Primary Students' Mental Computation: Strategies and Achievement

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Several studies have documented students' approaches to mental computation. Few, however, have explicitly linked strategy use to success or otherwise on mental computation tests. As part of the evaluation of a mental computation program in one primary school, students in Years 3 to 6 undertook pre- and post-tests of mental computation to measure performance and growth over time. Interviews were conducted with twelve of these students, three from each grade. The findings indicated that students who were procedural in their approach were less successful on the pre-test but generally showed growth on the post-test, following a strategy-based intervention. The implications of these findings are discussed.

Research in mental computation has tended to focus on identifying and describing students' strategies for addressing particular kinds of calculation, often within a framework of "number sense". Number sense is defined as having a "general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways" (McIntosh, Reys, Reys, Bana & Farrell, 1997, p.3). Mental computation promotes number sense through a development of understanding how numbers work and relate to each other. (McIntosh, 2005)

It is known that students use many different strategies for undertaking mental computation. In addition and subtraction, for example, four classes of strategy, Counting, Separation, Aggregation and Wholistic, have been described, each of which has a number of sub-categories (Heirdsfield & Cooper, 1997). Bana and Korbosky (1995) identified 17 different valid ways in which students responded to the question 24 ÷ 6. These studies were undertaken using interview approaches that allowed students' thinking and number sense to be explored. In a different approach, Watson and associates used error patterns from large scale mental computation tests to infer number sense (Watson, Kelly & Callingham, 2004). For example, in single digit multiplication and division (tables facts) the most common type of error was associated with a 'near' factor such as responding 49 to 7 x 8, or 5 or 7 to 54 ÷ 9. However, for the question 54 ÷ 6, with near factors of 8 and 10, the most common error was 8, and 10 was infrequent. In this context the number 10 doesn't 'make sense', and overall the responses suggested that students drew on some number sense since they gave answers that were reasonable in the context of the question.

The nature of the approaches used in computation has been categorised as *instrumental* or *relational* understanding (Skemp, 1976). Instrumental approaches are those which rely on the application of a particular process with no explanation that indicates an understanding of the underlying concepts. In contrast, relational understanding displays conceptual knowledge of how numbers work, and is closer to number sense. In this study, the terms *procedural* and *conceptual* are used to signify instrumental and relational understanding. Students successful in mental computation are likely to be those who have relational understanding and flexible approaches to using this understanding. Such students could be considered to have strategic knowledge of mental computation, being able to choose appropriate methods to solve different mental computation problems.

The study reported here is part of the evaluation of one primary school's mental computation program. The program is unusual in that it deliberately focuses on the development of generic strategies, such as visualisation or using doubles and near doubles, applied to different kinds of computation and operation. Over a two week period, the whole school, from kindergarten to Grade 6, includes some aspect of the target strategy in their mathematics lessons, applied to content appropriate to the age and stage of learning. The aim is to develop students who have a range of ways of approaching computation, and who can use these approaches strategically to solve problems – in other words, to develop number sense. The evaluation included a pre- and post-test of mental computation, classroom observations and interviews with 12 students from Years 3 to 6. It is the interview data that are reported here.

The aims of this component of the study were (a) to determine the extent to which students were able to articulate and describe their mental computation strategies, and (b) to relate students' strategy use to their success in the tests of mental computation. The first aim was one that the teachers at the school deemed important for their students' development, and which they had deliberately fostered in class. The second aim was to confirm the strategy-based approach that the school had decided upon.

Method

Classroom teachers selected the interview subjects to represent the spread of ability across their classes or who they thought had some unusual approaches to computation. Three students were chosen from each of Years 3 to 6; three students were girls and nine boys. As part of the bigger study, these students had undertaken a test of mental computation prior to the interview. Students in grades 3 and 4 answered 50 questions and students in grades 5 and 6 responded to 65 questions, which included some addressing part-whole numbers, as well as whole numbers. Some of the test items were a basis for the questions asked in the interviews.

