Where Did I Go Wrong? Students' Success at Various Stages of the Problem-Solving Process

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This paper reports one aspect of a larger study which looked at the strategies used by Grade 6 students to solve six non-routine mathematical problems This paper summarises the findings in relation to the students' success at various stages of the problem-solving process and the influence of metacognitive thinking on this success Results suggest that most difficulties arise from a lack of metacognitive activity during the *execution* and *verification* stages of the problem-solving process

A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of mathematical problems (Booker, Bond, Sparrow & Swan, 2004) Problem solving was the theme of the 1980's and the NCTM maintains that "solving problems is not only a goal of learning mathematics, but also a major means of doing so" (NCTM, 2000, p 4) Results from the TIMSS study conducted in 1999 (TIMSS Video Mathematics Research Group, 2003) however, revealed that Australian and American students (among others) were not performing well on problem-solving tasks Schoenfeld (1992) expressed concerns about students' problem-solving ability and in Australia, Lovitt (2000) maintained that the problem-solving movement had "failed" While the factors contributing to this varied, many researchers (e g , Wilson, 1998; Yeap & Menon, 1996) identified metacognitive thinking as being integral to the problem-solving process and crucial to problem-solving success This study looks at the process students undertake when solving non-routine problems and the extent to which they engage in metacognitive behaviours

The Problem-Solving Process

Problem solving requires a variety of skills including interpreting information, planning and working methodically, checking results and trying alternative strategies A successful problem-solver must incorporate a range of heuristic approaches (Schoenfeld, 1980) when solving problems and the heuristic plan which is most widely advocated by researchers (e g , Schoenfeld, 1980) as facilitating problem-solving is that derived from Polya (1957) This requires the solver to understand the problem, devise a plan, carry out the plan and look back and examine the solution obtained Garofalo and Lester (1985) suggested that students are largely unaware of the processes involved in problem-solving and pointed out that Polya's model does not encompass metacognition Their framework incorporated metacognitive behaviour at each of four stages It comprised a) *orientation*: strategic behaviour to assess and understand a problem, b) *organisation*: planning of behaviour and choice of actions, c) *execution*: regulation of behaviour to conform to plans, and d) *verification*: evaluation of orientation and organisation and evaluation of execution

Table 1 describes the framework used in this study to consider evidence of the extent to which students were successful at each of Garofalo and Lester's (1985) stages This paper reports on their success at each stage of problem-solving process Students' communication of findings was described in Beswick and Muir (2004)

Table 1

Framework used for Analysing Students' Problem-solving Attempts

Aspect of analysis	Relationship to Garofalo and Lester's (1985) framework
Understanding the problem	*Orientation*: To what extent was the student able to attempt a solution that evidenced at least some understanding of the nature of the problem?
Choosing and implementing a solution strategy	*Organisation*: Did the student choose a solution strategy that, if implemented correctly would result in the correct solution? AND *Execution*: To what extent was the student able to execute the chosen strategy?
Communication of findings (verbal and written)	This provided evidence of students' metacognitive activity throughout the process and a stimulus for some students to engage in *Verification*
Getting the answer	*Verification*: To what extent was the problem-solving attempt successful?

Considerable research effort has focussed on the difficulties students experience in the *orientation* stage of problem solving Silver and Smith (1980) highlighted the importance of the ability to recognise the structure of the problem, while Kaur and Blane (1994) identified difficulty at the *orientation* stage, rather than a lack of mathematical skill, as a cause of unsuccessful problem-solving attempts More recently Pape (2004) examined differences in the mental representations of problems of more and less sophisticated problem solvers These had important implications for the likely success of subsequently implemented solution strategies Other comparisons of expert and novice problem solvers have identified the propensity to seek to develop an understanding of problems in terms of core concepts, before attempting a solution, as characteristic of expert problem solvers (Bransford, Brown & Cocking, 1999) Bransford et al (1999) noted that experts recognise features and patterns of problems that are not noticed by novices

