CAS: Student Engagement Requires Unambiguous Advantages

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Encouraging students to develop effective use of Computer Algebra Systems (CAS) is not trivial. This paper reports on a group of undergraduate students who, despite carefully planned lectures and CAS availability for all learning and assessment tasks, failed to capitalize on its affordances. If students are to work within the technical constraints, and develop effective use of CAS, teachers need to provide assistance with technical difficulties, actively demonstrate CAS' value and unambiguously reward its strategic use in assessment.

Encouraging students to develop effective use of Computer Algebra Systems (CAS) is not trivial. An initial investment of time and thought is necessary for students and teachers to avail themselves of the facilities offered by CAS. This paper reports on a study where many students failed to capitalize on the affordances of CAS. These undergraduate students were loaned CAS calculators for their personal use throughout a semester course, with institutional value granted to CAS by acceptance of its use for all assessment tasks, including examinations. Despite this encouragement, it is clear that most students made very limited use of CAS. Survey responses suggest that students either did not perceive the affordances of CAS or did not view these as of sufficient value to warrant the initial investment necessary to learn to use CAS effectively. In this paper, first, we outline the study and, with reference to the literature, the rationale for using CAS. Next, we report and discuss some of the outcomes. Finally, we consider important implications of this experience.

The Plan and Theory

This study was undertaken with a group of undergraduate students studying an introductory functions and calculus course. These students had readily available access to CAS for all tasks and assessment. This technology was offered to these students because CAS not only provides the facility of scientific and graphical calculators, but also symbolic manipulation. Since their development in the 1970's and their introduction into some tertiary teaching and learning in the 1980's, the powerful technology of CAS has been recognised as highly valuable for doing mathematics and potentially valuable for teaching and learning mathematics. Studies involving both tertiary and secondary mathematics classes (for example, Heid, 1988; Atkins, Creegan & Soan, 1995; Lagrange, 1999) have supported the contention that the symbolic features of CAS offer the potential to free students from manipulation errors and thus allow them to quickly generate both exact and approximate results. Hopkins and Kinard (1998) reported on a study indicating "improvement in success rates in subsequent mathematics courses" where "students … worked with the new approach to learn *algebra* … to acquire an understanding of the underlying concepts, developing traditional algebraic techniques out of this understanding, and employing the TI-92 calculator to facilitate computations".

Careful consideration of the correct results of a series of examples, produced quickly with CAS, can allow students to recognise patterns and develop algebraic insight (Pierce, 2002: Artigue, 2002). Such studies (McCrae, Asp, & Kendal, 1999) have also demonstrated that the support provided by CAS, described by Kutzler (1994) as scaffolding, allows students to handle more complicated problems than most students can do by-hand at the same age or stage. This scaffolding effect could assist students with weak basic algebraic skills to successfully undertake a calculus unit that relies heavily on the accurate manipulation of algebraic expressions.

Pea (1997) argued that technology would provide the opportunity for distributed intelligence and indeed classroom observation has shown that students not only choose to use CAS to perform difficult or time consuming mathematical operations but commonly anthropomorphise CAS as a helpful friend and independent arbitrator (Pierce & Stacey, 2001). Students participating in the study, referred to in this paper, failed to adopt the use of CAS in this manner. Pea (1997) emphasised that the value of a software tool for mathematics learning does not depend solely on its inherent features but the context in which the activity takes place. This study provides insights that help us to understand factors that affect students' effective use of CAS.

The study was undertaken with a group of 43 first-year, undergraduate students enrolled in a 15-week course (13 weeks teaching, then examination period). Two staff members were involved in teaching the students: Teacher A presented lectures to the whole cohort and acted as tutor to two subgroups while Teacher B tutored a third group. Through the Texas Instruments loan program, students were provided with TI-89 calculators for their use, both in and out of class. These were issued in the first week of the course when Teacher A presented a class that focused very carefully on the basic menus, capabilities of CAS and the essentials of CAS syntax. In most lectures he explicitly taught required CAS syntax and in all lectures modelled its use for solving some problems. The students were told that they were free to use CAS for course material. Teacher A emphasized that CAS could and should be used to assist with all assessment tasks. Use of CAS in the examination was given special emphasis:

> Teacher A: … we did a practice exam in week 13 for revision and I actually pointed out to them where they should use their CAS calculators … just to check answers or to work through step by step.

