Essential Complementarities: Arguing for <u>an</u> Integrative Approach to Research in Mathematics Classrooms

The Annual Clements / Foyster Lecture

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Dichotomies such as teacher-centred versus student-centred classrooms, real-world versus abstract tasks, and even teaching versus learning can restrict mathematics educators and educational theorists in general to a fragmented view of the mathematics classroom. Constructing such dichotomies as oppositional offers a set of false choices, sanctifying one alternative, while demonising the other. International research offers insight into possible explanatory frameworks within which such choices are no longer oppositional or even dichotomous, but rather can be seen as complementary. The acceptance of such complementarities is a first step to an integrative theory of classroom practice and learning.

This paper constitutes the main text of the inaugural Clements/Foyster lecture. As the co-founders of an organization with the unifying acronym "MERGA", I would like to think that Ken Clements and John Foyster might approve of the theme of the paper. My main argument is that education, and mathematics education in particular, in attempting to make sense of the world of the classroom, has pursued the established Western tradition of dichotomising all aspects of our experience. This tendency was most explicit in the division of spirit and matter by Greek philosophers of the fifth century BC and became entrenched in the Cartesian division of body and mind. Much more recently, authors such as Fritjof Capra (1976) have drawn Western attention to Eastern philosophies in which the basic assumption is unity and interdependence, and yin and yang are seen correctly as complementary aspects of an essential unity.

Introduction

My contention in this paper is that it is in the examination of classrooms across a variety of cultural settings and school systems that we find our educational assumptions most visible and open to challenge. With the growing internationalisation of education, and as the education community gives higher priority to international research, it is timely to examine the insights that accrue from comparative analyses of classrooms that are situated in very different cultures. The contrasts and unexpected similarities offered by research in such culturally-diverse settings reveal and challenge existing assumptions and theories and make essential a reconstruction of some of our most basic dichotomies as complementary elements in more integrative theories. This questioning of the permanency of pervasive binary opposites is central to the 'deconstructive' stance adopted in this paper. The Derridean idea that language constructs difference is at the heart of both this paper and the postmodern position that "we live, not inside reality, but inside our representations of it" (Butler, 2000, p. 21).

International research in mathematics education has provided us with a wealth of detail about student achievement levels, curricular content, prevalent problem types, teacher beliefs, class size, lesson duration, homework, textbooks, teacher question types, utilisation of real-world contexts, and, more recently, fine-grained analyses of classroom practices and interactions. The descriptive documentation of similarity and difference (Clarke, 2003a and b) can only take us so far. The diversity that we find in international studies of mathematics classrooms provides us with a base from which to interrogate our own practices and the assumptions on which those practices are predicated.

Among the most central of these assumptions are various dichotomous categories that act to constrain our theorising about educational settings and the processes of interest there. In this paper, I address only four of these dichotomies in any detail: Teaching and Learning; Abstract and Contextualised; Teacher-Centred and Student-Centred; and the teacher's contemporary dilemma – To Tell or Not to Tell. Very simply: these are false dichotomies. It is my contention that unless we can integrate each pair of categories as complementary elements of a more inclusive theoretical framework, we will remain unable to account for the diversity we find in international studies of classroom practice. It is precisely the growing body of data from such international studies that provides us with the diversity that we need to interrogate and refine our current theoretical position with regard to classroom practice. In the discussion that follows, I attempt to demonstrate the potential value of such an integrative approach.

The integrative approach adopted in this paper has been variously inspired. For example, Bourdieu (1990) has argued that it is a mistake to see individuals as somehow located in a social structure that is external to them. Rather they are part of that structure, and the structure is part of them. In that sense, learning is not just socially-mediated it is fundamentally social in character and the patterns of social participation that provide much of the substance of this paper both facilitate the participants' learning and embody it. There are other dichotomies that I will not address here. Vygotsky decried the dichotomisation of "intellect and affect," when he wrote: "Their separation as subjects of study is a major weakness of traditional psychology" (Vygotsky, 1986, p.10). Also, it has already been argued persuasively by Cobb, Svard and others, that if we are to move forward, we must *conceive of* socio-cultural and constructivist theories of learning not as competing but as complementary (Cobb, 1994; Sfard, 1998, 2000). That they can be constructed so as to be in competition is evident. Each theoretical frame provides coherent accounts and explanations for particular forms of learning in particular settings. Any conception of either theory that precludes the other is arguably inadequate. The identification (construction) of a theory of learning compatible with a given situation may take the social or the individual as its starting point but ultimately will be obliged to make appeal to the other if a coherent account is to be constructed. This is aligned with Confrey's conception of the self as both autonomous and communal (Confrey, 1995). Complementarity within an expanded theoretical framework provides a suitably integrative resolution of the concerns with these dichotomies. For a more detailed argument of the benefits of an integrative approach, I move now to the first of the four dichotomies to be addressed in this paper.