Each student was interviewed twice. The first time was as part of a grade group in order to establish a relationship with the researcher. The second interview was an individual interview lasting approximately forty minutes. Mental computation questions were posed based on those in the mental computation test, but with extension questions designed to reveal students' flexibility of thinking and use of known facts. Students from Years 3 and 4 were asked eight starter questions, and those from Years 5 and 6 were asked 12 starter questions, the same eight as the younger students and four additional questions focussed on fractions, decimals and percents. Follow up questions were asked depending on responses to the initial question and questioning was stopped when it was obvious that the student could not answer further.

Responses to questions were classified as procedural or conceptual, depending on students' explanations of their strategies. For example, responses to $16 + 8$ such as counting on by 1 were classified as procedural, whereas a response that halved 8 and added a four to 16 to get 20 and then added the additional four was classified as conceptual, because it made use of the inherent structure of the number system.

Following the interviews, the students again undertook mental computation tests, along with their cohort. The post-tests were different from the pre-tests, and were given about 15 weeks after the initial testing. All tests had previously been used in an earlier study (Callingham & McIntosh, 2002) and levels of difficulty of the items established. These difficulty levels were used to anchor the tests given in this study so that direct comparisons of the students' performances over time could be made using Rasch modelling techniques (Bond & Fox, 2001, ch. 9). Students' measured ability at two points in time was obtained from the Rasch measurement scale, based on their performance in the pre- and post-tests.

Results

Summary results for the 12 students are shown in Table 1. These show ability measures in logits for the pre- and post-test measures, and the change for each student, sorted from lowest to highest ability at the pre-test measure. Two students were absent for the post-test. Names have been changed for confidentiality.

Table 1
Summary Results for 12 Students

		Ability (Logits)		
Name	Year	Pre-test	Post-test	Change
Fred	4	-4.41	2.20	6.61
Jim	3	-2.74	-0.83	1.91
Dan	5	-1.05	-0.42	0.63
Kaye	3	-0.54	0.41	0.95
Louise	4	-0.25		
Lewis	3	0.23	0.77	0.54
Chad	5	1.00	0.93	-0.07
Mike	5	1.69	2.24	0.55
Colin	4	2.65	3.06	0.41
Mark	6	2.90	3.33	0.43
Ben	6	3.10		
Carol	6	3.55	2.64	-0.91

The results indicate a wide range of ability across the individual grades, other than Year 6 who were all at the higher end of the scale. When the individual strategies were considered, however, some interesting differences emerged.

Interview findings

The lower ability students, Fred, Jim and Dan, were predominantly procedural responders. Fred's predominant strategy was counting on or counting back, often using his fingers. When asked how he worked out 16 + 8, which he did correctly, he responded "I saw the fingers in my head and counted them". Only when prompted did he say that he had started at 16. For 17 – 7 his initial response was two which he corrected to one after some thought. When he was asked how he worked it out, he said he just saw it on the paper and this gave him the answer. On prompting about the one, he changed his answer to ten, saying, "Ten has a zero".

Jim used similar strategies, using some form of counting on in most instances. He did, however, indicate the beginnings of number sense in his response to 12 x 10. He initially responded, "I know it's 100 plus something …" but finally got the correct answer by adding 10 twelve times, keeping track furtively with his fingers.

The middle range group of students, Kaye, Lewis and Louise, had more mixed approaches. Kaye applied predominantly procedural approaches, using a range of strategies such as skip counting. In response to the question Double 38, Kaye clearly described separating 38 into tens and units, adding each of these and then putting the answers together to get a correct response. This response appeared to indicate the start of some conceptual understanding.

Louise and Lewis both had a conceptual approach to the question 16 + 8. Lewis clearly described going to the nearest ten in response both to the initial question, and the extension question of 24 + 8. Neither student built on from the earlier answer, but applied the conceptual approach of partitioning the addend appropriately to bridge to the next multiple of ten. As the questions became less familiar, however, they tended to draw on learned procedures with varying success. To get the answer to 7 x 3, Louise added 7 and 7, and then counted on by ones. Lewis, in contrast, added 3 seven times, using his fingers to keep track. Lewis used the same process to answer the follow up question of 7 x 6. In contrast, Louise attempted to use different procedural thinking by drawing on a table fact of 6 x 6, which she said was 32, and adding another six, counting on by one to get 38. Neither student considered doubling the original answer, even when prompted.

