Developing Effective Teachers of Mathematics: Factors Contributing to Development in Mathematics Education for Primary School Teachers

Jean Carroll
Victoria University
jean.carroll@vu.edu.au

In response to issues of developing effective teachers of mathematics, this paper investigates the factors, which have contributed to growth related to mathematics teaching and learning for a group of Melbourne primary school teachers. The teachers were surveyed to determine the people or events that had contributed to their development. Major themes identified in the data were the importance of lifelong learning, relationships and reflection upon their classroom experiences. The themes are analysed in order to make recommendations for appropriate professional development for the teachers.

Much of the research into mathematics education in recent years has been driven by concerns about student achievement in mathematics. In response to these concerns there has been a refocusing of attention on the teacher. Teacher effectiveness has come under the microscope, reforms in teaching standards have been formulated and teacher professional development has been re-thought (AAMT, 2002; National Council of Teachers of Mathematics, 2000; Zaslavsky, Chapman, & Leikin, 2003).

The study reported in this paper investigates primary school teachers' views of the factors that have contributed to improvement in their teaching of mathematics, over the years. This study varies from many others reported in the literature in that it surveyed whole school staffs rather than just those teachers attending professional development programs or engaged in further study, as in many other published studies. This study was designed to enable all of the teachers in a school to participate, particularly those teachers who avoid mathematics professional development. Themes in the responses of participating teachers are identified and recommendations are made in the light of these for professional development.

Recent research into teacher effectiveness in mathematics has suggested that there are significant differences between teachers. Sullivan and McDonough (2002) found evidence that children from similar backgrounds had markedly different experiences at school. The different experiences could only be attributed to differences between teachers. Similar results were reported by Siemon, Virgona, and Corneille (2001, p. 99) in a Victorian study of middle schools in which they found that there was as much difference within schools, that is from class to class, as there was between schools, in student achievement. This suggests that individual teachers make the difference.

The increasing focus on developing teachers' abilities to deliver high quality student outcomes means that attention must be devoted to the issue of what constitutes effective professional development when considering the differences between teachers. We are becoming much more aware of what effective teachers of mathematics do in their classrooms (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997; Reynolds & Muijs, 2000) and the debate over what teachers should know to be effective teachers of mathematics continues (Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996, p. 403; Carpenter, Fennema, Franke, Levi, & Empson, 2000; Lowery, 2002). A study of "Effective teachers of

numeracy in UK primary schools" (Askew et al., 1997), which was designed to identify what teachers know, understand and do that enables them to teach numeracy effectively, found that it was hard to identify what aspects of the teachers' subject matter knowledge made a significant difference to student learning in numeracy. It was not as straightforward as their level of qualifications or the fluency with which teachers could list ideas contributing to numeracy.

The more effective teachers tended to demonstrate deeper understanding of the links between different numeracy concepts and could provide alternate meanings and representations. It was not the level of formal qualification, but the nature of the knowledge about the subject that was important. The researchers recommended that primary school teachers may need to develop fuller, deeper and more connected understandings of the number system (Askew et al., 1997).

The review of effective teaching conducted by Reynolds and Muijs (1999) provided useful insights into the characteristics of classrooms in which mathematics is taught effectively. This review investigated teacher effectiveness, mainly in primary schools in the United States and Britain identifying features of classrooms, which contributed to effective mathematics teaching. The United States study identified high opportunity to learn, an academic orientation from the teacher, effective classroom management, a high proportion of whole class teaching, and heavily interactive teaching as important attributes of effective teaching. The British review of research reinforced these factors and detailed the provision of a clear structure for lessons, rehearsal of existing knowledge and skills, and the use of a variety of activities on a topic as also important in effective mathematics learning. Classroom processes associated with low achievement, included too much emphasis upon repetitive number work, too much individualisation and too little fluency in mental calculation.

The developing insight into what effective teachers do, raises many questions about professional development. Questions such as how teachers develop their understanding of the processes of teaching and learning and how these understandings grow and change throughout their careers are important considerations for those planning professional development. These questions are by no means trivial and highlight the fact that much research has been done into student learning but we still have much to learn about how teachers develop their understandings. The study reported in this paper is an attempt to identify factors that have contributed to primary teachers' learning about mathematics teaching and to consider how these might be incorporated into planning for professional development.

