"I Type What I Think and Try It": Children's Initial Approaches to Investigation Through Spreadsheets

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How do children make initial sense of an investigative situation when approaching it through the pedagogical medium of the spreadsheet? This paper examines the ways groups of ten-year-old children made sense of number investigations explored in a spreadsheet environment, and how their preliminary responses were shaped, and their sub-goals framed, by the features of that setting. It also explores the manner in which this might filter their understanding and conjectures.

Investigation of a mathematical situation, whether one contrived as a 'school maths' model or one necessitated by a real life circumstances, requires an aspect of familiarisation. Polya (1945), was the first to formally articulate this 'understand the problem' stage in his four-step approach to problem solving, but contemporary mathematics educators maintain the validity of this initial step (Holton, 1998). What am I trying to find out? What information do I have? How do I gather more pertinent information? What picture is beginning to emerge? These questions may be part of that familiarisation process, and the individual's response to the mathematical phenomena that will condition the shape of the investigative process.

This familiarisation process isn't discrete from the solving process however, nor is it necessarily chronologically placed prior to the commencement of that process. Nunokawa, (2002) discussing Resnick's concept of sub-goals in solving more complicated problems, observed that these aspects were intertwined. He noted that the "settlement of sub-goals was supported by his [sic] understanding of the situation", but that also " the sub-goals settled by the solver influenced his understanding of the problem situation" (p. 204). Sub-goals are generated as part of the familiarisation and re-familiarisation of the problem, and where the learning is situated will influence the specificity of their production.

It is important to be aware of how using a spreadsheet might constrain the investigative process, by influencing the generation of sub-goals, as well as their previously identified potential to open up investigative opportunities (Calder, 2004; Drier, 2000; Ploger, Klinger and Rooney, 1997). Other researchers have found links between the use of ICT and the development of understanding in mathematics. Zbiek (1998) established that it enhanced students' ability to model mathematically. Chance, Garfield and delMas (2000) found that the use of ICT enriched the students' ability to problem solve and communicate mathematically; that it allowed the learner to concentrate more on conceptual understanding.

It is reasonable to surmise that these findings will correspond with children investigating with a spreadsheet, as there are some generic qualities of ICT that facilitate an investigative approach. Providing an environment to test ideas; linking the symbolic to the visual; linking the general to the specific; giving almost instantaneous feedback to changing data; being interactive; and giving students a measure of autonomy in their investigation are commonalities that facilitate investigation.

The current study is designed to explore how the pedagogical medium of a spreadsheet, used as a tool for investigation, might colour the learning experience, and how processing mathematics in this way might influence children's perceptions and understandings. This paper examines the ways participants approached the mathematical investigations as they negotiated the requirements of the tasks, and how this filtered their conjectures and generalisations. The research question can be more perspicuously put as: In what ways might investigating mathematical problems with a spreadsheet influence the understanding of the problem and therefore the approach taken to solve it? Critical to this is the participants' discourse as they negotiate the meaning of the tasks.

Recent social philosophers view understanding as being embedded in the social context within which it is conceived. While varying in their viewpoint regarding universal principles that might underpin behavior, both Foucault (1998) and Habermas (1976) nevertheless converge philosophically regarding the nature of discourse as an intervening agency in conceptual understanding. Ricoeur (1981) flavoured the hermeneutic perspective through defining discourse as spoken and written language. He parallels the relationship between spoken and written discourse, with action and the sedimentation of history. "History is this quasi-'thing' on which human action leaves a 'trace', puts its mark" (Ricoeur, 1981, p.209). In this case, the evolving history of the interpretation of the task, and the negotiation of sub-goals, is a collaboration of the discourse, and the corresponding action. A hermeneutic viewpoint allows the incorporation of the participants' discourse and actions, as the links between what they are saying, and what they are doing, is examined in terms of how they interpret the mathematical phenomena.

Hermeneutics can also be understood as the manifestation and restoration of meaning that a person makes sense of in a personal way, or as a demystification or reduction of illusion. These perspectives underpin the constructivist, and social constructivist theories of learning respectively.