Colin, Mike and Chad all gave predominantly conceptual responses. Colin described how he "… work[ed] with groups of ten on the hard ones", whereas the easy ones he just knew. He volunteered his approach to multiplying by nine:

> I take away one from the timesing number [to get the tens]. Then work out how many more to add on [to the number left] to get to ten. I just figured it out for myself.

Mike and Chad both had a good sense of doubles and used this effectively. Chad, for example, although adding 24 three times to correctly answer 24 x 3, recognised that he could simply double that answer to calculate 24 x 6, and multiply it by ten to get 24 x 30. As Year 5 students, both were also presented with some fraction, decimal and percent questions. The question 0.5 + 0.5 was answered by both students in terms of whole numbers, fives for Mike and fifties for Chad. Both answered correctly, applying a procedural rule to work out where to place the decimal point. The follow up question was 0.5 + 0.75. Mike was unable to answer this question but did answer the same question in fraction form by taking "… a quarter and adding it to a half. Half and a half is one, and add on a quarter." Chad, in contrast, correctly answered the decimal version, applying the whole number strategy as before but when faced with the fraction form, turned it into decimals and used the whole number strategy again.

The three students who scored best on the pre-test, Mark, Ben and Carol, were all from Year 6. Of these students, Mark and Ben appeared to draw on a mixture of procedural and conceptual understanding. In response to whole number questions that they would reasonably be expected to answer quickly they replied that they just 'knew it', but could describe general procedures for explaining the calculation to students who couldn't do it. For example, to the question 16 + 8, Mark said that he would explain it as "the same as three times eight, so three groups of whatever …". He used the same reasoning for 24 + 8. Ben, in response to the same question said "I saw 16 as three groups of five plus one, and eight as five plus three and I put the fives together and then the leftovers." These two students appeared to have a good grasp of number relationships and drew on these in strategic ways to obtain correct responses. In contrast, Carol was almost totally procedural in her responses. She answered 16 + 8 correctly by using her fingers to count on by ones, and her response to 7 x 3 was the same as that of Louise: add seven and seven and count on a further seven.

At times, however, the students appeared to apply procedures inappropriately or with little understanding. Mark, for example, in response to 12 x 10 responded, “It’s a multiple of ten so you add a zero to the original number”. He also applied a rule to correctly answer a follow up question of 1.2 x 10, saying “You move the decimal point one space. Times is going up so you move it backwards”, but was unable to clarify what he meant by going up or backwards. Ben, applying the same procedure, was clearer about the decimal point: “You add one zero so move the decimal point one place to the right.” Carol, however, although able to apply the same add-a-zero rule to whole numbers reverted to trying to use the written algorithm for “1.2 x 10. She used the written algorithm repeatedly in many calculations, either visualising it or, for more complex problems, tracing it on the desk with her finger. Despite achieving the highest performance on the pre-test, Carol was unable to answer correctly many of the questions at interview. If she couldn’t use the written algorithm she could not get the answer, or, alternatively, applied the algorithm incorrectly. This was particularly obvious with decimal, fraction and percent questions.

The item 125 – 99 was a useful indicator item for identifying students’ thinking. Of the three lowest achieving students, Fred didn’t “… get it”, Jim guessed 34 and Dan used an effective procedural count on by ten strategy of 99 add ten, add ten, add five and add one. Louise was also unable to attempt the question, while Lewis subtracted nine from 125 and then counted back by tens keeping track with his fingers, also a procedural response. Mike, in a procedural attempt, tried to subtract 90 and then take away a further nine, miscounting and getting an incorrect answer of 35. Kaye, however, demonstrated some number sense by recognising that 100 take 25 was “very easy” and that 99 was close to 100 but was unable to put this idea to practical use. Chad was able to use this concept by working from 100, recognising that 25 add one was 26. Colin used a similar conceptual strategy but explained it more clearly, saying that he was thinking “99, how many more to get to 125?” He correctly answered this and the follow up questions of 125 – 89 and 135 – 99. Ben used another conceptual strategy by subtracting 25 from 125, and then taking away another one, then making the jump to recognising that what he had taken away would be the answer. Carol and Mark, however, reverted to procedural strategies, Mark complaining that he would “… need paper”, and Carol counting on by one from 99 using her fingers.

