Structured Peer Interactions to Enhance Learning in Mathematics

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This paper reports on an action research study in which a composite class of Years 6-8 students were involved in a collaborative learning training programme. The intervention was designed to promote the students' engagement in verbal interactions. Categories of talk were analysed before and after the training sessions to determine both the amount and the cognitive level of talk. Students showed a gain in the mean amount of cognitive talk and higher cognitive talk after the intervention.

There have been questions raised about the prevalence and effectiveness of teaching and learning in the New Zealand classroom by using small groups in a collaborative learning situation (Education Review Office (ERO), 2000). This concern has also been raised in response to New Zealand's results in the *Third International Mathematics and Science Study - Repeat* (1998) that showed no significant improvements in mathematics achievement by New Zealand students during a period when a new mathematics curriculum advocating group work had been introduced as mandatory in 1993. The results also showed that countries which seldom used small groups achieved more highly than New Zealand students. The ERO report recommended a closer examination of small group teaching practices in New Zealand and further research on group teaching in terms of its contribution to developing mathematical understanding.

The current curriculum statement is underpinned by a social constructivist philosophy of learning in which communication plays a central role in a student's development of mathematical understanding. This philosophy of learning, which promotes discourse, reflects both Piaget's (1967) cognitive development theory and Vygotsky's (1978) social learning theory. The expectation within this teaching and learning context is that individuals should develop better mathematical thinking by discussing mathematical ideas with peers, giving explanations, responding to questions and challenges, listening to peers, making sense of others' explanations, and asking for clarification of ideas. The use of such conceptually orientated explanations, involving alternative solution strategies, assists in building robust knowledge structures, thus strengthening students' mathematical achievements (Fuchs, Fuchs, Karns, Hamlett, Dutka, & Katzaroff, 1996; King, Staffieri, & Adelgais, 1998; Stein, Grover, & Henningsen, 1996). In the construction of knowledge, cognitive conflict and resolution are seen as the mechanism for transforming thought (Piaget, 1967) and those children who participate in the activities and social dialogues of collective discourse are seen to develop higher mental functions more effectively (Vygotsky, 1978).

Structures to Promote Discourse and Learning

Current curriculum documents in many countries (for example, Australian Education Council, 1991; Ministry of Education, 1992; National Council of Teachers of Mathematics, 2000) emphasise and promote the communication of mathematical ideas. It is advocated that students should be provided with situations that allow them to construct and modify their mathematical knowledge through discourse (Yackel, Cobb, & Wood, 1991). Opportunities for students to communicate about mathematics arise when students work collaboratively on a problem (Artzt, 1996). Collaborative learning procedures are those that enable students to engage actively in the learning process through interaction and discussion with peers in small groups on inquiry tasks. It is a reciprocal process of mutuality where each other's reasoning and viewpoints are explored in order to construct a shared understanding of the task (Goos, 2000). Effective collaborative learning is not automatic. The situation requires structure with student-to-student interaction in small groups, individual accountability and responsibility, organised co-operation, and a common learning task or goal for the group (Davidson & Worsham, 1992). These skills of positive interdependence allow the synthesis of independent and collaborative contributions thus making learning more successful than competitive or individualistic models (Brown & Thomson, 2000; Qin, Johnson, & Johnson, 1995). The structuring of collaborative learning increases the level of on-task discussion and provides a mechanism so that students can negotiate meanings from other students' task-related conclusions (King, Staffieri, & Adelgais, 1998).

Johnson and Johnson (1987) have shown that as groups practise collaborative learning skills they develop through four stages: forming, functioning, formulating, and fermenting. The 'forming' skills are basic skills required for groups to function and include moving and talking quietly, using eye contact and group members' names, and encouraging all group members to participate. 'Functioning' skills are those skills which allow greater self-management within the group. Individual members maintain their given roles, all group members are included and encouraged, and the interactions are both courteous and positive. Students use 'formulating' skills to apply and analyse ideas and to ask for and listen to elaborations, justifications, and summaries from other group members. 'Fermenting' skills enable students to integrate ideas to form a concept or general principle. Students with these skills are able to question, critique and evaluate peers' ideas, and develop and integrate the ideas of others into a new concept or application. At this level students are also able to handle controversy in a positive and constructive manner.

The complex interplay of content, context and relationships in the nature of discourse makes it difficult to select an adequate tool to measure and analyse classroom interactions. So for this study, a theoretical basis is provided by Thomas' (1994) model on the nature of talk. Thomas' model, based on Bennett and Dunne's work (1992), was developed and used in New Zealand mathematics classrooms specifically to investigate the interactions of children as they worked in small groups independently of the teacher. Within this model the initial classification of talk was between task-related and non-task-related talk. Thomas then distinguished between cognitive talk that was directly relevant to the task, and talk which was socially orientated around management of materials or the group. She created a further subcategory that separated cognitively orientated talk into 'action talk', when the talk was focussed on the activity of the moment, and 'reflection talk' which was primarily associated with the discussion of ideas, explanations, and the clarification of understandings. Thomas found that the social and cognitive nature of the task itself impacted significantly on the kind of talk which occurred and that problem-solving tasks were more likely to engage the children in explanations and abstract discussion than games or production tasks (making a model or collecting data). Working in pairs also reduced the task-related social talk associated with organization.