Data monitoring students effective use of CAS was collected from observation of the 'introduction to CAS class', students worksheets, interviews with volunteer students, an interview with Teacher A, discussions with teacher B, analysis of students' examination scripts and from student surveys. In the next section we summarise and illustrate key aspects of this data, relating to students' use of CAS.

Reality in Practice

While 45 students enrolled in the unit only 31 sat the final examination. Twenty-one students completed a survey in week 8 and 16 completed a post examination survey. Students were asked to put their student identity number on all work and surveys taken for data collection but some students chose not to identify themselves thus precluding tracking and paired comparisons. However, the variety of data sources did provide rich information on students' adoption of CAS as a tool to support their doing and learning of mathematics.

Teachers' Class Observations and Use of CAS

Both teachers had view screens for the TI-89. Teacher B, who was familiar with the use of computer-based CAS software, reported that he had taken time to learn to use the TI-89. However, despite his initial verbal commitment to this study, in later discussion with the researcher and students, was openly negative about the facility offered by this technology. He discouraged its use to the point that two students returned their calculators half way through the course. They later re-borrowed these, from Teacher A, prior to the examination.

Teacher A reported using the view-screen in all lectures but in less than half of his tutorial classes. He was allocated to a tutorial room that was not conducive to the use of an overhead projector. The interview excerpt below describes his use of CAS in class explanations.

> Teacher A: …my standard approach would be to introduce a problem for a particular section of the theory, so for derivatives from first principles, for example, we would simply start with power functions and do derivatives of power functions. We started with the graph, on the calculator, and I'd show students how to get a tangent line and estimate the slope of the tangent line then we'd go through some theory. … my aim was to introduce the theory then use the CAS to give them an opportunity to approach the problems … without them [being] on top of the mechanics of the theory. … I could do the derivatives of some more complicated functions than they would normally have been comfortable with …, with hindsight [I think that], … the students who picked up the theory could use the CAS and they could understand how it linked together with the CAS. Students who did not understand the theory couldn't or wouldn't use the CAS in the same way, and they would tend to follow step by step instructions on how to use CAS. … [For] the first and second examples I did, I gave step by step instruction on how to do it on the CAS but then after that when they were left to their own devices. They [the students who did not understand the theory] got lost very quickly.

Teacher A reported that initially students were keen to use the graphical and arithmetic facilities of CAS. He observed that they quickly adopted the use of CAS for elementary calculus but seldom made use of the algebra facilities. Commonly, when presented with an unfamiliar problem, he observed students working by-hand to manipulate the algebraic expression into a familiar form and then, for example, using CAS to find a derivative. He observed differences in the approach of each of his tutorial groups:

> Teacher A: I had one tutorial group who were very demonstrative in that way who would ask and ask and ask [how to use CAS] and I had one group you wouldn't know if they had calculators or not … the group that used the calculators was mathematically stronger … The weaker students probably didn't use it as much and it's kind of strange because I gave both of those groups the same instruction on the CAS. …So if anything I expected it might work the other way round, that the weaker students might rely on the calculator but in fact I'd say that it was the stronger students who used the calculator more readily… I think they were much more comfortable to find out and use it [CAS] to do problems and to use it to investigate patterns. … I got the sense that the other group felt lost in the maths and the CAS calculator will only help them get further lost.

While, as indicated above, CAS use was expected in the examination it was also made clear to students that, while correct CAS use would be rewarded with some marks, this was not sufficient.

> Teacher A: I was just astonished when I marked the exams and [they had] come up with the wrong answer. … I wouldn't have been very astonished if they had given the right answer but no steps but the fact that they [gave wrong answers or did not attempt questions was unexpected.] … they were told that they would gain some marks for a right answer but at least 50% of the marks for any question would be assigned for the steps in between.