Method

The data in this study were collected, as part of a larger study of primary teachers' views of mathematics teaching and learning. The data reported in this paper were collected using a survey instrument designed to enable teachers to express their views on their own teaching and learning of mathematics and this paper specifically reports on the teachers' views of the factors which have contributed to the development of their understanding of mathematics and mathematics teaching. The survey also collected biographical data regarding years of experience, mathematics education and type of teacher training. The sample was selected by approaching the principals of 20 schools, randomly chosen from a middle class suburban Melbourne region. The 13 principals who agreed to have their staff participate in the study actively encouraged their teachers to be involved. The teachers were

asked to provide written responses to the question: Which things, people or events have been most influential in your teaching of mathematics?

The written responses to the question were analysed by combining all responses to the question, then reading the responses as a whole and identifying themes in the data. This method of analysis was derived from the Phenomenographic approach described by Marton (1993). To provide an indication of the strength of themes in the data, percentages have been used in the results. The percentages represent the proportion of the one hundred and eight teachers who mentioned a particular theme in their response and provide an index of the importance attached, by the teachers, to particular themes in the data. As any one response may contain references to a number of themes, the total percentage of respondents to any one question is often much greater than one hundred. Teachers are identified in the results section by a respondent number which is sometimes abbreviated, for example Respondent 16 becomes R16.

Results

One hundred and eight primary school teachers from the 13 schools completed the survey. Female teachers outnumbered males by more than seven to one, with ninety-five female and thirteen male teachers responding. The teachers' experience ranged from first year out to thirty-two years teaching, with the average being fourteen years in the classroom.

The question: “Which things, people or events have been most influential in your teaching of mathematics?” was designed to investigate the factors to which these teachers attributed changes in their teaching. Teachers reported multiple influences over a wide time span. Some of the experiences recounted as influential were current while other significant experiences occurred years ago. A typical response describing professional development courses, the role of colleagues, changes in attitudes and the role of students is given by Respondent 51 in describing the range of factors which over the years have contributed to changes in her teaching:

> A growing realisation over the last ten years that my pupils and I can be more relaxed and have fun with maths, if I keep in mind that it is pointless to persist in concepts if children are not ready for them. Therefore I am more confident and relaxed and the children also. Certain colleagues over the years - just useful hints.

The results are summarised in Table 1. Planned professional development courses and conferences were said to be most influential by these teachers. Colleagues, teachers working together, sharing and learning from each other, rated as significant influences. Teachers’ histories arose also, in the descriptions of influences. Almost one third of the teachers mentioned their own school experiences were important, while a quarter of the sample referred to teacher training experiences as influential in the teaching of mathematics.

Most of the influential events described by the teachers were positive. A minority of teachers recalled negative experiences from their school days, but on reflection they had been able to use these to improve their teaching by ensuring that they were never like the teachers who were remembered.

Table 1.
Which Things, People or Events Have Been Most Influential in Your Teaching of Mathematics?

Categories	Percentage
Planned professional development such as Maths association PD, Ongoing professional development courses, In-services,	42.6
Own school experiences	31.5
Colleagues, visiting other schools and classrooms, key groups	28.7
Teacher training experiences	25.9
Curriculum materials, textbooks, curriculum guides	18.5
Experiences with children	9.2
Professional reading	6.5

Professional Development Courses and Conferences

Professional development programs, in-service courses and Mathematics Association of Victoria conferences were said to be influential in the teaching of mathematics for 43% of respondents. In answering this question 23% of the sample referred to programs they had attended. Such programs were conducted over a period of time and involved teachers in discussing current approaches, sharing good ideas, reading research papers, and trialing material in classrooms while working with a trained facilitator. Many teachers named the program facilitator as a person who contributed to changes they had made in their teaching.

The following responses were typical: "Recently professional development courses have been most influential in my teaching" (R19), "My tutor, who made maths a very practical hands on experience" (R39) and "Professional development programs gave me a whole range of strategies and activities to expand my understanding of mathematics" (R46). Respondent 57 explained how the EMIC professional development program (an ongoing program) had contributed to changes in her understanding and beliefs:

> Definitely my participation in the EMIC course with a wonderful tutor - A practicing primary teacher. She made it clear to me that children must learn by doing and I began to integrate mathematics into as many other areas of the curriculum as I could. She altered my mindset to this subject.