Efforts to improve students' ability to solve problems have focussed on their metacognitive activity and strategies for facilitating its development Most of these studies have involved secondary students (e g , Goos, 2002; Williams, 2000) In a study, unusual in that it involved grade 3 students, Goldberg and Bush (2003) observed that, in contrast to the behaviour of expert problem solvers, when given a novel task in class, children were very likely to 'jump' into the problem with one strategy, continue the strategy without looking back and finish without examining the solution This occurred even among subjects of an intervention aimed at improving metacognition While Goldberg and Bush (2003) acknowledged the possible influence of the nature of the problems used in her study it seems reasonable that younger students would exhibit more such non-expert behaviour than older students Whatever its origin such behaviour can result in errors at every stage of the problem-solving process ranging from misunderstanding the problem (*orientation*), choice of an ineffective strategy (*organisation*), and/or a solution that does not work but is not identified as such (*execution* and /or *verification*) In another study of metagonition and primary students' problem-solving activity, Wilson (1999) found that the sheer number of metacognitive acts exhibited by grade 6 students did not necessarily relate to their success in solving the problems

This paper reports on the difficulties experienced by primary students at each problem-solving stage (Garofalo & Lester, 1985), and the extent to which students showed evidence of metacognition throughout the process Through analysing the stages at which the errors

occurred, this study adds to the research on successful problem solving by younger students and identifies directions for future problem-solving instruction for this age group

The Study

The study involved four grade 6 students from each of five primary schools that varied in their socio-economic status, size and location The selection of schools was aimed at addressing reliability and validity issues that may be raised in relation to a small sample size (Patton, 1990) Grade 6 students were targeted because though still in primary school, they could be expected to have literacy levels sufficient to understand the questions and articulate their solution processes

Six mathematical problems were chosen based on the researcher's classroom teaching experience with regard to their appropriateness to the selected age group, and their potential to be answered using a variety of strategies (see appendix A)

Students were interviewed using a semi-structured approach (Burns, 2000) A copy of each problem was presented and read aloud to the child by the researcher and each student was then asked to respond to the question and make a written recording of any working out used in this process Reading the problems ensured that the students knew what each problem said but in no way assisted them with the *orientation* stage that concerns comprehension in terms of the mathematical demands of the task rather than mere decoding of words Following the completion of each problem, each student was asked if they had encountered a similar problem before, and to verbally explain what they had done The researcher asked prompting questions when and if clarification was required Written work was collected and the interview was audio-taped

The rating scales shown in Table 2 were used to separately rate each of the students' responses in terms of Understanding the problem, Choosing and implementing a solution strategy, and Getting the answer Similar scales were also used by Adibnia and Putt (1998) in their study of problem solving by year 6 students Judgements of the level of understanding and the efficiency of Choosing and implementing a plan were not dependent on a correct answer A student may have received a rating of '2' for example, for the first two stages, yet not have produced a correct answer

Table 2

Rating Scales for Assessing Students' Problem-solving Attempts

Stage	Ratings		
Understanding the problem	0 – Completely misinterprets the problem	1 – Misinterprets part of the problem	2 – Complete understanding of the problem
Choosing and implementing a solution strategy	0 – No attempt or completely inappropriate strategy	1 – Partly correct strategy based on part of the problem interpreted correctly	2 – Chooses a strategy that could lead to a correct solution if implemented properly
Getting the answer	0 – No answer, or wrong answer based on an inappropriate strategy	1 – Makes copying or arithmetic error; partial answer for a problem with multiple answers or answer labeled incorrectly	2 – Get correct solution

Results and Discussion

The 20 students attempted all six problems The rating scales were applied to the 120 solutions and are presented in this section with discussion about the results for each of the stages of the problem-solving process Examples of some of the responses are presented to demonstrate particular solution characteristics and to highlight the use of metacognitive thinking Student comments are also used as evidence of strategy use The respective numbers of the ratings recorded at each stage are presented in Tables 3, 4, and 5

Table 3

Number of responses for each Problem, receiving each rating for Understanding of the Problem

Problem	No rated '0'	No rated '1'	No rated '2'	No of correct solutions
1	0	10	10	5
2	2	3	15	7
3	0	4	16	13
4	2	3	15	8
5	1	12	7	4
6	0	3	17	10
Totals:	5 (4%)	35 (29%)	80 (67%)	47 (39%)

One third of the responses scored 0 or 1, indicating complete or some misunderstanding of the problem Eighty responses received a rating of '2' indicating a sound understanding of what the problem was asking them to do Only 47 of these responses, however, resulted in correct answers, due to difficulties at other stages of the problem-solving process

When asked about whether they had encountered a similar problem the students, like those in Huang's (2004) study, tended to judge similarity on the basis of surface features or contextual information Some students, however, were able to articulate their recognition of the mathematical structure of the problem as evidenced in Amali's response to problem 1: "Yeah I've done this kind of thing before with cards and combinations of clothes and things " In this instance Amali evidenced behaviour characteristic of expert problem-solvers and, consistent with the literature on the *orientation* stage, it assisted her with planning and implementing a correct solution strategy

