Talking and Writing About the Problem Solving Process

Kim Beswick
University of Tasmania
<Kim.Beswick@utas.edu.au>

Tracey Muir
University of Tasmania
<Tracey.Muir@utas.edu.au>

This paper reports one aspect of a larger study which looked at strategies used by Grade six students to solve six non-routine mathematical problems. One focus of the study was the relative effectiveness of students' written and verbal communication in revealing their thinking during the problem solving process. The results suggest that students may benefit from instruction on communicating their thinking in writing and that emphasising writing as a guide to students' thinking may disadvantage lower ability students.

Recent curriculum reforms in some Australian states (e.g. Department of Education Tasmanian, 2002) have highlighted the importance of thinking, communicating and instilling deep understanding in our students and have seen a re-emphasis on mathematical problem solving as an important mechanism for enhancing these skills. Problem solving is, of course, not a new idea in mathematics education. Over half of a century ago, the importance of problem solving was recognised (Brownell, 1942, cited in Suydam, 1980) and its importance was emphasised strongly throughout the 1980's (Suydam, 1980). Polya (1949) and others (e.g. Branca, 1980; Schoenfeld, 2002) maintain that problem solving is the goal of mathematics learning while the National Council of Teachers of Mathematics (2000) go further saying that problem solving "is not only a goal of learning mathematics but also a major means of doing so" (NCTM, 2000, p.4).

Similarly, communication is and always has been an important part of mathematical problem solving (Willoughby, 1990) and a key to gaining an insight into students' understanding. This study examined both students' written and verbal communications, in response to a number of mathematical problems, in terms of what they revealed about students' problem solving processes.

Written and Verbal Communication in Problem Solving

Flewelling and Higgenson (2003) explain how communication relates to Polya's (1957) four-step problem-solving model. Students can talk about the problem to understand it better and to clarify the method. They can listen to the other people's ideas, draw pictures, use manipulatives, write words and symbols to represent the steps involved in solving the problem, share results of computer or calculator operations, and decide upon the best way to describe and explain how the solution was reached (Flewelling & Higgenson, 2003). Furthermore, Mousley (1999) found that many of the indicators that teachers use to gauge the extent of children's understanding relied on students' ability to communicate. They included such things as an ability to articulate understanding, to respond to probing questions, to explain concepts in their own language, and the ability to teach the relevant concept to others (Mousley, 1999).

The benefits of writing about the executive processes of problem solving are well documented (Williams, 2003; Pugalee, 2001; Azzolino, 1990; Clarke, Clarke & Lovitt, 1990). However, some students express a dislike for writing explanations and justifications in mathematics and according to Bicknell (1999), many students find writing explanations a difficult and tedious exercise. In contrast with this even quite young children are able verbally to articulate their thinking (Franke & Carey, 1997). Kaur and Blane (1994) found that it was often difficult to diagnose the difficulties experienced by students in solving problems by examining their written solutions alone. The limitations of relying on written recordings of solutions to gauge students' understanding were clearly illustrated by Clements and Ellerton (1985). Their study aimed to determine whether or not students who gave correct answers to pen and paper tests had a comprehensive understanding of the mathematical concepts and relationships which the tests were designed to measure. They found that about one quarter of children's responses could be classified as (a) correct answers given by students who did not have a sound understanding of the mathematical knowledge, skills, concepts and principles behind the question, or (b) incorrect answers given by students who had partial or full understanding.

According to Huniker and Laughlin (1996), students become aware of what they really know and what more they need to know as they talk about their experiences and test their new ideas with words. With regard to interviews aimed at revealing students' thinking, Stoyanova (2000) emphasized the importance of framing 'good' questions in order for teachers to understand their students' problem solving processes, and indicated that a dialogue needed to occur between teacher and student in order to elicit student responses. Ginsburg (1987) also advocated the interview as an effective means to enable students to communicate their mathematical thinking and Norbury (2003) recommended its use particularly with younger children.

Despite the fact that problem solving is a complex process and that children's problem solving or thinking ability cannot simply be assessed through pen and paper means, the difficulty of conducting one-to-one interviews in classroom contexts means that this remains the norm.