The Reification of Fundamental Dichotomies: Teaching and Learning

Learning and teaching represent the most fundamental and pervasive dichotomy around which our understandings of classroom practice have been constructed. Stepping outside the constraints of culture and language, we find that this central distinction is conceived very differently by differently communities. In fact, the distinction between teaching and learning is very much an artefact of language.

Previous research, and much of our theorizing, has tended to dichotomise teaching and learning as discrete activities sharing a common context. I have argued elsewhere (Clarke, 2001) that this dichotomisation is a particularly insidious consequence of the constraints that language (and the English language, in particular) imposes on our theorising and that such dichotomisation misrepresents both teaching and learning and the classroom settings in which these most frequently occur. There is no intention to challenge the separate integrity of “teacher” and “learner” as labels for individuals engaged in particular practices or discourse modes. It is just that classrooms are more effectively understood as sites for bodies of mutually-sustaining practice that in combination characterise a process we might call (in English) “teaching/learning”.

The consequences of choosing not to dichotomise teaching and learning are far-reaching. Perhaps the most compelling illustration of the dangers of dichotomisation can be seen in the comparison of two translations of the same paragraph by Vygotsky.

> From this point of view, instruction cannot be identified as development, but properly organized instruction will result in the child's intellectual development, will bring into being an entire series of such developmental processes, which were not at all possible without instruction. (Vygotsky, 1982, p. 121, as quoted in Hedegaard, 1990, p. 350)

Compare this with the following translation.

> From this point of view, learning is not development; however, properly organized learning results in mental development and sets in motion a variety of developmental processes that would be impossible apart from learning. (Vygotsky, 1978, p. 90)

The pivotal assertion that must be understood is whether Vygotsky was asserting the impossibility of certain forms of intellectual development “without instruction” (which presumes an actively interactive more competent other) or “apart from learning” (which on one level seems a tautology, but which could also be interpreted as equivalent to the assertion that properly organized interaction with the environment is essential for certain forms of development to occur). This distinction is non-trivial, since it calls into question the significance of the mediation of another more able individual (the teacher/instructor). Given what we know of the significance Vygotsky attached to the role of the teacher, it would appear that the most appropriate reading of the major premise is “a variety of developmental processes would be impossible without instruction.” This accords with the significance attached, in the passage quoted below, to the child’s interaction with “people in his environment” rather than just with all aspects of that environment, with or without the mediation of others.

The ‘conflicting’ translations arise because of a duality of meaning in the original term employed by Vygotsky. This duality has been noted previously, but its significance seems to have been given scant consideration in the interpretation and application of Vygotsky’s work. As we have seen in the two translations above, the same term (“obuchenie”) is translated both as “instruction” and as “learning” and clearly shares with corresponding terms in other languages the capacity to invoke both teaching and learning, as these are named in English. Once this duality of meaning is recognized, our reading of Vygotsky and our theorizing about the teaching/learning process are greatly enriched. For example, in one of the most famous passages from the translated Vygotsky, the word “learning” can be replaced by the word “teaching” and the resultant text is still meaningful – but, perhaps, with a different meaning.

> We propose that an essential feature of learning [*teaching*] is that it creates the zone of proximal development; that is, learning [*teaching*] awakens a variety of developmental processes that are able to interact only when the child is interacting with people in his environment and in collaboration with his peers. (Vygotsky, 1978, p. 90)

If our framing of “instruction” in language presumes a complicit “learner,” whose “learning” is inextricably entwined with an “instructive” setting, then our interpretations of the activities of the classroom are more likely to identify communal practices and the progressive participation in a common discourse as essential features, than to fragment the classroom into teaching and learning activities undertaken separately by individuals.

Speakers of Russian are not alone in their use of a term that combines both teaching and learning. In Japanese, "tagushushido" combines teaching and learning in the same way. In Dutch, there is one term that means both learning and teaching: "leren". To distinguish between the practices of teaching and learning, the Dutch say "leren van" to signify "learning" and "leren aan" to signify "teaching". In French, the term "didactique", and particularly Brousseau's use of that term (Brousseau, 1996), invokes a mutuality of responsibility and participation not always found in American or Australian interpretations of the classroom.