This project also draws on Bloom's Taxonomy of Cognitive Processes (1956). It was found to be a useful structure for differentiating the complexity of cognitive talk. In this study lower cognitive talk included recall, repetition or rehearsal of previous information. Higher order cognitive talk included application and analysis at the fermenting level of collaborative skills, and synthesis and evaluation at the formulating level of collaborative skills.

Research Design and Collaborative Skills Intervention

The research design was based on an action research model in a classroom setting that as Elliott (1991) states is "The study of a social situation with a view to improving the quality of action within it" (p.61). The cyclic approach (Kemmis & McTaggart, 1981) involved planning, acting, observing, and evaluating with adaptations being made after group discourse and reflection. This paper reports on aspects of the third cycle of a peer tutoring and collaborative learning action research project.

A composite class of 25 Year 6, 7 and 8 children from a rural full primary school participated in a collaborative learning skills intervention programme aimed at increasing the frequency and quality of student verbal interactions. Two teachers with joint responsibility for the class mathematics programme participated in the study. A third teacher combined the dual roles of trainer in collaborative skills and researcher. Data were collected and analysed for a group of 15 students; four girls and two boys from Year 6, and three girls and six boys from Year 8. These students were present for the entire training programme. The mathematics teaching during the period of the study involved addition and subtraction of whole numbers, and fractional knowledge and operations. The draft Diagnostic Interview for the Numeracy Project (Ministry of Education, 2002) was used to assess academic gains and to identify learning needs within the class.

The study involved an intervention designed to develop verbal interactions that should lead to higher order cognitive talk during group work in mathematics. Students practised four strategies that assisted in developing group skills of forming, functioning, formulating, and fermenting (Johnson & Johnson, 1987). These were:

Wait and give individuals time to think for themselves;

Be specific with feedback and encouragement;

Give help when asked in the form of a specific strategy, idea or question rather than an answer; and

Support agreement or disagreement with evidence.

In order to reduce the task-related social talk during the intervention students initially worked in pairs. One member of the pair adopted the role of coach and was responsible for the use of instructional strategies to promote mathematical learning. The role of coach was alternated so that individuals could develop a greater awareness of the effects of their feedback. As the level of collaborative skills developed within the class the students began to work in groups of three or four on problem-solving tasks. The students worked on the problem individually for a specified time and then shared solutions and evidence to support conclusions with their partners using Lyman's (1992) 'Think-Pair-Share' strategy. When the partners reached an agreed solution they presented their solutions and evidence to another pair. The students were encouraged to compare their solutions and reasoning with other presentations. In this way multiple solution paths were viewed and compared, and flawed solutions were challenged and discarded. Any answer could be challenged as long as the challenge was supported by reasoned argument. Groups were encouraged to reflect and comment on their own learning and the effectiveness of the discussion within the group.

The classroom teachers supported the development of the collaborative group practices by modelling these interactions with the students and by asking the students to reflect on their group interactions. The teachers discussed with each other the principles of quality task selection (Stein, Grover, & Henningsen, 1996) and selected 'rich' problem solving tasks which maintained the students in their *Zone of Proximal Development* (Vygotsky, 1978). This was done prior to the recording of baseline data to provide an optimal learning environment for the students' pre and post test measurements. The effectiveness of the programme was assessed in relation to both teachers' and students' perceptions about the programme and achievement gains in the stages of the Numeracy Development Project (Ministry of Education, 2002) and also gains in the amount and quality of talk. It is this latter measure that is reported in this paper.

In order to increase the validity of the research and to provide a deeper understanding of the collaborative interactions data were triangulated through the use of multiple research instruments: audio recordings; questionnaires; anecdotal observations; and diagnostic interviews. The collaborative groups were taped during engagement on a problem-solving task one week before and one week after participation in the four week collaborative training programme. The tapes were transcribed and analysed into categories of talk based on Thomas' model (1994) except task-related talk which was subcategorised as lower or higher cognitive talk. As with Thomas' study, the data analysis within these subcategories proved difficult. Within one student's 'turn' there was sometimes a range of talk incorporating features of 'higher' and 'lower' cognitive talk. In order to remain consistent within the analysis, cognitive talk which contained features of both 'higher' and 'lower' talk was deemed as 'higher' talk, and the 'lower' cognitive talk was seen in a supporting role.

Results and Discussion

This paper reports the results from one cycle of the classroom-based action research study. This first set of results is based on an analysis of the type and amount of talk from all students in the sample according to six classifications. These collated results, reported using the mean of all participants, are shown below in Figure 1. Although there was a gain in the mean percentage of task-related talk and level of cognitive talk before and after the intervention this was not significant.

All Talk 100%
⇓ ⇓
Task-related Talk 92% (95%) | Non Task-related Talk 8% (5%)
⇓ ⇓
Task-related Cognitive Talk 82% (90%) | Task-related Social Talk 10% (5%)
⇓ ⇓
Lower Cognitive Talk 61% (54%) | Higher Cognitive Talk 21% (35%)

*Before(After)

Figure 1. Summary of mean percentage of talk before and after the intervention.