In-service days and shorter professional development sessions were also influential for the teachers. These were usually mentioned in conjunction with a presenter that the teachers felt was most effective. For example, "particular memorable challenging workshops, people including …"(R102) and "in-services taken by people like …(R38). Twenty-two prominent presenters of professional development were named as being particularly influential in this category.

Own School Experiences

Personal experiences as school students were mentioned by 32% of the teachers as being influential in their mathematics teaching. Seventeen percent of these participants described experiences that were both positive and negative. Positive school experiences were recounted by 51% of the teachers in this category. Respondent 3 recalled at secondary school, "... a year ten and eleven teacher who fostered a love of maths" while respondent

108 recalled that “One of my high school teachers would have to be one of the most influential teachers I have had. He took the time to go through every step/procedure involving maths and would make sure you understood what he meant”.

Those whose recollections of school experiences were positive spoke of their own enjoyment and love of learning mathematics in school and of teachers who allowed them to explore first, developed their understanding and who showed patience. Respondent 16 provided an indication of how school experiences influence her teaching when she wrote:

> My fifth form maths teacher … loved maths and his enthusiasm carried across to the class. He showed how even the most complex problem could often be presented graphically to make it more understandable. This was a long time ago but I still use this idea when I’m teaching problem solving maths.

Negative school experiences were referred to by 31% of the respondents in this category. Respondent 11 remembered “... being a remedial student of maths while in primary and secondary school” as a significant influence, while respondent 13 recalled her “... own feelings of inadequacy in maths at primary school and its effect on my further maths learning”. The longer term effects of negative school experiences in learning mathematics can be seen from respondent 22’s statement “I found Maths difficult in high school and realise now how much it affected my self-esteem”.

Recollections of negative school experiences were of feelings of inadequacy, fear of being thought “dumb” and low self esteem. An example of how difficulties during school days influence teaching today can be seen in respondent 32’s reaction to this question: “ I struggled in the area of maths and therefore don’t want my children to develop the same negative feeling toward maths”.

Colleagues

Working with colleagues in various ways was said to be influential for 29% of the participants. Mentioned in this category were teaching colleagues who passed on hints, shared ideas and were involved in team planning. Some teachers mentioned colleagues who had provided them with effective modeling of good practice and others, like respondent 12, spoke of the benefits of working with experienced teachers, “In my first year teaching the infant mistress took lots of maths lessons in my grade while I watched. I still remember and use many of her methods”. The positive effect of encouragement from other teachers was also considered an important influence.

Teacher Training Experiences

One quarter of respondents mentioned teacher training experiences as being influential in their teaching. For example:

> At teachers’ college my maths lecturer realised, although capable in other subjects, I obviously had a problem with maths. He took me through all the basics until I suddenly understood why I was doing each formula. I realised then that until basics are learnt, complex maths, cannot and should not be attempted. I also realised that basics can be relearned at any age (R22).

The teachers in this category stated that their teacher training had influenced their teaching in positive ways such as development of understanding of concepts that previously had been learned by rote. Many teachers referred to a particular lecturer who had made a difference.

Curriculum Materials

Textbook series and other print resources that provided information and activities involving current approaches, as well as curriculum support materials and the Curriculum and Standards Framework were regarded as influential by 19% of the teachers.

Experiences with Children

Learning from children, about how they think and learn, was mentioned specifically by 9% of the teachers. Some examples of teachers' descriptions of their learning are: "The children - as one gets to know them and listens to how they arrive at solutions or construct patterns one adapts to maximise and build onto previous understanding" (R5), "Grade 6 high achievers extended my mathematical teaching skills. They demanded variety, high interest levels and extended skill development" (R20), and "The children themselves have influenced the way I now teach maths. I am constantly more aware and amazed at the different ways children work through problems" (R41).

Other Influences

Seven percent named professional reading as significant for them, for example: "A variety of books dealing with current approaches to teaching of mathematics have widened my horizons of modern style of teaching of mathematics" (R19) and 4% wrote of the importance of their fathers as positive influences on their mathematics learning: "My father has a real bent for number and he always encouraged my sister and I as well as my brothers to look at problems and try to work them out for ourselves" (R17) and "My father was my best mathematics teacher" (R 29).