Pre- and Post-test Comparison

The pre- and post-tests were undertaken by students approximately 15 weeks apart. Two of the interviewed students, Louise and Ben, were absent on the occasion of the post-test, so complete data were available for ten students. Since the two tests were anchored to the same values through Rasch measurement techniques (Bond & Fox, 2001, ch. 9), the results could be directly compared. The changes in performance observed were considered in the light of the students’ strategy use and initial performance.

The change between pre- and post-test results for the ten students for whom data were available is shown in Figure 1. The ability measure is in logits, the unit of Rasch measurement, and improvements are shown as positive change. The ten students are organised in order of performance on the pre-test, from Fred (weakest) to Carol.

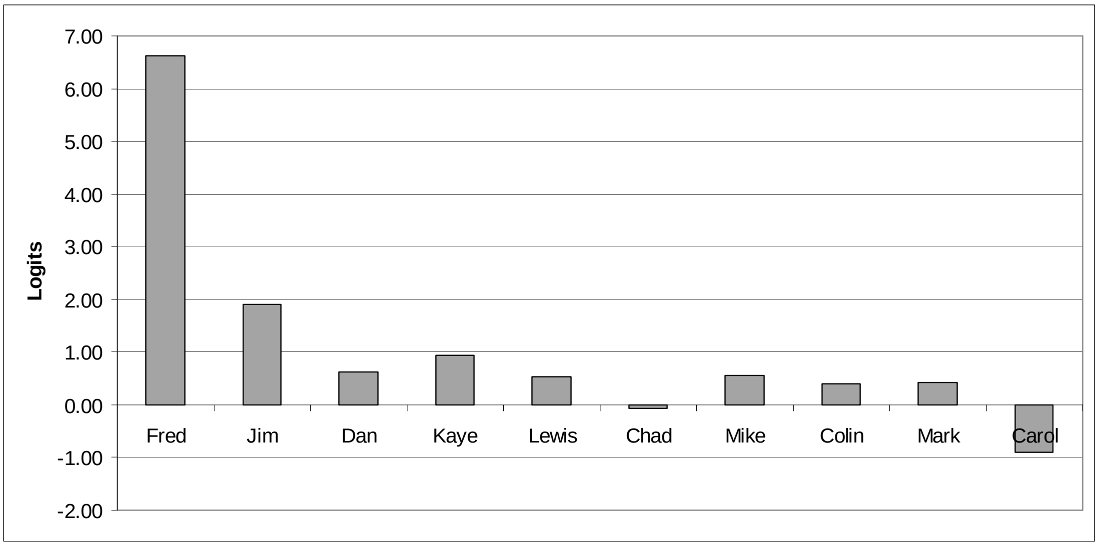

Figure 1. Change in measured ability from pre- to post-test for interview students.

The lowest ability students, Fred and Jim, showed the greatest improvement overall. Both these students were coming off a low base and improvement may be easier to show in this situation. In the first test however it was noticeable that Fred in particular had failed to attempt many questions and his greatly improved performance was probably due to a greater willingness to make an attempt. This may suggest improved confidence in this student. One student, Carol, showed a marked decrease in performance. Carol was a procedural responder who relied heavily on written algorithm use. Chad, who appeared to show almost no change in performance, was a student in Year 5. His interview responses seemed to be conceptual, demonstrating good number sense, but he did not translate this into growth. His test responses, however, showed several errors explainable by counting on errors, such as answers out by one, or responses having a near factor. This may suggest that under test conditions he used learned routines rather than the conceptual understanding displayed in a less pressured situation where he had more time to think.

Discussion

One aspect of the interviews that was very noticeable was the articulate manner with which the students were able to describe their strategies. No student responded “I don’t know”, when asked to explain how the answers had been calculated. All but the lowest achieving students had some kind of strategy that allowed them to attempt all questions, with few unable or unwilling to have a go. The classroom approach of focusing on discussion and strategy development appeared to have provided these students with a vocabulary and ways of describing their mathematical thinking.