In the middle of the last century, the biologist von Uexhull put forward the proposition that a spider's web is the spider's model of the fly. This whimsical imagery conceals a powerful reasoning technique similar to reverse engineering. From the structure of a spider's web: the spacing and strength of the strands, the location and size of the web, and from other characteristics of the spider's web, we can deduce much about the fly. Classrooms are a little like the spider's web. From the way in which a teacher structures the classroom (and the practices for which it is the setting), we can infer much about that teacher's (and that society's) model of the student. The types of resources provided, the type and duration of the various activities, the forms of interaction that are encouraged and discouraged, all offer insight into the teacher's conception of what Steffe called "the epistemic student", the student as constructor of knowledge. Within the confines of accepted practice and available resources, teachers attempt to construct classrooms to afford and constrain particular activities. What Brousseau (1986) has brought to our attention is the reciprocality of the construction of classroom practice. Learners (that is, students) engage in practices that afford and constrain teacher actions, and the actions of their classmates. Social interaction by an individual within the classroom presumes that the individual has a model of the other classroom participants and can, to some extent, anticipate their capabilities, their needs, their expectations and their responses. What is clear is the extent to which classroom practice is a jointly constituted body of negotiative social interactions that is best investigated and understood in terms of the mutuality and reciprocality of its constituent activities and of its co-construction as Teaching/Learning.

Empirically, the integration of Teaching and Learning has been addressed in analyses of patterns of participation in mathematics classrooms in a variety of countries as part of the Learner's Perspective Study (LPS). In particular, the classroom practice that I have designated elsewhere by the Japanese term "Kikan-Shido" (or "Between-Desks-Instruction") has provided a powerful example of a "whole class pattern of participation" (Clarke, 2004). In making the claim that Kikan-Shido could be so described, I needed to demonstrate that it had a recurrent form, recognisable to those participating in it. This is not to say that the meanings attributed to the activity by those participating in it were correspondent. Individuals can participate in a practice whilst being positioned differently within it, and whilst attributing different characteristics to the activity. That is, without being identical, the participants' descriptions of the activity make it clear that they are talking about essentially the same form, but they may attribute quite different functions to that form. The other essential element is the need to demonstrate that all participants can shape the particular body of practice signified by Kikan-Shido. That is, that the pattern of participation is co-constructed.

Without reproducing the argument in full here, any theory of classroom practice must conceive of the activities in the classroom as co-constructed. Kikan-shido as it has been reported (Clarke, 2004, and O'Keefe & Clarke, 2004) is clearly a dance done by teachers and students, where the steps are improvised according to need. The participants in the classroom, teacher and students, are complicit (co-conspirators) in this improvisation. Any characterisation of classroom practice must encompass the complementary actions of teacher and students within a single integrative frame. Acceptance of this point has implications for the research designs by which we study the activities occurring in classroom settings.

Dichotomies of Task: Context and the "Relevance Paradox"

Suppose that one society seeks to develop understanding and proficiency in mathematical proof, attaching significance to the development of those forms of reasoning and argumentation idiosyncratic to mathematics, while another attaches greater priority to equipping its people with an understanding of mathematical procedures and proficiency in utilising these in everyday practical situations, while a third society emphasises (and rewards) concept development, mathematical creativity and collaborative problem solving. There is no reason why these goals are incompatible or mutually exclusive, but they do reflect a valuing of different aspects of mathematical activity, and a curriculum that prioritised one such goal would not necessarily resemble a curriculum that prioritised another. The evaluative comparison of the consequences of such differently targeted curricula (as in international studies of student achievement) is a problematic exercise, whereas the comparative study of the methods and success of each society in addressing its local curricular goals has the potential to be mutually enriching as one community learns from the practices of the other and adopts and adapts some of its goals and methods for local use.

> Many countries, especially Korea and the Netherlands, emphasised solving problems . . . Japan, Sweden, and the United States emphasized 'recalling' mathematical information, and Hong Kong and Israel emphasized 'justification and proof'. (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997, p. 136)

In a recent analysis of LPS data from Sweden and China, Katja Svan examined the "Relevance Paradox" postulated by Mogens Niss (1994), in which the objective relevance of mathematics in society was contrasted with its subjective irrelevance as perceived by many students. Svan was not comparing 'mathematics teaching' in Sweden and China, but rather looking at the beliefs and values communicated and held in two very different classrooms: one in Shanghai and one in Uppsala. Both classrooms were addressing the same mathematics topic (coordinate systems and graphing linear functions).

Svan's analysis contrasted the Chinese and Swedish mathematics classrooms from the perspective of the emphasis given by the teacher and the students to the real-world relevance of the mathematics being learned. In the Swedish classroom, the students demanded that the teacher justify the relevance of what was being taught, and the teacher provided lengthy justifications on several occasions. It was clear that the Swedish teacher felt that the demonstration of relevance was a reasonable expectation and accepted responsibility for providing this. Despite the teacher's efforts, students were outspoken in their lack of belief in the relevance of the mathematics they were studying. Both the Swedish classroom data and post-lesson interview data seemed to provide a powerful illustration of Niss's relevance paradox.