Students' Use of CAS in the Examination

Students were permitted to use CAS as they pleased in the examination. Its likely beneficial use had been demonstrated in a revision class (as discussed above) and the examination paper included a 'hint' in two places: "You should check your results using a CAS calculator". However, analysis of their examination scripts only revealed obvious evidence of use by one third of the students and most of these did not exhibit effective use of CAS as described by Pierce and Stacey (in press).

We saw work such as

Student A

$$f(x) = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{-2}{3}\left(3x^4\right)^{-\frac{5}{3}}$$

and

Student B

$$f(u) = \frac{u}{1-u^2}$$

$$f'(u) = 1 - u^2$$

as strongly indicating that students did not demonstrate the use of CAS. While others, like the student who when finding the derivative of $\ln(\sin(x))$ worked by-hand to the incorrect solution of $(1/x)\cos x.\sin x$ only to follow this with the CAS solution of $1/\tan x$, used CAS in an unthinking manner. Even students, who on questionnaires reported using CAS for all exam questions, obviously did problems by-hand and did not indicate that they had checked their results. This was evidenced, for example, by Student C who wrote:

$$g(s) = \frac{1}{s^2}\,, \qquad G(s) = \frac{1}{\frac{1}{3}s^3} = \frac{3}{s^3}$$

If this student did use CAS they either did not anti-differentiate $1/s^2$ or were unable to make sense of the result and only opted to demonstrate their by-hand working.

Students' Comments from Survey

Two- thirds of the students who responded to the first survey reported that they experienced some 'common problems' and others said that they just did not know how to use the CAS. The most common problems listed were: use of menus, finding commands and use of correct syntax, including correct use of brackets. Three students also made direct reference to the difficulty they experienced in making the transition to CAS from the graphics calculators they had used at school.

Responses to the potential use of CAS were diverse. Some students felt that CAS was useful: 'when I can't work it out in my head', 'when I can't get the answer in the back of the book'. While others were only happy to use CAS provided they 'knew how to do it manually'. One student said that CAS was helpful to explore algebraic concepts 'because the ease of producing a visual example of a function, and thereby gaining insight into concepts, without the tedium of mechanics'.

Eight students, from across the three tutorial groups, gave emphatically negative responses when asked if they had used CAS to explore algebra concepts. This was indicated by exclamation marks or the use of very large writing. Reasons given for this behaviour were that they: did not know how to use CAS; preferred to use a graphics calculator; only did what was necessary to complete this course. The majority of students claimed that they had used CAS for tutorial or assignment problems, and a few students had used CAS to 'play around', or to do the 'algebra required for other units'.

Data from Interviewed Students

In week 12, all students were offered an opportunity to share their opinions and experience of using CAS. Only five students agreed to be interviewed. Examples of work from Students A, B and C's examination scripts were shown above.

Student A, a mature age student, found that the resolution and size of calculator graph screens was a problem and that zooming in this environment was fiddly and annoying. His solution was to use a computer-graphing program for these problems. He had, however, worked with parameters in the algebra module of CAS, changing values systematically to see what he could learn. He saw the value of CAS to 'look for what you want to see or learn rather than focusing on details of routines'. In his experience 'too often with conceptual stuff ─ going through the mechanics ─ you lose sight of the concepts.'

Students B and C were accustomed to using a TI-83. Student B experienced difficulty in locating even comparable utilities on the TI-89. He admitted that he had not made the transition to CAS and felt strongly that students should 'learn algebra' and 'just use the calculator to check'. He had not considered using CAS to assist learning except perhaps to use CAS to 'find an answer and then work backwards'. Student C said that he only used CAS for differentiation.

Student D found that the modelling of CAS use in lectures, along with explanations of syntax, was very helpful. He felt CAS use was fairly self-explanatory and, unlike other students, felt that having previously used a graphics calculator was an advantage. He felt quite competent to use CAS to find answers, especially derivatives, more quickly, but had no idea how CAS might be used to discover a rule.