While there is a tension between the pedagogical manifestation of these viewpoints, they can also be seen as complementary. Cobb (1994) advocates that, "the social withdrawal perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the process by which they do so" (Cobb, 1994, p.13). Brown (2001), saw the formations of understanding evolving from both individual and collective interpretations of mathematical stimuli. These understandings develop through social activity and discourse, with all the historical, political and cultural influences that such an interpretation implies. It appears they are intimately entwined if one considers that an individual's construction can only occur within a social framework. It follows then that identical stimulus enacted upon in various pedagogical media will lead to different contextualisation, and hence understanding.

By examining the participants' discourse as they engaged in the tasks; by observing their actions; and by analysing their reflections, insights were gained into the ways investigating mathematical problems with a spreadsheet might influence their understanding of the problem. As they negotiated the requirements of the tasks, a more fulsome picture of the ways participants framed their conjectures and generalisations evolved.

Participants

The participants were drawn from year six students, attending five of the campus's partnership schools. There were four students from each school who have been identified through a combination of problem solving assessments and teacher reference. There were eleven boys and nine girls. The schools were from a wide range of socio-economic areas (decile one to decile nine), with two of them being full primary (years 1 to 8) schools.

Approach

A qualitative methodology was used. Ethnographic research is concerned less with predictive generalisations, than with the formation of generalised descriptions and the interpretation of events. The researcher's perspective is not the sole contributor, and there is also the need to gain understandings of the learning occurring at an individual level, and the possible reasons for this. That is, the understanding of actions or implications rather than causes. This also indicates the need for elements of an interpretative paradigm. To gain insights into, and an understanding of, the learning that might occur for individuals, observations in the learning environment and interviews with participants were used to provide important information.

The participants worked on a programme of activities using spreadsheets to investigate mathematical problems, predominantly suitable for developing algebraic thinking. The students participated in four one-hour sessions, once a week, over four weeks, using spreadsheets to investigate mathematical problems. This included some instruction on using spreadsheets as well as using them as a tool to explore the problems. They were observed, and their investigation was printed out or recorded. The conversations of the participants, while they negotiate both the context, and the investigation of the interventions, were taped and transcribed. These then become the discourse to be analysed. Checking was done to ensure accuracy of the transcription, but an interpretative constituent is implicit to discourse analysis, and the researcher needs to be mindful of misunderstanding. It is, nevertheless, an effective way to gain critical insights into the participants' thinking. They were also interviewed in groups, and some individually. Observations made in situ were recorded.

Results and Discussion

There were three areas to consider in response to the research question:

1. How did the children negotiate their understanding of the task, and were their initial responses shaped by the computer environment?
2. In what manner did this initial familiarisation/exploration process lead to generalisation, posing of conjectures, and resetting of sub-goals?
3. In what ways did investigating in a spreadsheet environment fashion the children's approach to investigation in general?

The first set of data refers to the following activity.

Investigate the pattern formed by the 101 times table by:

- Predicting what the answer will be when you multiply numbers by 101

- What if you try some 2 and 3 digit numbers? Are you still able to predict?
- Make some rules that help you predict when you have a 1, 2, or 3, digit number. Do they work?
- What if we used decimals?

It was noticeable that the children were willing to immediately enter something into the spreadsheet. There was little attempt, in general, to negotiate the task situation through discussion or pencil and paper methods, although some individual processing of the task requirements must have occurred. For example:

2. A So, we've got to type in 101 times.
3. B How do you do times?
4. A There is no times button. Oh no, wait, wait, wait.
5. B There is no times thing. Isn't it the star?
6. A =A1*101. Enter.

This approach was confirmed with responses in the interview:

A I preferred thinking something about what I needed to do, then take it and highlight it down and then the whole table is there, which would help me.

C What we did is we tried a few formulas. To start off with we like typed in a few formulas that we thought it might be, and then went through and got the correct one.

D Because of the spreadsheet, we went straight to formulas, looked for a pattern; for a way to make the spreadsheet work.

It appears the actual spreadsheet environment provided the impetus to take this initial approach. Not only did the use of spreadsheets lead them to explore in a seemingly stylised procedure, it also lead to an immediate form of generalisation. To generate a formula that models a situation is to generalise in its own right, but to consciously look to fill down ("highlight it down"), or create a table of values is also indicative of an implicit cognisance of a pattern; of an iterative structure that is a way into exploring the problem. A and B continue:

7. B 202.
8. A Now let's try this again with three. What number do you think it will equal? 302?
9. B No, 3003. Oh no 303.