By contrast, in the classroom in Shanghai, mathematics tasks tended to be very abstract in character and the teacher made no effort to demonstrate or argue for the real world applicability of the mathematics being studied, and the Chinese students did not appear, either during the lesson or in interview, to require this sort of justification of the content being studied. However, in the post-lesson interviews, the Chinese students expressed consistently strong beliefs in the utility of mathematics in general and in relation to the specific mathematics they were studying. One Chinese student said:

> I think basically, I should grasp the fundamental points that are necessary for students and also I have to use these points in my everyday life. (Shanghai School 1, Lesson 4, post-lesson student interview)

Svan concluded that analysis of the interviews with 15 of the Chinese students showed that there was a shared belief that mathematics was useful not only in future work and study, but also in their current everyday lives. It is not clear how the students developed those beliefs as they were not introduced to anything but abstract mathematics during the lessons.

Svan has christened this the “Expanded Relevance Paradox” (Svan & Clarke, in preparation) and means, by this term, to refer to the paradoxical character of application-oriented mathematics teaching associated with subjective irrelevance and pure mathematics-oriented mathematics teaching associated with subjective relevance.

In summation: The majority of the tasks in the Swedish classroom were ‘word problems’ and involved contexts from everyday life, more or less relevant to the students. Despite the teacher’s very public commitment to demonstrating the relevance of the content, the students strongly questioned its utility. The students in the Shanghai classroom experienced teaching and tasks that focused on abstract mathematics, yet the students appeared quite certain of the immediate and future relevance of the content.

Clarke and Helme (1998) identified the importance of recognising context as a social construction, and distinguished the ‘Figurative Context’ invoked by the task from the ‘Social Context’ in which the task was undertaken. As reported by Clarke and Helme, students appear to attend to the figurative context to different degrees.

> Context in our view is neither a neutral background for the negotiation of mathematical meanings, nor merely a catalyst mediating between task content and the individual’s mathematical tool kit. Rather we should speak of the personal task context as an outcome of the realization of the figurative context within the broader social context. (Clarke & Helme, 1998, p. 130)

There is a recent commitment in South Africa to contextualising the curriculum around themes of societal significance, such as substance abuse or HIV-AIDS. Analysis of student-student interactions in the South African classrooms studied in the LPS project, led Sethole, Adler and Vithal (2002) to conclude:

> The context AIDS, is not understood as a ‘veneer’ to mask the mathematical intentions of the lesson but a genuine context to be engaged. To this end, and drawing from Skovsmose’s notion on critical mathematics, the new practice may be seen as an inescapable consequence of blurring the boundary between the mathematics and the everyday. (Sethole, Adler, & Vithal, 2002, p. 11)

The Relevance Paradox proposed by Niss (1994) is based on a dichotomisation of the function of mathematics in society and in the classroom, and postulates a dislocation between these two contexts that is experienced by students as a lack of connection (subjective irrelevance). The LPS data set problematises this schism in two startling ways: firstly, Chinese students appear to have constructed the missing connection independent of explicit classroom modelling or advocacy by the teacher; and, secondly, the South African initiative removes the need for connection by dissolving the distinction between the classroom and the everyday. In the terms employed by Clarke and Helme, the distinction between the figurative and the social, always tentative, has been effectively dissolved in China, through a perspective in which the significance of classroom activity derives from its situation in a broader cultural context that does not require re-fabrication at the local level of the classroom, and also in South Africa, where the minutae of mathematical content are subordinated to a macro-social agenda that reconstructs the nature and purpose of classroom activity in socio-cultural rather than solely mathematical terms. Resolution in both settings arises from avoidance of the dichotomisation of real-world and school mathematics, by viewing students as simultaneously members of complementary communities of practice within a broader integrative socio-cultural context.

Reconceiving the Teacher-Centred/Student-Centred Dichotomy

Moving now to the third of the dichotomies. Popular in recent educational literature as descriptors of classroom practice are the terms ‘teacher-centred’ and ‘student-centred.’ These terms vary in definition and in use, but they represent a key dichotomy driving much of contemporary Western educational (particularly pedagogical) reform. From one perspective, they appear to offer mutually exclusive alternatives with regard to the location of agency in the classroom. Western educational reform advocates student-centred classrooms, and research in Western settings confirms the value of practices associated with these classrooms (Chazan & Ball, 1997; Clarke, 2001).

