Assessing Primary Students' Knowledge of Networks, Hierarchies and Matrices using Scenario-Based Tasks

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This paper investigated primary students' (N = 127) knowledge of the properties of Networks, Hierarchies, and Matrices using a set of scenario-based tasks. Firstly, the results revealed differences in students' knowledge of the various properties for each diagram, and differences in their knowledge of some properties across the various diagrams. Secondly and unexpectedly, the performance of older students' was not significantly higher than younger students. Implications for instruction about diagrams and avenues for future research are discussed.

Mathematicians have long recognised the value of diagrams as cognitive tools and the use of diagrams has resulted in major mathematical breakthroughs, such as the Pythagorean discovery of irrational numbers (H. Simon, 1995). However, in the Information Age, all citizens, not only those who pursue mathematically-oriented careers, need to be diagrammatically literate in order to create and interpret various data representations. Diagram literacy involves knowing about diagrams and being able to use that knowledge in various mathematical situations (Diezmann & English, 2001). Diagrams are an important cognitive tool in mathematics because they: facilitate the conceptualisation of the problem structure (van Essen & Hamaker, 1990); provide a wholistic representation of information (e.g., Novick, 2001; Tufte, 1997, Winn, 1987); support inference-making (Lindsay, 1995); and foster visual reasoning, which is complementary to sequentially-oriented reasoning (Barwise & Etchemendy, 1991). Three particularly useful diagrams in mathematics which have unique spatial structures and have broad applicability in mathematics are the Network, Hierarchy and Matrix (Novick, Hurley & Francis, 1999). Henceforth, these diagrams are referred to as spatially-oriented diagrams.

Instruction in Diagram Use

Although there has been a dramatic upsurge in diagrammatic research and theory development across a range of disciplines (e.g., Glasgow, Narayanan & Karan, 1995), there have been repeated calls for research and theory development to inform instructional practice in school mathematics (e.g., Diezmann, 1999; Shigematsu & Sowder, 1994; Yancey, Thompson & Yancey, 1989). Students need explicit instruction in diagram use because they are reluctant to employ diagrams (e.g., M. Simon, 1986); lack the expertise to use diagrams effectively (e.g., Dreyfus & Eisenberg, 1990); and experience difficulties in diagram use that inhibit rather than facilitate their problem solving performance (Diezmann, 1995; 1999). As the selection of an appropriate representation is a critical step in reasoning about information (Novick, 2001), effective instruction in diagram use needs to address students' knowledge of the properties of diagrams. This instruction should commence in the primary years of schooling because the ability to interpret visual representations, such as diagrams, is fundamental to numeracy (Department for Education and Employment, 1998).

Distinguishing Properties of Spatially-Oriented Diagrams

Recently, a cohesive framework of 10 properties of spatially-oriented diagrams has been proposed and tested (Novick, 2001; Novick & Hurley, 2001). Novick and Hurley confirmed the existence of these properties with college students using a set of scenario-based tasks. However, they found that only six of the ten properties were sufficiently discrete to be readily investigated (see Table 1, Column 1). Each of these properties differs according to the particular spatially-oriented diagram (see Table 1, Columns 2-4). These scenario-based tasks used a common context (e.g., medical scenarios), to avoid students selecting their responses on the basis of the cover stories rather than on the structural information. The first sentence or two of the task set up a cover story. The next sentence or two focused on a particular property of a diagram. The final sentence indicated that someone wanted a diagram for a purpose relevant to the cover story. The students were then asked to (1) select the diagram best suited to the story from two diagrams and to (2) justify their selection and (3) non-selection of particular diagrams. This research with college students provides a model for investigating primary students' knowledge of the properties of spatially-oriented diagrams, and hence, has the potential to inform instructional practice. Thus, the purpose of this paper is to ascertain how assessment of primary students' knowledge of the properties of spatially-oriented diagrams can inform instruction.