The Study

The study examined the use of problem solving strategies by Grade six students and their ability to communicate their thinking both verbally and in writing. The specific research question addressed in this paper is: Does reliance on written recordings of the problem solving process differentially disadvantage students according to their mathematical ability?

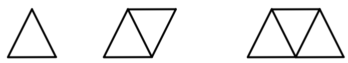
Subjects

Four grade six students were selected from each of five primary schools, making a total of twenty students (12 girls and eight boys). Grade six students were targeted because they could be expected to have a sufficient level of literacy in order to be able to record their solutions in writing. Teachers in each school were asked to nominate the four students who would participate in the study. Each teacher nominated with respect to mathematical ability, one 'above average', two 'average' and one 'below average' student, maintaining a gender balance if possible. This nomination was based on the professional judgment of the individual teachers, but in most cases was also supported by the students' Year Five Numeracy Testing results which rated their overall numeracy in relation to other students in the state. The total sample was thus made up of five 'above average', ten 'average' and five 'below average' students with respect to mathematical ability.

The Problems

Six mathematical problems were chosen based on the researcher's classroom teaching experience with regard to their appropriateness in terms of the degree of challenge offered to the selected age group, and their potential to be answered using a variety of strategies both across the problems and for any given problem. The problems are listed below.

1. Jenny is making towers of cubes using red, blue, yellow and green. How many different towers can she make by changing the order of the colours?
2. Jim got into a lift. He went down five floors, up six floors and down seven floors. He was then on the second floor. At what floor did he get on?
3. Susan worked at an apple orchard. When she was sorting the apples for sale, she noticed that two out of every seven apples had worm holes. If there were 70 apples in the basket, how many could be expected to be ‘good apples?
4. Some children were playing with some rabbits in a yard. I tried to count them and found that there were 30 legs and 11 heads. How many children and how many rabbits were in the yard?
5. At a meeting of the Good Friend’s Society, everyone begins by shaking hands with each other once. If there were ten people at the meeting, how many handshakes were there?
6. I made some triangles using matchsticks. I used three matches to make one triangle, five to make two triangles and seven to make three triangles. If I continued in this way, how many matches would I need to make 12 triangles?



Procedure

Students were interviewed using a semi-structured approach (Burns, 2000). A copy of each problem was presented and read aloud to the student by the interviewer. The student was then asked to solve the problem and to make a written recording of any working that they used in this process. Potentially helpful concrete materials such as cubes, counters and matchsticks were available to be used as the students chose and in relation to any of the problems. Following the completion of each problem, each student was asked to verbally explain what they had done, referring to their written solution as they wished, while the interviewer tape recorded their responses, made observational notes and asked prompting questions when and if clarification was required.

The rating scales shown in Table 1 were used to separately classify each of the students’ written and verbal responses in terms of the clarity with which they communicated the student’s thinking, and the correctness of the solutions obtained. These scales were also used by Adibnia and Putt (1998) in their study of problem solving by year six students. Judgments of the clarity of the communication were made independently of an assessment of the correctness of their solutions. That is, very clear communications of incorrect solutions, as exemplified by Belinda’s verbal response to Problem Two, were rated ‘2’. Belinda said:

> I added up 5, 6 and 7 and got 18, except that I thought that can’t be right because I don’t think there’s an 18th floor … then I wrote down a different sort of pattern and I wrote the first D (for down) and I wrote 5 and then 6, then 7, then … I did 57 how many 6’s … and I just wrote all the lines to 57 and counted 6 and put a line around them and then I got 9 remainder 3 so I just added 9 and I put 3 with it and got 12. So he got on at the 12th floor.