They were immediately into the business of predicting and confirming in a confident, relatively uninhibited manner. They began to pose conjectures, and test them in an informal approach:

10. A OK. Now you try a number.
11. B My lucky number 19.
12. A That'll be one thousand, nine hundred, and nineteen.
13. B Equals. So we need to think of a rule.
14. A Its like double the number. Its nineteen, nineteen.
15. B What about 20? Oh you'll get 2020.

The ability to predict, form a conjecture then test it is indicative of a robust generalisation process. In this case, and with others in the study, the children chose a particular path because they were using the spreadsheet. The shape of their investigation was determined by the particular pedagogical approach. They were also able to quickly move beyond the constraints of the prescribed task, forming a fresh generalisation.

20. B Oh try 1919.
21. A One, nine, three, eight, one, nine. No make it 1818 and see if its 1818.
22. B Look eighteen, 3, 6, eighteen.
23. A Before it was nineteen, thirty six, nineteen; write that number down somewhere and we'll try 1919 again.
24. B Yeh, nineteen, 3, 8, nineteen. That's an eight.
25. A What's the pattern for four digits? It puts the number down first, then doubles the number and repeats.

It is clear they were using a visual referent to the theory that is evolving. They were not looking at the procedure that is producing the number patterns, just the actual visual sequence itself. Lines 22, 23 and 24 of the transcript imply that possibility through their naming of the products as, for example, eighteen, 3, 6, eighteen. They were seeing the number as three or four discrete visual elements, rather than thinking of a consequence of an operation. Their concluding generalisation confirms this also in line 25. It could well be with appropriate scaffolding the pattern may be investigated in a more fulsome manner, but again the data implies that the spreadsheet environment influenced their approach to the investigation. It filtered the path to, and the nature of, their conjectures, and their subsequent conclusions were shaped in visual rather than procedural terms.

It is also noteworthy that the characteristic of spreadsheets to produce immediate responses to inputted data assisted the further development of their emerging theory; it facilitated the risk taking aspect of the investigative process (Calder, 2004). As well, it led them to promptly set a new sub-goal in the investigation.

This is similar to the data produced with another investigation involving exploring dividing one by other numbers.

When we divide 1 by 2, we get 0.5, a terminating decimal.
When we divide 1 by 3, we get 0.33333…., a recurring decimal.
Investigate which numbers, when we divide the number one by them, give terminating, and which give recurring decimals.

In the first case they negotiated to gain some initial familiarisation of the task.

E One divided by one is one - it should be lower than one.
F Try putting one divided by two, and that should be 0.5

They then entered 1 to 5 in column A and =A1/1 in column B to get:

1. 1
2. 2

3. 3
4. 4
5. 5

This posed an immediate tension with their initial thoughts and fostered the resetting of their sub-goal.

E Is it other numbers divided by one or one divided by other numbers?
F Lets recheck. She entered =A1/4 and got the following output:

1 0.25
1
1

F Umm, we're not going to get change…we'll have to change each one.

They appeared to intuitively feel there should be a way to easily produce a table of values to explore. The spreadsheet environment was shaping the sense making of the task and the setting of their sub-goals. Critically, it was enabling them to immediately generalise, produce output, then explore this visually. They explored other formula e.g. =B1/(4+1), before settling on: =1/A1. They generated the following output:

1. 1
2. 0.5
3. 0.33333…
4. 0.25
5. 0.2
6. 0.16161616…
7. 0.1428514285…
8. 0.125 etc.

E So that's the pattern. When the number doubles, it's terminating. Like 1, 2, 4, 8 gives 1, 0.5, 0.25, 0.125.
F So the answer is terminating and is in half lots. Lets try that =0.125/2; gives 0.0625- which is there. (Finds it on the generated output from above)

The structured, visual nature of the spreadsheet prompted the children to pose a new conjecture, reset their sub-goal, and then allowed them to easily investigate the idea of doubling the numbers. The table gave them some other information however.

F 1 divided by 5 goes 0.2, which is terminating too. (Long pause)

After further exploring, they reshaped their conjecture, incorporating their earlier idea.