For example, Clarke (2001) provided examples of student-student interactions that demonstrated the potentially significant role that students might play in the collaborative generation of knowledge in the mathematics classroom.

> A feature of Karen’s role in the Lauren/Karen dyad was to pose questions of Lauren and of herself. Some evidence can be found to suggest that Lauren was the more mathematically capable student. Nonetheless, the successful culmination of the dyad’s problem solving efforts must be attributed, in part, to the Karen’s persistent framing of task-related questions. The effectiveness of such self-scaffolding as a component of dyadic problem solving will derive significantly from the appropriateness of such questions and the extent to which one learner attends to the questions (and other contributions) of the other. (Clarke, 2001, p. 310)

In a parallel analysis of student cognitive engagement, Helme and Clarke presented evidence for the significance of student-student interactions in promoting high-level cognitive engagement and consequent learning.

> We would argue that student-student interactions appeared to offer more scope for high-level cognitive engagement than teacher-student interactions, both in whole-class instruction and in interactions with small groups. (Helme & Clarke, 2001, p. 191)

On the basis of this evidence, student agency for knowledge generation was accorded a high level of significance in the Australian classrooms analysed in this study (Clarke, 2001), and the results of this study could be interpreted as providing further support for the advocacy of the “student-centred” classroom, a key element in the recent reform agenda of most Western educational systems.

By contrast, Asian classrooms have been typified as teacher-centred by both Western and Asian researchers, yet the students in these classrooms are highly successful in international studies of student achievement (‘The Asian Learner Paradox’) (Leung, 2001). Recent research in Chinese classrooms suggests that classroom practice is misrepresented by such a dichotomy (Huang, 2002) and that a theoretical framework is needed by which the ‘teacher-centred’ and ‘student-centred’ characteristics of classrooms can be more usefully characterised and investigated, without the assumption of an absolute dichotomy.

> “How can teacher dominance and student-centeredness coexist and work well in Chinese mathematics classrooms?” (Huang, 2002, p.226).

There is general assumption in most literature that classroom discourse encompasses any form of interaction that takes place in a classroom. Nevertheless, research involving classroom interactions has tended to focus on either the teacher’s talk (Wilson, 1999; Young & Nguyen, 2002) or teacher-students’ interactions in either whole class (e.g., Klaassen & Lijinse, 1996, and Seah, 2004) or group discussion (e.g., Knuth & Peressini, 2001). There have been very few studies, if any, that took into account the role of student-student private interactions in generating knowledge in the classroom. Clarke and Seah (2005) adopted a more integrated and comprehensive approach, by analysing both public interactions in the form of whole class discussion and interpersonal interactions that took place between teacher and student and between student and student during between-desk-instruction. Interpersonal student-student interactions available for analysis were restricted to a focus group of up to four students. While this approach did not allow all interactions that took place in the classroom to be studied, it provided an avenue to track the generation of knowledge that could occur in both the public and interpersonal domains.

Analysis was carried out on video and post-lesson interview data related to mathematics lessons in Hong Kong, Melbourne, Shanghai and San Diego. All teacher classroom utterances and all statements by focus students, together with post-lesson interviews with teacher and students were transcribed and translated into English. The classroom transcript of each lesson was scanned for terms or phrases that expressed, represented, illustrated or explained mathematical concepts or understandings. These terms or phrases were referred to as “math-related terms”. These might take the form of conventional mathematical terms such as ‘gradient’ or everyday expressions such as ‘slope’ or ‘steepness’.

The occurrence of each term was then displayed in a tabular form analogous to the resource utilization planning charts of engineers (Table 1). These math-related terms were classified into three categories:

- Those ‘primary terms’ that corresponded to the teacher’s stated instructional goals (in lesson plan or interview),
- Those ‘secondary terms’ that were subordinate to or supportive of the teacher’s main instructional goals (usually previously-introduced or familiar terms which served to explicate the meaning of the terms central to the lesson’s intended focus),
- Those terms that appeared infrequently and fleetingly in the course of classroom discussion (in either public or interpersonal statements). These were referred to as ‘transient terms.’

If these math-related terms are thought of as resources drawn upon during the collaborative process of classroom knowledge construction, then the analogy is not inappropriate. Table 1 has been significantly abridged for reasons of space: Only the first 6 minutes of the lesson are displayed and only a subset of the lesson’s math-related terms are included. The terms are separated within the table by bold lines into the three categories and a brief description is provided of the classroom activity coincident with the occurrence of the various terms. Each vertical column corresponds to one minute and the occurrence of each term is designated by speaker (T = teacher; Andrea, etc = student), by time-code (eg 06:13, seconds and frames, within the designated minute) and by "P" if the utterance was an 'interpersonal' rather than a 'public' utterance.