Table 1
Properties of Spatially-Oriented Diagrams

Properties of Diagrams	Network	Hierarchy	Matrix
1. Global structure: the general form	lacks formal structure	an organisational structure	a factorial structure
2. Number of sets	one set of information	no limit on sets of information	ideally two sets of information
3. Item/link constraints: how items link together	no constraints	organizational structural constraints	factorial structural constraints
4. Link type: links between items are best conveyed by a particular diagram	flexible links	directional links	associative non-directional links
5. Linking relations: one-to-many links, many-to-one links or both	both linking relations	either linking relation but not both	not salient, but can have both linking relations
6. Transversal: the possible paths	multiple paths connect item "A" and "B"	only one path connects items "A" and "B"	paths are not relevant

Design and Methods

This paper reports on an investigation into primary students' knowledge of six properties of spatially-oriented diagrams. It is part of a larger study on primary students' knowledge of these properties and the influences on the development of that knowledge. The larger study employed an accelerated longitudinal design (Willett, Singer & Martin, 1998) in which two differently-aged populations are being studied for a three-year period. The advantage of the accelerated longitudinal design is that it shortens the length of time for longitudinal research by tracking differently-aged cohorts over a relatively short time period (Willett et al., 1998). The investigation reported here focuses on the performance of two cohorts of primary students in the first year of the larger study. The two research questions investigated are:

1. Which properties of spatially-oriented diagrams contribute to primary students' understanding of particular diagrams?
2. Do older primary students possess more knowledge of properties of particular spatially-oriented diagrams than younger primary students?

Participants

A total of 127 students from a large metropolitan primary school with a mixed demographic population participated in this investigation. Cohort A (n = 67) and B (n = 60) comprised students who were in Grade 3 (approximately 8-9 years) and Grade 5 (approximately 10-11 years) respectively in the first year of the larger study. These participants comprised all students at these year levels whose parents or guardians gave permission for them to participate.

Properties of Diagrams Measure

Primary students' knowledge of the properties (i.e., *Global Structure, Number of Sets, Item/Link Constraints, Link Type, Linking Relations, Transversal*) of spatially-oriented diagrams was measured using a set of scenario-based tasks based within the context of an Amusement Park. This context was selected because it is of interest to primary students and provided scope for the development of a broad range of tasks. Fifteen Amusement Park tasks were produced to test students' knowledge of up to six properties of diagrams for the Network, Hierarchy and Matrix (see Table 2). These tasks were developed from the theoretical descriptions of the properties of spatially-oriented diagrams and modelled on scenario-based tasks for college students (Novick, 2001; Novick & Hurley, 2001). No tasks were produced for three of the property and diagram combinations due to design difficulties. The six Network tasks that tested students' knowledge of each of the six properties of spatially-oriented diagrams are presented in the Appendix. The specific property investigated with those tasks is shown in brackets after the title. The property-focusing sentence has been underlined for illustrative purposes but was not underlined for students. Below each Amusement Park task, the following sentence was presented: "Which type of diagram do you think would best show the information given?" Students were then asked to select the appropriate diagram from two labelled diagrams (Matrix or Network or Hierarchy). In one of these diagrams, the property was represented, and in the other diagram the property was not represented. Only two (correct/incorrect) spatially-oriented diagrams were presented for each scenario. Each pair of diagrams was featured a similar number of times. The chance factor (.50) in the selection of the "best" diagram was addressed in the larger study by requiring the students to justify their selection and also to explain why they did not select the remaining diagram. Students' justifications are beyond the scope of this paper but are discussed elsewhere (Diezmann, in press). Two versions of the set of 15 tasks were produced with tasks counterbalanced to minimise the effect of the order of tasks on the results.

Table 2
Amusement Park Tasks that Test Knowledge of the Properties of Networks, Hierarchies and Matrices

Properties	Network Tasks	Hierarchy Tasks	Matrix Tasks
Global structure	*Lost Property*	*Amusement Park Activities*	*Lunch Orders*
Number of sets	*Final Attraction*	NA	*Amusement Park Shows*
Item/link constraints	*Friends*	*Playground Video*	*Free Vouchers*
Link type	*Illness*	NA	*Sandwich Bar*
Linking relations	*Clown Show*	*Animal Trainers*	*Visitor Guides*
Transversal	*Adventure Rides*	*Helpful Staff*	NA

Data Collection and Analysis

Data on students' performance on the 15 scenario-based tasks were collected over two individual interviews to avoid undue fatigue. Students' performance on these tasks was scored as "1" for a correct response and "0" for an incorrect response. Each student's responses were marked independently by two scorers with an inter-rater reliability of 99.9%. Scorers subsequently reconciled disagreements in scores. To investigate students' knowledge of the properties of specific diagrams (i.e., Research Question 1), the frequency of each cohort's performance on these tasks was analysed using binomial tests. To examine relatedness of student age to the properties of specific diagrams (i.e., Research Question 2), chi-square tests (comparing Grade 3 and Grade 5 performance) were used.