Table 1
Rating Scales for Assessing both Written and Verbal Communication, and Correctness of Solutions

Clarity of written and verbal communication of solution and process	Correctness of solution
0 – Very unclear, very incomplete 1 – Reasonably or partly clear with omissions or weaknesses 2 – Clear and appropriate communication of findings, reasoning, method	0 – No answer or incorrect answer based on an inappropriate strategy 1 – Incorrect due to transcription or computational error or partial answer based on an appropriate strategy 2 – Correct answer based on an appropriate strategy

Results and Discussion

The respective numbers of written and verbal responses receiving each rating are shown in Tables 2 and 3. The ratings obtained for the correctness of answers are shown in Table 4. (It should be noted in relation to Table 4 that a rating for correct solutions obtained by the use of an inappropriate strategy or by guessing is not included as no responses fitting this category were obtained.)

Table 2
Ratings of Written Responses by Ability Level

Rating	Above average		Average		Below average	
	No.	%	No.	%	No.	%
0	2	7	20	33	20	66
1	10	33	23	38	4	13
2	18	60	17	28	6	20
Total numbers	30		60		30	

Ninety three percent of written recordings by above average students conveyed at least something of the students' thinking. This was the case for only two thirds of responses from average ability students and just one third of those from below average students.

Table 3
Ratings of Verbal Responses by Ability Level

Rating	Above average		Average		Below average	
	No.	%	No.	%	No.	%
0	1	3	1	2	1	3
1	2	7	16	27	18	60
2	27	90	43	72	11	37
Total numbers	30		60		30	

More than ninety percent of the verbal explanations provided by students at each of the ability levels were at least partially clear in their communication of the students' thinking.

Table 4
Ratings of Solutions in Terms of Correctness

Rating	Above average		Average		Below average	
	No.	%	No.	%	No.	%
0	1	3	3	5	11	37
1	10	33	32	53	16	53
2	19	63	25	42	3	10
Total numbers	30		60		30	

Only five percent or less of the solutions produced by average and above average students rated '0', while more than one third of the solutions produced by below average students received this rating. From Tables 2, 3 and 4 it is clear that more able students are more likely than other students to produce clear and appropriate written records of their problem solving activity, more likely to be able to clearly verbalise their thinking, and more likely to obtain correct answers.

The numbers of '0' ratings for written recordings obtained at each ability level is related to the numbers of students in each ability category who obtained '0' for the correctness of their solutions. Although the correctness or otherwise of the solution was not taken into account in rating the written or verbal responses it is clear that the students' ability to engage with and attempt a problem would influence what they were able to convey either in writing or verbally.

Verbal communications rated more highly than written responses for all ability levels. This is consistent with the other findings (e.g. Kaur and Blane, 1994) and also unsurprising given that the students communicated verbally, with their written recordings available to remind them of the processes they undertook and to assist with structuring their responses. Nevertheless it is noteworthy that just one student at each ability level produced a verbal response that rated '0'. That is, in all other cases (97.5%), regardless of the ability of the student, the verbal communication provided an at least partially clear explanation of the student's thinking. This is important given that written responses alone would have provided potentially useful insights into the students' thinking (i.e. ratings of '1' or '2') in just 65% of cases with the overwhelming majority of this difference accounted for by average, and particularly below average, students. The following examples are illustrative of the differences between the written and verbal communications of students considered to be of below average mathematics ability.

In response to Problem Six, the triangle problem, Jack wrote only the incorrect answer, 23, which was rated '0'. However, when asked to verbalise his thinking he revealed that his reasoning in relation to the problem was sound and that his incorrect answer was simply the result of a computational error. He said:

> I just started um, with, how it said, how it's got on the sheet there and just kept adding it on, like, um, just added the top bit and then added the bottom bit to it and it worked out to 10 triangles first and then I got another um, 4 toothpicks and just added them on and made triangles and kept going until 23.

In response to Problem Five, the handshake problem, Peter wrote an incorrect answer, 41, at the top of the page and at the bottom of the page he wrote the numbers nine down to one vertically. There was no indication of the purpose of these numbers but nevertheless they were relevant to an appropriate solution of the problem and hence his written communication was rated ‘1’. Peter’s verbal explanation rated ‘2’ and revealed that he did understand what the problem was asking and had chosen an appropriate solution strategy. He said:

> I got ten matchsticks and I got one out and he would shake the other matchsticks’ hands, every matchstick’s hand once, so that’s nine times and one of the others would shake the other matchsticks’ hands eight times and then another one would shake them seven times and another one would shake other hands and so forth and so forth.