Table 1
The Distribution of Responsibility for Knowledge Generation

Mathematical Idea/Term	0 – 1 mins	1 - 2 mins	2 - 3 mins	3 - 4 mins	4 - 5 mins	5 - 6 mins
Current Activity	(0:00 to 2:57) T reviewed the things learnt in the previous lesson with the class; drawing x- & y-axes (coordinate axes), locating the coords. of a pt & features of 2 pts having the same abscissa.			(2:57 to 8:19) T discussed how to find coords. and marked points on the blackboard: (1) find the quadrant; (2) draw a perpendicular to x-axis and a perpendicular to y-axis; (3)locate coords. of pt.		
Coordinate(s)	T (17:15)	T (06:26) Eve (07:15) T (09:15) T (50:01)	T (07:13) T (03:19)			T (27:19)
Area						
Abscissa		Anthea (30:14) T (32:05)		T (03:19) T (34:11)		T (27:19)
x-axis	Sam (43:17) Eve (52:26) T (56:03)	T (32:05)		T (34:11)	T (08:15)	Anthea (29:15)P
Ordinate		Simon (48:17) T (50:01)		T (24:13)	T (08:15)	T (27:19)
y-axis	Sam (43:17) T (52:09) Eve (52:26)	T (09:15)			T (49:29)	Eve (30:12)P
Transient Terms	Eve (51:04): Coordinate axis.	T (50:01): … rectangular coordinate plane.	Anthea (18:22)P: rectangular plane			Eve (30:12)P location

The capacity of this analytical approach to distinguish between classrooms is most evident in a comparison of eighth-grade mathematics classrooms in Shanghai and Hong Kong, since both sets of classrooms could be described as being embedded in a Confucian-heritage culture. The style of teaching in both Shanghai schools analysed was such that the teachers generally provided the scaffold needed for students to reach the solution to the mathematical problems without "telling" them everything. Hence, one could find quite a few math-related terms, which the teacher had not taught, that were introduced by the students during public discussion. A particularly powerful example of this devolution of responsibility occurred when the teacher in SH2-L04 (Shanghai School 2, Lesson 4) drew the class's attention to an alternative method of solving simultaneous equations being used by a student which the teacher described as more 'elegant' than the standard (textbook) method.

Students in the Hong Kong classes studied were generally not given the same opportunities to contribute during lessons, in comparison with classes in the other three cities studied (Shanghai, Melbourne and San Diego). The teachers generally stated very explicitly every step for solving the mathematical problems discussed. In other words, students were guided through the steps for each problem type with very little opportunity for original thought or input into class discussion. Where a new math-related term was introduced into whole class public discussion, this was either done by the teacher or by a student in response to very explicit prompting from the teacher. There were, however, math-related terms that occurred for the first time in interpersonal conversation between students, but were not subsequently voiced in the public arena.

As examples of ‘Asian’ classroom practice, in several respects the Hong Kong and Shanghai lessons analysed displayed more extreme differences in practice than those evident from comparison of ‘Asian’ and ‘Western’ lessons. Within the sets of lessons analysed for each city, significant variation was evident from the perspective of the distribution of responsibility for knowledge generation. The practices of the classroom in Shanghai School 2 provided some powerful supporting evidence for the contention by Huang (2002) and Mok and Ko (2000) that the characterization of Confucian-heritage mathematics classrooms as teacher-centred conceals important pedagogical characteristics related to the agency accorded to students; albeit an agency orchestrated and mediated by the teacher.

> A unique teaching strategy consisting of both teacher’s control and students’ engagement in the learning process emerges in Chinese classrooms. (Huang, 2002, p. 227)

Once the distribution of responsibility for knowledge generation is adopted as the integrative analytical framework, the oppositional dichotomisation of teacher-centred and student-centred classrooms can be reconceived as reflecting complementary responsibilities present to varying degrees in all classrooms. The deconstruction of the teacher-centred/student-centred dichotomy has specific consequences for teacher practice. In particular, one of the most contentious entailments of this dichotomy can be revisited; the legitimacy of teacher “telling.”

To Tell or Not To Tell: Dichotomies of Practice

One common interpretation of the constructivist manifesto (i.e., that “knowledge is the result of a learner’s activity rather than of the passive reception of information or instruction,” von Glasersfeld, 1991, p. xiv) has been that it became no longer legitimate for teachers to “tell” students anything. This position is not a logical consequence of adherence to constructivist learning theory, which suggests that students inevitably construct their own mathematics, whatever the classroom situation (Cobb, 1995). However, Telling or Not-Telling have been constructed oppositionally with such success that publications on contemporary pedagogy (such as Wood, Nelson & Warfield, 2001), while usefully discussing many pedagogical strategies, see no need to address any strategies that might be construed as analogous to “telling” and even articles that purport to address the issue (such as Chazan & Ball, 1999) offer teachers little insight into how (and, as importantly, when) their mathematical knowledge might be articulated explicitly to the benefit of their students.