Student E found difficulty setting graph windows and worked by flicking though the menus or asking advice from another class member. She used CAS to 'change things around … to experiment with graphs for the assignment' and reported that she made 'quite a bit of use of CAS because it was available' - always in her bag.

None of these students' examination scripts showed clear examples of effective use of CAS. It was not surprising that Students B and C, who admitted that they had not engaged in the task of learning to use CAS, displayed difficulty even with functional use. However the examination scripts of Students A, D and E, which include many lines of incorrect by-hand working, also suggest that they did not use CAS, made errors or ignored CAS output. While these students volunteered to be interviewed their survey responses and examination scripts were not atypical. Some possible reasons for these outcomes are explored in the discussion below.

Discussion

The range of responses to the availability of CAS demonstrated in this study indicates a number of important considerations to be addressed if CAS is to become a valuable instrument for doing and learning mathematics. These students' surprising non-use of CAS, despite the teacher's well-considered efforts, provides an impetus for valuable reflection. In the behaviour of these students we see clear examples of the difficulty of focusing students' attention on the affordances of CAS and the complexity of issues affecting their assimilation of this artefact in their work practices. Pea (1997) reminds us that :

> For the many hoped for goals of education, we presuppose the success of the social contractibility of affordances – that one can get a learner to attend to the pertinent properties of the environment … there can be considerable variation in the ease with which one can show a learner how to exploit

> those means to form a system of distributed intelligence for achieving that task. This will vary with the learner's background experiences, the obviousness of the mapping between the learner's desire or goal, and the assimilation of the artefact as means toward it. … Culture and context contribute towards its achievement.

The framework for Effective Use of CAS (Pierce & Stacey, in press) provides a structure for analysing the results of this study. First, consider the technical aspect. Those students who refused, or were discouraged from using CAS, certainly experienced technical difficulties. We see this in comments like: "Not familiar with this calculator, poorly interfaced, not user friendly", "Didn't use it to do graphs as I didn't know how". Most students who did engage with CAS reported some technical difficulties with menus, commands or syntax. The range of errors and lack of correct functional CAS use, evidenced on the examination scripts and comments on the student surveys such as " [a common problem was] to be able to tabulate data then graph it" and "finding functions in the menus, what they mean [and] when to use them". This evidence, along with Teacher A's observation that his mathematically weaker students made little use of CAS, suggests that many students lacked the required semantic knowledge to interpret the menus of available commands.

Second, consider the personal aspect of Effective Use of CAS. Previous experience has demonstrated the importance of positive attitude (Pierce & Stacey, in press). This finding was reinforced in this study. Teacher B's negative attitude certainly contributed to a pervading negative attitude amongst the students despite the efforts of teacher A to present a positive role model. From the students' perspective, the institutional value (Artigue, 2002) placed on CAS was ambiguous. It was expected that the modelling of CAS use in lectures, combined with its accepted value for assessment tasks, would have provided sufficient incentive for students to work to overcome the initial technical hurdles. In spite of this, it seems the parallel emphasis on by-hand skills, which matched their valued school experience discouraged students from persevering to achieve effective use of CAS. The reflection of Teacher A recorded below indicates that, in the future, he would change the emphasis in his teaching with CAS.

> Teacher A: I think I'd probably concentrate … less on the syntax, … and focus more on the integration between things like the algebra, the derivatives and … try to get across that in each step you do, it [CAS] can make it easier. Next time I'd use it more in the investigative patterns … actually use CAS to show the development of the concept - specific at the start and working to a general solution at the end. … CAS is really useful for this but [we] need good examples.

Clearly, teacher privileging, as described by Kendal and Stacey (2001), played an important role in determining student attitudes and therefore behaviour. As evidenced above, some students made minimal effort to become even functional CAS users. While Teacher A saw students using graphs to explore concepts in class, he reported that few students made use of the algebra facility in that way. This concurs with the post examination survey on which fewer than half of the students reported using CAS to experiment or try out ideas for solutions. Few students made strategic use of CAS.