When asked what he then did with those numbers, he responded that he added them up and got forty one. He had correctly identified that the numbers from one to nine needed adding, he simply made an error when calculating their total. Interestingly, Peter’s use of matchsticks in this problem is one of just 14 of a possible 120 instances in which concrete materials were used. Of these two were by above average students, four by average students, and eight by below average students.

Simon’s written and verbal communications of his solution to Problem Two, the lift problem, both rated ‘2’ and were typical of those of students of above average ability. His written communication is shown in Figure 1.

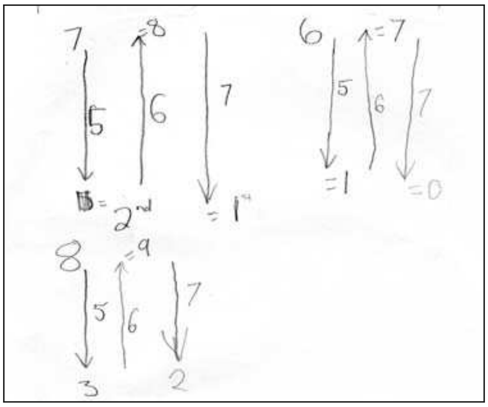


Figure 1. Simon’s written communication for Problem Two.

His verbal explanation was as follows:

> I just started at a number – I just thought I’d pick a number and then just try something that there’d be a fair chance of getting, like around the answer … I started with 7 and I went down 5 which equalled 7, no 2 sorry, and then went up 6 and that equalled 8, and came down 7 and that was only 1, so then I started at 6 and went down 5 and that was 1 and went up 6 and it equalled 7 and down to zero, so I knew that it had to go up in the numbers, so I went 8, went down 5 and got 3, up 6 and got 9 and then down 7 and got 2.

The extent to which the verbal communications of the various ability groups rated more highly than their written communications can be compared by considering the total ratings achieved for each of the verbal and written communications for all responses from each ability group of students. These figures are shown in Table 5. In each case the total rating for responses from average students is halved so that all can be considered relative to a possible maximum of 60. Table 5 also shows the difference between the total ratings for written and verbal communications as a percentage of the total written rating, and the total ratings for correctness of solutions achieved by each group. In this column the total rating for responses from average students is also halved.

Table 5
Total Ratings for Written and Verbal Communications by Ability Level

Ability group	Written communication	Verbal communication	Correctness of solution	Verbal - Written as a percentage of written
Above average	46	56	48	22%
Average	28.5	51	41	79%
Below average	16	40	22	150%

While the absolute differences between the total ratings for the written and verbal communications from above average, average and below average students are respectively 10, 22.5 and 24, both these figures and the percentage improvement on the total written communication ratings of each ability level's total verbal communication rating, must be considered in relation to the room for improvement upon their written responses available to each group. Clearly since, as shown in Table 2, 60% of the written recordings produced by above average students rated '2' compared to 28% and 20% respectively for those of the average and below average students, there was far less scope for their verbal responses to rate more highly than their written. It is clear, however, both from Table 5, and from the examples of Jack, Peter and Simon, that having the opportunity to verbally communicate their thinking seems to be considerably less important for more able students compared with students of either average or below average ability.

In addition, there is a much closer correspondence between the total ratings for correctness of solutions and written communication for the responses of above average students than for those of other students. Above average students who were able to successfully solve a problem were usually able to clearly communicate their thinking in writing, however average students were less effective in their written communication even when their solutions were correct, and below average students struggled with both solving the problems and with communicating their thinking in written form.

Conclusion

Students in all ability groups in this study were able to better express their thinking verbally than in writing. It therefore seems that all might benefit from instruction aimed at helping them to develop efficient and meaningful ways of recording their thinking in writing, and that strategies that build upon students' ability to verbalise their thinking might be effective. It appears that this is particularly true of average and lower ability students. The findings also highlight the importance of talking to students, even those of middle school age who might be expected to be competent writers, about their solutions and the reasoning behind them in order to gain insight into their thinking. There is evidence that this may also be especially important for less able students.