Definitions of “telling” have been based on the *form* (i.e., whether or not the teacher is making a declarative statement or other type of assertion) rather than on the *function* of the teacher’s action. A teacher’s communicative act must be addressed from the related perspectives of the teacher’s intention, the nature of the act, and the interpretations of the act by the recipients or audience. By focusing on function (intention, action, and interpretation) rather than form, we overcome some of the difficulties experienced in analyzing the efficacy of teacher practices from a constructivist perspective. Constructivist learning theory has been extrapolated to the domain of teaching practice, and “constructivist teaching” has been set up in opposition to “transmissive teaching” (Richardson, 2001, for example). Criticism of transmissive teaching has an extensive history and has sometimes led to simplistic exhortations to avoid “telling” without serious discussion of those teaching actions that involve directly introducing new ideas.

Clarke and Lobato (2002) (and subsequently Lobato, Clarke & Ellis, 2005) have proposed a theoretical reformulation of teachers’ communicative acts in terms of function rather than form. This reformulation is founded on the distinction between “eliciting” and “initiating.” By focusing on function (intention, action, and interpretation) rather than form, some of the difficulties experienced in analysing the efficacy of teacher practices from a constructivist perspective are overcome. Such a framework offers a more incisive tool for the analysis of the teacher’s contribution to classroom discourse. In particular, it offers a language in which to frame the devolution of the responsibility for knowledge generation from the teacher to the student, or, alternatively, the concentration of that responsibility in the teacher. For example, teacher acts that take the form of a question but have the function of telling can be identified and the responsibility for the initiation of a new mathematical idea can be correctly located with the teacher rather than the responding student. Equally, as has been argued above, the capacity of the student to contribute to the generation of knowledge can be recognized, and classrooms can be compared according to the extent to which the student is accorded the opportunity to make this contribution. The fundamental consideration is the distribution of responsibility for knowledge generation.

Clarke and Lobato (2002) asserted the importance of interweaving the two “functions” initiating and eliciting. Since it is the development of the students’ mathematics that we aspire to promote, it is the students’ mathematics that takes priority. It is our contention, however, that the teacher’s mathematics can also find legitimate voice in the classroom in the interest of stimulating the development of the student’s mathematics. Initiating/eliciting is not a simplistic dichotomy like “tell/not tell”—it’s not an either/or. Both categories of action are necessary and their use is interrelated. Eliciting has typically been defined in terms of the form of the communicative act (e.g., asking questions such as “Could you explain your reasoning?”) or in terms of the degree of student involvement (e.g., the use of student-centered activities). Elicitation occurs when the teacher wants to learn more about students’ images, ideas, strategies, conjectures, conceptions, and ways of viewing mathematical situations. When the teacher’s communicative act *functions* to facilitate the expression of the student’s mathematics, then this constitutes “eliciting.” In order to provide experiences that might challenge students to reorganize their thinking, teachers need to develop models of their students’ mathematical realities (Simon, 1995; Steffe & Thompson, 2000). The adequacy of these models will depend on the teacher’s ability to elicit the student’s mathematics.

Initiating is most profitably used in conjunction with eliciting. Elicitation occurs when the teacher wants to learn more about students’ images, ideas, strategies, conjectures, conceptions, and ways of viewing mathematical situations. Initiating is often preceded by eliciting, so that the teacher can gather information about students’ thinking before making a judgment whether to work with and structure the students’ ideas or to introduce new information. Initiating involves the insertion of new ideas into the conversation, ideas that the teacher assumes will be interpreted in many different ways rather than passively received. Once the teacher engages in initiation, she then steps back and elicits to see what the students did with that information. Both actions have their function within the teacher’s promotion of student conceptual development. The mutuality and complicit nature of these interactions bring us back to the spider’s web, the epistemic student, and the co-constructed nature of teaching/learning. The agenda that frames such classroom activity is initially the teacher’s agenda, but this agenda is iteratively modified in response to the progress of the ensuing classroom discussion in order to accommodate the students’ prior and emerging understandings (see Lobato, Clarke & Ellis, 2005, for specific examples).