Discussion

Three factors arise as particularly significant is this analysis. These are that; teachers' professional learning is a lifelong process starting from their own school days, relationships with significant others were crucial for the development of these teachers, and reflections on their teaching experiences contributed to the teachers' development.

Lifelong learning

As the teachers looked back over their learning during their careers they were able to identify significant events that stood out as points of growth or change for them. The events dated back to their schools, related to the teacher development and occurred over years of teaching. Quite often these events had an emotional component such as the fostering of a love of maths by a parent or teacher, a determination that students should not struggle with maths, as the teacher had, or the encouragement of a colleague. Teachers described their gradual coming to know about teaching mathematics as a result of their years of working with and observing students. Planned professional development activities were also important in the growth of these teachers. Teachers recalled the impact of professional development programs which had they had attended years before.

From these teachers, we learn that professional development is part of lifelong learning and involves the development of new mindsets and attitudes, the development of insight into experience, and the taking of personal responsibility for learning about teaching.

Relationships

It is clear from their responses that interactions with significant others has had a considerable influence on the teachers in this study, and consequently on the mathematics education of the many children involved. The most consistent aspect of the development of these teachers is that their change experiences were person-centred. Sixty-six people were named as contributing positively towards change. Tutors, teachers, lecturers, PD presenters and colleagues were credited as catalysts for change. Professional growth for these teachers was most often linked to significant and particular people.

The teachers also spoke of the importance of their own school teachers and the experiences of their own schooling. School teachers were remembered because they provided good models and fostered an appreciation of mathematics. Other school teachers were so bad that they inspired a resolve in their pupils never to subject children to such experiences. Colleagues are referred to as important influences, who share ideas, model good practice, help with planning and programming, and provide encouragement. Teacher training experiences were influential mainly in the context of a meaningful relationship with the lecturer, rather than in terms of the content or methods that were learned. Associated with the relationship was a positive change in the teachers' attitudes to include the ideas that maths is fun or that they could be successful in mathematics

Reflection on Practice

These ongoing personal relationships combined with the learning that flowed from their experiences in teaching provided the teachers with a powerful medium for professional development. It was apparent, that the teachers reflected upon their experiences of teaching and learning mathematics. An important aspect of their ability to reflect on their experiences was the presence of a colleague, mentor or facilitator with whom to discuss their reflections.

Conclusions

Teacher professional development occurs in the context of lifelong learning with experiences over a long period of time contributing to teachers' knowledge of, beliefs about and attitudes towards mathematics teaching and learning This study suggests that professional development was perceived by the teachers to occur through interaction with significant others. Professional relationships, which seemed to have a personal component, and persisted over a period of time, were conducive to effective professional development for teachers. The personal interactions of the teachers in this study contributed towards the development of positive attitudes and changes in the teachers' beliefs about teaching.

The learning flowing from the experience of teaching provided a powerful medium for professional development. It was apparent from this study, that teachers reflect upon their experiences of teaching and learning about mathematics. This process was facilitated for those teachers who had participated in ongoing professional development programs. The results of this study suggest that teachers, such as those in this study, may benefit from professional development programs that value the understandings and affective factors that teachers bring to the task, and that help teachers to reflect on their experiences. Such programs will be most beneficial if they occur within a supportive social context.

Three issues stand out as cause for reflection on and further study by teacher educators. Firstly, only seven percent of the teachers admitted to engaging in professional reading

relating to their teaching of mathematics. This relates perhaps to their conception of professional learning as a social activity rather than a solitary one. Secondly, it was significant to note that only nine percent of the teachers specifically mentioned learning from children as a source of professional growth, although the idea was mentioned more generally in other responses. These two facts suggest that there is potential to further enhance the teachers' abilities to learn from their experiences, in contexts where their experiences are related to relevant professional reading. The research here suggests that there is scope for teachers to increasingly take personal responsibility for generating and refining their understandings of the classroom situation. A third issue of importance in teacher development is that only 25% of the sample mentioned teacher training as an influence on their development. This raises the question of whether teachers had just forgotten their training experiences or were they really insignificant for three quarters of the sample. Given the huge investment in teacher training this issue is one that requires further examination.

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