References

Adibnia, A., & Putt, I. (1998). Teaching problem solving to year six students: A new approach. *Mathematics Education Research Journal, 10*(3), 42-58.

Azzolino, A. (1990). Writing as a tool for teaching mathematics: The silent revolution. In T. Cooney & C. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s, 1990 Yearbook* (pp. 92-100). Reston, VA: NCTM.

Bicknell, B. (1999). The writing of explanations and justifications in mathematics: Differences and dilemmas. In J. Truran & K. Truran (Eds.), *Making the difference,* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide, pp. 75-83). Sydney: MERGA.

Branca, N. (1980). Problem solving as a goal, process and basic skill. In S. Krulik & R. Reys (Eds.), *Problem solving in school mathematics 1980 Yearbook* (pp. 3-8). Reston, VA: NCTM.

Burns, R. (2000). *Introduction to research methods* (3rd ed.). Melbourne: Longman.

Clarke, D., Clarke, D., & Lovitt, C. (1990). Changes in mathematics teaching: Call for assessment alternatives. In T. Cooney & C. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s, 1990 Yearbook* (pp. 118-129). Reston, VA: NCTM.

Clements, M.A., & Ellerton, N. (1985). Assessing the effectiveness of pencil-and-paper tests for school mathematics. In B. Atweh & S. Flavel (Eds.), *GALTHA, MERGA 18,* (Proceedings of the 18th annual conference of the Mathematics Research Group of Australasia, Darwin, pp. 184-188). Sydney: MERGA.

Department of Education Tasmanian. (2002). *The essential learnings framework 1.*Hobart: Author.

Flewelling, G., & Higgenson, W. (2003). *Teaching with rich learning tasks*. SA: AAMT.

Franke, M., & Carey, D. (1997). Young children's perceptions of mathematics in problem solving environments. *Journal for Research in Mathematics Education, 28*(1), 8-25.

Ginsburg, H. (1987). *Assessing the arithmetic abilities and instructional needs of students*. Texas: Pro-Ed.

Huinker, D., & Laughlin, C. (1996). Talk your way into writing. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in mathematics K-12 and beyond - 1996 Yearbook* (pp. 81-88). Reston, VA: NCTM.

Kaur, B., & Blane, D. (1994). *Probing children's strategies in mathematical problem solving*. Paper presented at the AARE Conference, University of Newcastle, 27th Nov-1st Dec. 1994.

Mousley, J. A. (1999). Perceptions of mathematical understanding. In J. Truran & K. Truran (Eds.), *Making the difference* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide, pp. 388-396). Sydney: MERGA.

NCTM. (2000). *Executive Summary: Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Norbury, H. (2003). Early year's numeracy interview – an assessment tool for P-4. In W. Moroney & P. Brinkworth (Eds.), *Springboards into numeracy,* (Proceedings of the national numeracy conference, pp. 119-124). Adelaide: AAMT.

Polya, G. (1949). On solving mathematical problems in high school. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics 1980 yearbook* (pp. 1-2). Reston, VA: National Council of Teachers of Mathematics.

Polya, G. (1957). *How to solve it* (2nd ed.). USA: Princeton University Press.

Pugalee, D. (2001). Writing, mathematics and metacognition: Looking for connections through students' work in mathematical problem solving. *Schools Science and Mathematics*, *101*(5), 236-245.

Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13-25.

Stoyanova, E. (2000). Empowering students' problem solving via problem posing – the art of framing good questions. *Australian Mathematics Teacher*, *56* (1), 33-37.

Suydam, M. N. (1980). Untangling clues from research on problem solving. In S. Krulik & R.E. Reys (Eds.), *Problem solving in school mathematics - 1980 Yearbook* (pp. 34-50). Reston, VA: National Council of Teachers of Mathematics.

Willoughby, S. S. (1990). *Mathematics education for a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Williams, K. (2003). Writing about the problem-solving process to improve problem-solving performance. *The Mathematics Teacher*, *96*(3), 185-187.