The findings about the students’ performance and growth from this small scale study are, at first sight, somewhat paradoxical, however. Although those students of lowest ability, Fred and Jim, were predominantly procedural thinkers they showed the biggest gains. It seems likely that the approach adopted by the school was providing support to these lower performing students. Substantial growth was also shown by Kaye, a Year 3 student who demonstrated the beginnings of number sense in her responses.

Carol's superior performance on the initial test seems somewhat surprising. She did, however, demonstrate accurate and speedy use of the written algorithm, and extensive use of a limited procedure of counting on by one, which she did quickly. Her lack of growth, however, suggested that she was coming to the limit of her strategy use, and that the program did not appear to have benefited her. Although the school was implementing a strategy-based approach to developing mental computation competence, this had been in place for only two full terms at the time of the pre-test. In previous years, the school had placed a very heavy emphasis on algorithm development, with medals awarded for success. It is possible that the older students were somewhat less flexible in their thinking because of the emphasis in the past on specific algorithm use, and thus were less able to take on the strategies, resulting in lower growth overall. This comment could also be applied to Dan in Year 5 who, although a low achieving student initially, did not show the gains made by other similar students. It should also be noted that the period between pre- and post-test measures was very short, approximately three months, because of practical considerations in the school, and some students may not have had time to grasp the new approaches.

In general in the interviews students tended to respond conceptually to questions with which they could reasonably be expected to be very familiar. Those students of lower achievement, for example, used conceptual strategies, such as bridging to the next ten, when faced with questions such as 16 + 8, but fell back on more procedural approaches, such as counting on or attempting to build on known facts that were incorrectly recalled, when faced with more difficult questions. This situation was also perceived for students who achieved at high levels, such as Ben and Mark, who responded in terms of learned rules particularly to computations involving part-whole numbers. These findings imply that learning rules may be a bridge to developing conceptual relationships among numbers, provided that they do not prevent students also developing and building upon emerging intuitive knowledge. Carol, for example, had exemplary control of the written algorithm to the point where she could use no other strategy apart from some simple counting. Her weaker performance in the post-test indicated that these strategies had not served her well over time. Colin was a younger Year 4 student who recognised and used confidently a wide range of approaches that demonstrated good number sense. Although he did not show large growth, one reason might have been that, as a younger student, he undertook a shorter test and may not have been able to demonstrate fully his competence. Kaye, who appeared to be hovering between conceptual and procedural approaches, showed substantial growth, being one of the highest Year 3 performers in the post-test.

These findings have implications for teaching. They suggest that students use learned procedures initially but that these develop into conceptual approaches under the right conditions. Providing students with some explicit, structured approaches that they can use to attempt the problem posed would appear to benefit less able students in particular. The program developed by the school was consistent across all grades, and gave students a variety of tools that they could apply in flexible ways to solve mental computation problems. Inflexible methods, such as those used by Carol, are unlikely to lead to growth of understanding, but discussion and deliberate examination of a range of strategic approaches appears to be useful.

Implications for Teaching

This was a small scale study, and as such, care should be taken in interpreting the results. The findings are, however, in line with other research such as the suggestion that delaying the introduction of written algorithms is beneficial to students (McIntosh, 2005). The clear and deliberate focus on strategy development across the school that was observed in classrooms and demonstrated by the students during interviews does appear to have benefits for most students. Whether particular strategies are more useful than others, or whether specific strategies may be more effective in different grades remains a matter for conjecture and further research.

The fluency with which students talked about their methods, and the range of strategies that they displayed as a group, suggested that the explicit approach that the school was taking towards mental computation development was successful. Even when they got incorrect answers, the students were confident in their ability to attempt the problem, and used an approach that made sense to them personally. Although sometimes inefficient and inaccurate, these attempts were potential building blocks to more strategic approaches.

The interplay between rule-based procedures and conceptual understanding, particularly for mental computation involving part-whole numbers, deserves further exploration. In particular, the school-based conditions under which the development of confidence and number sense that allows strategic use of a range of methods to be used for mental computation needs to be examined further.

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