Results and Discussion

The findings for the two research questions are discussed in turn.

1. Which properties of spatially-oriented diagrams contribute to primary students' understanding of particular diagrams?

The binomial analyses compared the observed frequency of students selecting the correct diagram against chance (.50). Table 3 presents the results of these binomial tests for the properties of each diagram for both Grade 3 and Grade 5. Included in the table are the observed proportions of correct scored (in italics), the significance levels (two-tailed) of the bionomial tests (in parentheses), and symbols indicating whether the observed proportions were significantly greater than (>), less than (<) or not significantly different (=) from chance. The symbol "NA" is used in the table to indicate where results were not applicable due to the absence of a task for a particular property and diagram combination.

Students' ability to select the correct diagram to represent a particular property varied according to the property, the diagram and the age cohort. On the *Global Structure* property for the Matrix task, both cohorts' performances were greater than chance (.50) as was Grade 3 students' performance on the Network task (see Table 3). Grade 3 and Grade 5 students' performances on the Hierarchy task and Grade 5 students' performance on the Network task of only equal to chance were lower than expected. The *Global Structure* property should reflect the general form of the information contained in each cover story (see Appendix). On the *Transversal* property for the Network and Hierarchy tasks, both cohorts' performances were greater than chance. Similarly, on the *Link Type* property for the Network task, both cohorts' performances were greater than chance. Conversely, on the *Linking Relations* property for the Hierarchy task, the Grade 5 students' performance was greater than chance, however the Grade 3 students' performance was only equal to chance. On the *Number of Sets*, *Item/link Constraints* and *Link Type* properties for the Matrix tasks, both cohorts' performances were greater than chance.

Table 3

Students' Performance on Each Property for Particular Diagrams Relative to Chance

Properties	Network		Hierarchy		Matrix	
	Grade 3	Grade 5	Grade 3	Grade 5	Grade 3	Grade 5
Global Structure	> *.70* (.001)	= *.48* (.897)	= *.40* (.14)	= *.55* (.519)	> *.81* (.000)	> *.67* (.014)
Number of Sets	= *.49* (1.0)	= *.52* (.897)	NA	NA	> *.78* (.000)	> *.72* (.001)
Item/link Constraints	< *.28* (.001)	= *.38* (.093)	= *.61* (.087)	= *.53* (.699)	> *.70* (.001)	> *.78* (.000)
Link Type	> *.90* (.000)	> *.85* (.000)	NA	NA	> *.70* (.001)	> *.78* (.000)
Linking Relations	= *.46* (.625)	< *.30* (.003)	= *.60* (.143)	> *.65* (.028)	= *.58* (.222)	= .62 (.093)
Transversal	> *.64* (.028)	> *.75* (.000)	> *.64* (.028)	> *.78* (.000)	NA	NA

These results indicate that the *Transversal* and *Link Type* properties appear to be the best indicators of students' understanding of spatially-oriented diagrams because, in the Network and Hierarchy and Network and Matrix respectively, the students' performance was greater than chance (.50) for both cohorts. The *Transversal* property was not tested for the Matrix nor was the *Link Type* property tested for the Hierarchy. The similarity in students' performance on the *Transversal* property for the Network and Hierarchy tasks is not surprising given that a Hierarchy is a specialised form of Network. However, the reason for the similarity in students' performance on the *Link Type* property for the Network and Matrix tasks is unclear and needs to be investigated.

2. Do older primary students possess more knowledge of the properties of particular spatially-oriented diagrams than younger primary students?

To examine the relatedness of student age to properties of specific diagrams chi-square tests were conducted. These results revealed two points of interest. Firstly, a significant difference was only reached on one of the fifteen tasks, namely the *Global Structure* property for the Network task (χ^2 (1, N = 127) = 6.27, p = .012). Secondly, the direction of this difference was not as expected with Grade 3 students (n = 67; 47 correct responses) outperforming Grade 5 students (n = 60; 29 correct responses).

These findings suggest that the two additional years of schooling for Grade 5 students did not enhance their knowledge of the properties of these diagrams. Cohort or time period effects (Willett et al., 1998) provide possible explanations for the lack of significant difference in performance in favour of Grade 5 students. For example, the Grade 5 cohort might not be as capable as the Grade 3 cohort (i.e., cohort effect). Additionally, the Grade 5 students might have recently been taught something that interfered with their knowledge of the properties of diagrams (i.e., time period effect). For example, students might have been taught about a co-ordinate grid, which is visually similar to a matrix, but conceptually dissimilar.