Where do we see the purposeful alternation of elicitation and initiation most clearly? One example can be found in the classroom in Shanghai, already referred to above. Unlike an Australian classroom, the students in this classroom rarely ever talked directly to each other – classroom conversation was always mediated by the teacher - yet the students were clearly learning most effectively. Part of the explanation came in the interview after the lesson. The teacher said: “Don’t teach them mechanically, don’t teach them mechanically, let them brainstorm, enhance their flexibility.” And, “I was not afraid that students had all sorts of questions. I just let them appear . . . Sometimes if you restrict them from doing this or that, their problems won’t appear, right? But the problems will appear tomorrow, even if they didn’t today, right?”

This is an articulate summary of the heart of the contemporary reform agenda in Western education and demonstrates a commitment to the purposeful elicitation of the students’ mathematics. But, for cultural reasons, the opportunities for student discussion of the content were provided in a teacher-led whole class approach. With regard to the value attached to the students’ mathematics, once elicited, in the lesson referred to earlier, this same teacher said to the class, “Look at Shiqi’s solution! This is much better than the usual method. Everyone copy this down.” As was evident in the analysis of the distribution of responsibility for knowledge generation in this classroom, the responsibility was shared between teacher and students and, in so far as the teacher’s intentions could be put into effect, the classroom discourse was a purposeful alternation of initiation and elicitation.

It is in this manner that the utilisation of the distribution of responsibility for knowledge generation provides an integrative, explanatory framework that problematises teacher-centred and student-centred characterisations of the classroom and resolves the false opposition of dichotomous practices by replacing them with a conception of alternative interrelated (and fundamentally complementary) classroom practices.

Concluding Remarks: Alternatives to Dichotomisation

This paper has attempted to make two general points: (i) International comparative research can provide the means to interrogate our most fundamental assumptions – in this case concerning some basic dichotomies that have pervaded our theorising about classroom practice and learning; and, (ii) Such dichotomies can be reconceived as reflecting complementary elements within more integrative theoretical frameworks.

In order for international research to support the interrogation of such fundamental assumptions, we must be sensitive to issues of cultural authorship: of representation and of voice. In commenting on the proliferation of OECD-initiated international comparative research projects, Cohen characterised the OECD as “a club of 29 of the world’s richest countries” (Cohen, 1998, p. 4). Even when less affluent countries participate in international studies, it is frequently as the objects of investigation rather than as partners in the research. Research is frequently conducted from a ‘Western’ perspective and evaluates the practices it studies by ‘Western’ criteria. A notable and most welcome exception is the recent “insider’s perspective” on Chinese mathematics teaching and learning (Fan, Wong, Cai & Li, 2004). Once we have achieved more equitable representation of all interested nations in international research programs, we need to ensure that the perspectives of all participating cultures inform the design and analytical frameworks employed, and that the voices of all participating cultures are evident in the reports that arise from such research.

The detailed collaborative study of international policy and practice in mathematics education, and of the products of that policy and practice, should be undertaken in anticipation of insights into the novel, interesting and adaptable practices employed in other school systems and of insights into the strange, invisible, and unquestioned routines and rituals of our own school system and our own mathematics classrooms.

One important manifestation of cultural authorship is the situatedness of our advocacy of any particular classroom practice. Hatano and Inagaki (1998) remind us that the adaptation of pedagogical practice requires consideration of both the practicality of technical implementation and the extent to which the beliefs underlying the adapted practice are in harmony with local cultural values. Fuller and Clarke (1994) made a related point:

> The next generation of [research] questions pertains to how these tools are culturally situated and understood in the eyes of teachers and pupils, including how these tools help to structure the classroom's social rules. (Fuller & Clarke, 1994, p. 144)

The cultural positioning of pedagogical practice is an essential precursor to its adaptation and application in other settings.

Oppositional dichotomies such as teacher-centred versus student-centred classrooms, real-world versus abstract tasks, and telling versus not-telling offer mathematics educators false choices, sanctifying one alternative while demonising the other. International research offers insight into possible explanatory frameworks within which such choices are no longer oppositional or even dichotomous, but rather can be seen as reflecting strategic and interrelated pedagogical decisions, dependent on purpose and context, that must be understood in cultural terms before they can be related to any setting outside their classroom of origin.

The perils of oppositional dichotomies extend to research methodology. Happily, the utilisation of "mixed methods" designs (Johnson & Onwuegbuzie, 2004) is the subject of increasing advocacy, and complementarity is replacing incommensurability (Clarke, 1998, 2001). This paper has attempted to demonstrate the capacity of international classroom research to problematise and reconceive some of our most fundamental dichotomies. In each case, the alternative that is being offered to the prevalent oppositional dichotomies is an integrative perspective in which such alternatives are seen as complementary and interrelated aspects of a broader conception. Further research in applying such integrative frameworks must employ similarly integrative methodologies.

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