However, overall there was a high level of task-related talk. This result is consistent with several other studies (Bennett & Dunne, 1992; Higgins, 1994, Johnson, Johnson, Roy, & Zaidman, 1985; Thomas, 1994; Young-Loveridge, 1989). In a comparison of subgroups, Year 6 students who had been involved in the pilot study made more gains in the amount of task-related talk than the Year 8 students who had not been involved in the pilot. The girls made more gains in task-related talk compared to the boys. It is noteworthy that the groups that made the greatest gains also had the lowest initial scores.

An analysis was also conducted for the distribution of talk between members in a group. The groups with two members shared the percentage of talk equally with little change after the intervention. In one group of three there was a notable difference in the results when after training a dominant group member decreased her contribution to a more equitable level. In the other groups members continued to contribute almost equal percentages of talk. A paired t-test was used to analyse the proportion of talk contributed to by individuals before and after the intervention. However, overall the intervention programme appears to have made little difference to the proportion of talk contributed by group members.

The data obtained from the analysis of each student's contributing talk compared to a mean expectation for each group member suggests that some students had not mastered the forming and functioning skills of positive interdependence. This was based on the expectation that every member should contribute equally to group discussion. Group size also appeared to influence whether a student contributed an equal share to the discussion. Students were more likely to contribute equally when there were only two members.

The levels of talk for each student were also compared before and after the collaborative skills training using a paired t-test. There was a significant gain in the amount of higher cognitive talk after the intervention ($t = 2.40$, with 14 d.f., $p < 0.05$). The levels of higher cognitive talk were further analysed according to gender and class level. These results are summarised in Table 1.

Table 1
Mean Percentage of Higher Order Cognitive Talk Before and After the Intervention

Grouping	Higher Order Cognitive Talk (Before Intervention)	Higher Order Cognitive Talk (After Intervention)
Year 6 (Pilot Study)	29.5%	51.8%
Year 8	19.8%	33.4%
Boys	22.3%	36.4%
Girls	22.3%	32.9%

The pilot study group had a more sustained training period so it would be expected that their results would show a greater improvement. For three students whom the teachers describe as "hard working with excellent study habits" their overall percentage of task-related talk and overall cognitive talk decreased but their proportion of higher order cognitive talk increased after the intervention. A possible explanation is the training assisted them in becoming more in 'tune' with strategies that enabled them to complete problems more efficiently and successfully with less talk.

The teachers felt that the students had been actively engaged in developing and practising the collaborative learning skills that should lead to higher cognitive learning. One of the teachers felt that the structure had provided a vehicle for the quieter and less confident members of the class to make a contribution and to be heard. The other teacher thought that the collaborative skills programme had given confidence to students who had strong interpersonal skills but were weak in mathematics; such students they felt were taking more risks and experiencing greater academic success as a result. However, the students had different perceptions about the programme. They thought the training sessions were not for learning and practising collaborative learning skills but about having extra time to rehearse problem solving and to gain mathematical knowledge.

This raises the issue of transparency; the researcher believed that she had implemented the skills training programme in such a way as to make the collaborative framework transparent. However, in the reflection process it appears from the student feedback that the focus remained on the mathematical task and the solution strategies and not on the group process of collaboration and the quality of group talk. The reported study is limited by several factors. These include the limited timeframe and the small sample that is not followed during a lengthy sustained period. It is also difficult to make explicit the links between cognitive talk and mathematical achievement gains.

Conclusions and Implications

Despite the limitations mentioned this small classroom-based study gives encouragement to the notion that improved levels of achievement can occur when students are taught specific skills for enhancing the quality of interactions in group discussions. Gains in learning occurred during the period of the study but it can only be inferred that one of the contributing factors was the improvement in the levels of cognitive (and higher order cognitive) talk engaged in by the collaborative groups after the group training intervention. There were significant gains in the mean percentage of higher order cognitive talk and gains in the amount of task-related talk and overall cognitive talk for most students.

The action research nature of the intervention encouraged the researcher to reflect on her teaching practices within the classroom and to focus in particular on making interpersonal skills explicit through teaching and modelling in order to develop successful collaborative group work. This study was an attempt in a small way to make change, to increase the quantity and more importantly the quality of collaborative learning practices within small group situations. If students are going to be working collaboratively on group tasks that require interpersonal communication then the teacher plays an important role. This includes selecting the task and then being alert to the discussion and the patterns of behaviour and discourse.

With the increasing recognition given to classroom discourse and communities of inquiry within the mathematical classroom, teachers should explicitly teach collaborative group practices that foster the development of mathematical learning among all students. Observations of students' verbal-oriented behaviours (explaining, questioning, demonstrating, justifying) and awareness of levels of cognitive talk can provide a revealing view of how effectively students learn in collaborative situations. It is a real challenge to explore students' cognitive development in a group-based context. Given that not only students but also teachers may have had little training in collaborative learning techniques it also signals a need to develop this aspect of teachers' professional practice.

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