For the Learning of Mathematics, 23*(3). 5–10.