Table 4 shows that just over three quarters of the students' responses included choosing a strategy that would have been successful if implemented properly Given that one third of responses represented misunderstandings of the problem at least some of these choices must have been the result of good fortune Nevertheless it seems that the *organisation* stage of the problem-solving process presented little difficulty and that the *execution* stage proved more troublesome

Responses to Problem 1 are illustrative Nineteen of the 20 students chose either to make a list, draw a diagram, or use materials – all of which could have resulted in successful solving of the problem if implemented correctly One particular student, Julie, whose written recording is shown in Figure 1, combined drawing and making a list to solve the problem (she received a rating of 2 for Understanding the problem, 2 for Choosing and implementing a solution, strategy and a rating of 1 for achieving the Correct answer) Julie drew blocks of cubes and wrote letters for the colours in them, but she did not do this in a

systematic way When she had filled up the space, she stopped and counted how many she had drawn and wrote 17

Table 4
Number of responses for each Problem, receiving each rating for Choosing and Implementing a Solution Strategy

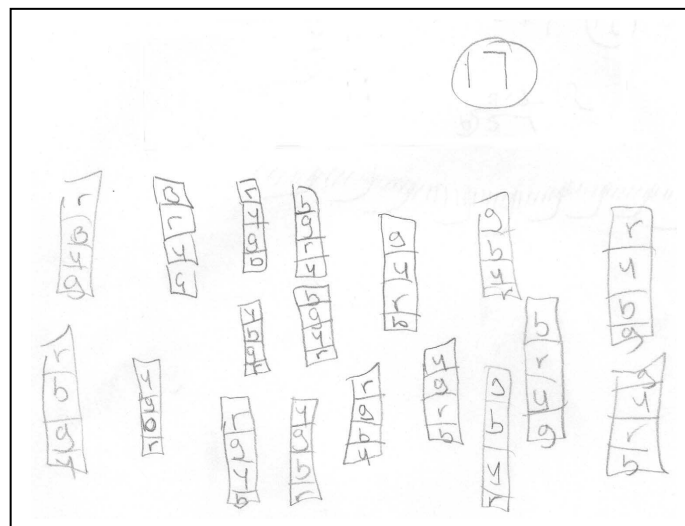| Problem	No rated '0'	No rated '1'	No rated '2'	No correct
1	1	1	18	5
2	2	4	14	7
3	0	2	18	13
4	4	1	15	8
5	3	9	8	4
6	0	1	19	10
Totals:	10 (8%)	18 (15%)	92 (76%)	47 (39%)

Figure 1 Julie's written communication for problem 1

She explained: "I drew four squares and then I did the initials for the letters, and I just done them, first I done them in that order and then I wrote them backwards and upside down " When asked how she kept track of the ones she had already done, she replied: "Um, sort of didn't really keep any track (laughs) just sort of, just looked around "

Julie's response is indicative of a lack of metacognition in the *execution* stage It and others like it in this study, add to Goldberg's (2003) evidence that Schoenfeld's (1992) finding that high school and college students, tended to "read, make a decision quickly and pursue that direction come hell or high water" (p 62) also applies to younger students

The students in this study also showed an overwhelming tendency to persevere with the strategy they selected initially, even when it was not working Hayley, for example, when asked if there was another way to solve Problem 1, indicated: "I'm not sure, this is the way I always do it " Melanie's written response to Problem 4, shown in Figure 2, provides some evidence of metacognitive thinking through her adjustment of the figures Nevertheless, she was really using 'trial and error' rather than using her guesses to improve her next attempt, and did not consider trying an alternative solution, such as drawing a

diagram which may have assisted her Her written responses show that she tried the combinations of seven rabbits and four children, and five rabbits and six children She recognised that the answer was not correct, but "did not know what to do " After trying a variety of combinations, she conceded defeat and stated: "I tried to find out how many twos were in 30, but that was 15 and I thought that four can't really equal into 15, so I just tried to…and I just couldn't come up with an answer "
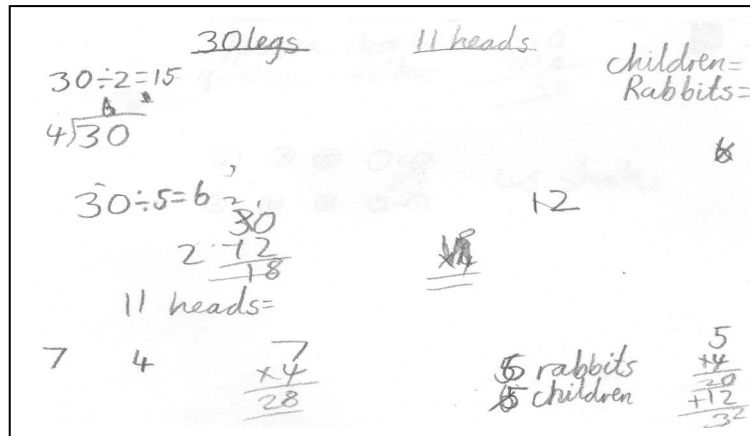