Conclusions and Implications

This assessment of students' knowledge of the properties of particular diagrams informs instruction about diagrams in five ways. Firstly, both cohorts' performance of equal to or less than chance (.50) on approximately half the tasks and the lack of significant difference in favour of <u>older</u> students on all tasks suggest that there is a need for explicit instruction in diagram use. Secondly, the differences between students' performances on various properties of the same diagram, and four of the six properties across diagrams (i.e., *Global Structure*, *Number of Sets*, *Item/link Constraints*, *Linking Relations*) suggest that diagrammatic knowledge is complex. Hence, it is fallacious to assume that students' knowledge of one property for a particular diagram indicates either knowledge of all properties of that diagram or knowledge of the same property across all diagrams. Thus, the curriculum needs to include attention to the various properties of each spatially-oriented diagram. Thirdly, because performance on the *Transversal* and *Link Type* properties appears to be robust across the tested diagrams and age cohorts, these properties may be foundational to developing students' knowledge of particular diagrams and might be easiest to grasp. Fourthly, because overall performance on the properties of the Matrix is superior in both cohorts to their performances on the properties of the Network and Hierarchy, the Matrix might be the easiest starting point for students with limited diagram knowledge. Finally, a set of scenario-based tasks can be used for diagnostic or strategic teaching purposes to identify which properties students know and don't know for each of the spatially-oriented diagrams.

Six avenues for further investigation have emerged from this study. The first two relate to subsequent data collection and analysis in the larger study. The latter four avenues relate to future studies. Firstly, there is a need to monitor the performance of the cohorts over time to establish whether the results of this investigation are reflected in trend data. Secondly, there is a need to explore the reasons for students' selection and non-selection of particular diagrams to gain insight into why students' performance is above, below or equal to chance (.50). Preliminary work on students' justifications has been reported elsewhere

(Diezmann, in press). Thirdly, as indicated earlier, there is a need to investigate the similarity in students' performance on the *Link Type* property for the Network and Matrix. Fourthly, there is a need to replicate this study to confirm these results. Cohort or time period effects could have impacted on the results of this investigation (Willett et al., 1998). Fifthly, the study could be further strengthened by the development and testing of isomorphic tasks to validate the Amusement Park tasks. Finally, proposals that inform instruction, such as commencing diagrammatic instruction with Matrices, need to be tested.

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Appendix

Lost Property (Global Structure) People who have lost something at the Amusement Park can visit the Lost Property Office to check if their items have been handed in. When someone asks about a lost item, the workers check the Lost Property shelves, but they do not keep a list of what has been handed in. <u>People might talk to different workers about their lost items if they visit the office more than once.</u> The Manager of the Lost Property Office wants a diagram showing all the people who spoke to Mr Jones about his lost glasses.	**Clown Show** (Linking Relations) The Show Manager at the Amusement Park has bought a little car for the new Clown Show. The Manager has decided that the funniest clown should drive the little car. <u>Each clown has been asked to watch at least two clowns do their best tricks and vote for the funniest clown. Each clown must be watched by at least two other clowns.</u> The Show Manager would like a diagram showing which clowns watched each others' acts.
Final Attraction (Number of Sets) Students and adult helpers from Smithtown School only had enough time left after lunch to visit one more attraction at the Amusement Park. <u>So during lunch, all the students and adults chatted with each other to try and decide which other exciting attraction they each wanted to visit.</u> The Organiser of the school trip would like a diagram showing which students and adults chatted to each other about the Clown Show.	**Friends** (Item/link Constraints) The classroom teacher is organising the children into pairs for their trip to the Amusement Park. <u>She has found that the children are happier when they are with a friend. Although the teacher could put any of the children together, whether they are friends or not, she wants the children to be happy during the trip.</u> The teacher would like a diagram showing which children are friends with each other.
Adventure Rides (Transversal) The Gold Ticket to the Amusement Park allows children to go on many different adventure rides. <u>They must begin with Space Traveller and end with Wild Water, but they can choose the order of their other rides.</u> The Ride Manager would like a diagram showing all the possible choices that the children might make about the order of their rides.	**Illness** (Link Type) A ride operator at the Amusement Park became ill while he was at work. <u>He may have infected other ride workers. These workers, in turn, may have infected other people.</u> The Health Director at the Amusement Park would like a diagram showing which people might give the illness to other people.