E If you take these numbers out they double and the answer halves.
F That makes sense though, if you're doubling one, the other must be half.
Like 125 0.008; 250 0.004.

E What’s next. Let’s check 500
F Let’s just go on forever.

They generated a huge list of output; down to over 4260.

F 500 0.002; 1000 0.001.

Although they didn’t fully explore the relation of the base numbers to the multiples of ten without scaffolding, they have made sense of, explored, and generalized aspects of the investigation. The pedagogical medium through which they engaged in the task has clearly had some influence on the contextualization and approaches they have taken. The children responded in a corroboratory manner in the interviews, when asked: “When you saw the problem how did you think you would start?”

E Re-read to get into the math’s thinking, then straight to a spreadsheet formula.
F Thought of a formula
G I type what I think and try it

It is also clear from their discourse and responses in the interviews, that the spreadsheets have provided not only a unique lens to view the investigation, but have possibly drawn a distinctive response in terms of investigative practice.

F Using a spreadsheet made it more likely to have a go at something new because it does many things for you. You have unlimited room. You can delete, wipe stuff out.

C Columns make it easier- they separated the numbers and stopped you getting muddled. It keeps it in order, helps with ordering and patterns.

A It helps when you look at patterns. You just type it in and see the whole pattern.

Conclusions

This paper attempts to enrich the evolving picture of how children using ICT to investigate mathematical situations, might shape their investigation in particular ways. Specifically, how using a spreadsheet as an investigative tool, might influence the understanding of the problem, and therefore the approach taken to solve it.

The data supports the supposition that the availability of the spreadsheet led to the children familiarising themselves with, then framing the problem through a visual, tabular lens. It is clear also that it evoked an immediate response of generalisation, either explicitly through deriving formulas to model the situation, or implicitly by looking to fill down, or develop simple iterative procedures. Tension, arising from differences between expected and actual output, and opportunities, arising from possibilities emerging from these distinctive processes, led to the setting and resetting of sub-goals. These, in turn, further shaped the understanding of the investigative situation, and the interpretation of mathematical conjectures.

The children also identified speed of response, the structured format, ease of editing and reviewing responses to generalisation, linking symbolic and visual forms, and the interactive nature as being conducive to the investigative process.

While this particular medium has unfastened unique avenues of exploration, it has as a consequence fashioned the investigation in a way that for some children may have constrained their understanding. What might get lost is a question for further research.

References

Brown, T. (2001). *Mathematics education and language: Interpreting hermeneutics and post-structuralism,* Dordrecht: Kluwer Academic Publishers.

Calder, N. (2004). Shaping understanding: How does investigating in a spreadsheet environment affect the conversations of initial training students doing mathematics. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville, pp.143-150). Sydney: MERGA.

Chance, B., Garfield, J., & delMas, R. (2000). Developing simulation activities to improve students' statistical reasoning. In M.O.T. Thomas (Ed.), *Proceedings of TIME 2000.* Auckland: The University of Auckland and Auckland University of Technology.

Cobb, P. (1994). Where is the mind? Constructivist and socio-cultural perspectives on mathematical development, *Educational Researcher, 23(7)*, 13-19.

Drier, H. S. (2000). Investigating mathematics as a community of learners. *Teaching Children Mathematics,* Feb., 358-362.

Foucault, M. (1998). *Aesthetics* (Edited by Paul Rabinow). London: Penguin.

Habermas, J. (1976). Systematically distorted communication. In P. Connerton (Ed.), *Critical sociology.* Harmondsworth: Penguin.

Holton, D. (1998). *Lighting mathematical fires.* Carlton: Curriculum Corporation.

Nunokawa, K. (2001). Interactions between subgoals and understanding of problem situations in mathematical problem solving. *The Journal of Mathematical Behaviour*, *20*(2), 187-205.

Ploger, D., Klinger, L. & Rooney, M. (1997). Spreadsheets, patterns and algebraic thinking. *Teaching Children Mathematics,* Feb., 330-334.

Polya, G. (1945). *How to solve it.* Princetown: Princetown University Press.

Ricoeur, P. (1981). *Hermeneutics and the human sciences.* Cambridge: Cambridge University Press.

Zbiek, M. (1998). Prospective teachers' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education, 29*(2), 184-201.