The students' previous school experience of mathematics valued by-hand techniques. Those students, who had recently studied year 12 mathematics, also placed great value on graphics calculator techniques and were reluctant to embrace the new technology. Artigue (2002) describes a three-stage process for the evolution of instrumental genesis.

> During the first phase … the graphical application plays a predominant role …; the symbolic application HOME plays a marginal role (essentially the computation of the derivative). In a second phase, … HOME becomes more involved in the computation of exact values for the function and its

> derivative, for calculating limits, or checking some graphical results, with a role of support to the graphical work. In the third phase … the symbolic application becomes the predominant tool in the solving process, jointly with paper & pencil work

Despite 13 weeks of teaching, of the students, in this study, who engaged with the use of CAS most were operating at the first stage, using CAS only for graphing and computing some derivatives. No student had progressed to the third stage where CAS becomes the dominant tool in the solving process along with by-hand work. These students' lack of engagement with the CAS-calculator is interesting because previous cohorts studying a similar course with DERIVE did not demonstrate the same level of reluctance (Pierce & Stacey, 2001).

Just making CAS available is not sufficient for students to adopt its use. Clearly Teacher B's negativity had a strong impact. Even Teacher A inadvertently gave his students ambiguous messages, advocating its use in lectures but seldom demonstrating its use in tutorials. In addition, although he had expected students to use CAS in the examination, his emphasis on marks for 'showing steps' was interpreted by the students as meaning 'show by-hand calculations'. That 'steps' could have meant an explanation of their plan or reasons for using CAS was not apparent to the students.

In conclusion, we summarise some of the key lessons that can be learnt from the experience of those involved with this study.

Implications and Conclusions

Learning to use CAS as a functional and pedagogical aid is a significant undertaking. The results of this study reinforce previous findings (see for example Kendal & Stacey, 2001; Pierce & Stacey, in press) that highlight the dominance of the affective aspect of CAS use over the technical aspect. The experience of the teachers reported in this paper emphasises the impact of their privileging. The high value of by-hand algebraic manipulation demonstrated by the teachers meant that the students saw CAS as an optional extra. In particular, the students' examination scripts indicate that they had not become effective users of CAS. It appears most students tried to emulate the by-hand working they had seen demonstrated in class. In addition, some students did not see that CAS offered sufficient advantages over a graphic calculator, valued in their final secondary school mathematics, to warrant the time and cognitive effort required to become effective users of this new technology.

Careful consideration of a number of issues is necessary before introducing CAS to a classroom. These issues include the purpose of its use, how it will be valued and whether the outcomes will be worth the investment required to establish technical competence. Of paramount importance, if students are to realise the potential of CAS for enhancing their understanding of concepts and encouraging higher order thinking, is for teachers to consider the messages which will be conveyed by both their words and actions. As Teacher A commented from his experience, attention to the nature of problems set and the value placed on CAS for their solution are critical factors in teaching students how to learn with CAS. This takes time, modelling of strategic CAS use and carefully directed guidance on technical aspects and exploration, with problems that explicitly encourage the use of CAS. Insufficient institutional value can result in limited CAS use, such as only employing it as an answer machine just like the 'back of the book' as these students did. If students are to benefit from the availability of CAS the technical aspects of its use must be given sufficient attention both at the beginning and when new mathematics is introduced. Effective, strategic use of CAS should be consistently modelled and explicitly valued.

This study highlights the challenge for both teachers and students adopting the use of new technology such as CAS. First there is an initial overhead in learning to work with the specific CAS interface. Next if students are to access the power of CAS to assist in both doing and learning mathematics teachers need to give attention to technical details of the mathematics and the CAS, to help students build the knowledge required to use CAS independently. Finally, throughout the whole process unambiguous value should be given to the alternative solution methods afforded by CAS. In particular this means rethinking what is meant by 'showing steps' or showing working' and the consequences for marking. New, acceptable standards need to be established and clearly communicated to students if they are to be expected to work towards developing effective use of CAS.

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