Figure 2 Melanie's written representation for problem 4

Garofalo and Lester (1985) asserted that many novice problem-solvers needed to develop an effective monitoring process in order to regulate strategy use In particular, many of the students in this study were able to select appropriate strategies but failed to employ self-correcting mechanisms or monitor their progress while working through the problems

Table 5
Number of responses for each Problem, receiving each rating for Getting the Answer

Problem	No rated '0'	No rated '1'	No rated '2'
1	1	14	5
2	2	11	7
3	2	5	13
4	5	7	8
5	5	11	4
6	0	10	10
Totals:	15 (13%)	58 (48%)	47 (39%)

Table 5 shows the ratings of responses in terms of the correctness of the answers obtained Each of the 39% of answers that were correct scored '2' on the rating scale and each was accompanied by ratings of '2' for each of the other aspects A further 48% of responses had the potential to be correct, but the majority of students wrote their answers down and did not check them for reasonableness Even when asked how they could check if their answer was correct, many respondents could not identify a means for doing this, and many responded with, "I just think it's right " In relation to Problem 2, for example, some students identified that they could use their answer to check that it corresponded with the floors, but did not initiate this checking process This behaviour is consistent with

Wilson, Fernandez and Hadaway's (1993) observation that developing the disposition to look back is very difficult Despite the difficulty, the results of this study suggest encouraging students to engage in the *verification* stage may well be a key to improving students' problem-solving performance The more successful problem solvers in this study did show a greater willingness to verify their solutions In Problem 6, for example, Kasey made the triangles with matches and then "checked it on my piece of paper by drawing the triangles and counting the lines " Greg also checked his triangle answer by "drawing more triangles just to check if my theory was right "

Conclusions

Thirty-nine percent of the students in this study experienced difficulty in the *orientation* stage of the problem-solving process Many students also had difficulty with the *execution* stage, and there was a distinct unwillingness to engage in *verification* In each of the *execution* and *verification* stages, the impact of metacognitive thinking or its absence was evident Most students seemed not to monitor their progress, reflect on the appropriateness of the strategy they had chosen, or display any inclination to try an alternate strategy even when frustrated by their lack of progress Students were similarly unwilling to reflect on the appropriateness of the answer they obtained or to attempt to confirm it using an alternate method In terms of understanding the problems very few students recognised structurally relevant similarities to problems they had already encountered

The results confirm many earlier findings, and that the focus of research effort on metacognition is appropriate This study provides evidence that many of the findings in relation to older students' problem-solving also apply to younger students In terms of the teaching of problem-solving, it may be that instruction has focussed on explicit strategy teaching and has been effective in providing students with a repertoire of approaches to problems, resulting in little difficulty with the *organisation* stage Perhaps more attention in this arena needs to be focussed on helping children to identify mathematically relevant structural features of problems and to the development of metacognitive skills including the inclination to reflect on answers and the processes that gave rise to them

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Appendix A

1 Jenny is making towers of cubes using red, blue, yellow and green How many different towers can she make by changing the order of the colours?	2 Jim got into a lift He went down 5 floors, up 6 floors and down 7 floors He was then on the second floor At what floor did he get on?
3 Susan worked at an apple orchard When she was sorting the apples for sale, she noticed that 2 out of every 7 apples had worm holes If there were 70 apples in the basket, how many could be expected to be 'good apples?	4 Some children were playing with some rabbits in a yard I tried to count them and found that there were 30 legs and 11 heads How many children and how many rabbits were in the yard?
5 At a meeting of the Good Friend's Society, everyone begins by shaking hands with each other once If there were 10 people at the meeting, how many handshakes were there?	6 I made some triangles using matchsticks I used 3 matches to make 1 triangle, 5 to make 2 triangles and 7 to make 3 triangles If I continued in this way, how many matches would I need to make 12 triangles? (Diagram of the triangles included)