Student Misconceptions about Projectile Motion[1]

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Historical conceptions of projectile motion have varied from the Aristotelean through impetus theory to Newtonian mechanics; but its standard mathematical treatment is only possible within the Newtonian framework. This paper reports a study suggesting that many Australian Year 12 mathematics students do not conceptualise projectile motion within that framework, but rather use a variety of Aristotelean, impetus and Newtonian conceptions. The implications for the teaching of projectile motion are briefly discussed.

It's not what you don't know that hurts you. It's what you know that ain't so! Mark Twain

In New South Wales, projectile motion is taught in Year 11 Physics and in Year 12 Extension 1 Mathematics. These syllabi are not linked in any way and instruction often gives students the impression that they are studying two distinct topics. In both subjects, performance on questions that test understanding is poor (see, for example, the examiners' comments on the 1999 Higher School Certificate mathematics examination [Board of Studies NSW, 2005]). Students often learn standard techniques by rote, but when questions become more difficult they resort to their intuition, which fails them because of their misconceptions (Gunstone, 1991). This paper analyses student conceptions of projectile motion and links those ideas to historical views of projectile motion.

Historical views of projectile motion

Aristotle (4th century BC) believed that an external force is needed to maintain the motion of an object. To account for the movement of projectiles that are not in direct contact with any observable mover, Aristotle suggested that air rushes around the moving object and pushes it forward.

The Greek philosopher John Philoponus (6th century AD) argued against the Aristotelian theory of motion and introduced the impetus theory (Boyer, 1991). The essence of his theory is that the act of setting an object in motion imparts to the object a force, called an impetus, that maintains the motion. This force allows the object to move in the direction in which the mover starts it. Since a projectile has no obvious external force, the impetus is said to be internal to the object.

The 11th century Islamic scholar and scientist Ibn-Sina held that the impetus is self-expending (Boyer, 1991). When the impetus is diminished or runs out, the natural heaviness of the object supplies a downward force and the object falls straight down. This version of physics is frequently shown in cartoons such as Road Runner. Ibn-Sina's theory extends to a stone thrown at an angle. From his perspective, the stone would travel along an oblique line until the impetus is exhausted, when it would momentarily stop. Then its "natural gravity" would impart an impetus, causing it to fall straight down.

Albert of Saxony (14th century) amended Ibn-Sina's theory by introducing a transition phase. In the firing of a cannon, he believed, there is a first phase when the impetus provided by the cannon is greater than the weight of the cannon ball; so the ball moves in a

[1] This paper reports part of a PhD study (Prescott, 2004) undertaken at Macquarie University by the first author under the supervision of the second.

straight line. During the second phase, as the initial impetus reduces the downward force has an increasing influence on the object, causing the object to fall below its original path. In the third phase, the impetus is spent and the cannon ball falls straight down.

French philosopher Jean Buridan (14th century) believed that the impetus is sapped by external influences such as air resistance or friction. Buridan also believed that an object dropped from a moving carrier does not acquire impetus.

Galileo Galilei (early 17th century) originally supported the notion that the force of a throw must be greater than gravity or the object will immediately fall (Kozhevnikov & Hegarty, 2001). In his *Dialogues Concerning Two Sciences*, Galileo puts forward the impetus idea through his character Sagredo (an intelligent layman) who says:

> So therefore the impressed force may exceed the resistance of gravity so slightly as to raise it only a finger-breadth; and finally the force of the projector may be just large enough to exactly balance the resistance of gravity so that the body is not lifted at all but merely sustained. (Hawking, 2002, p. 525)

Later, Galileo theorised that the trajectory of a projectile could be thought of as two independent motions: one component consisting of uniform motion in a horizontal direction and the other component consisting of vertical motion under acceleration due to gravity. By combining these two motions, he was the first to deduce that the trajectory of an ideal projectile is a parabola.

Later in the 17th century, Isaac Newton devised a universal theory of mechanics that validated Galileo's treatment. Newtonian mechanics, including the famous three laws of motion, is now the accepted way of modelling projectile motion. The crucial difference between Newtonian mechanics and impetus theory is that, whereas impetus is the cause of the motion and is internal to the object, in Newtonian mechanics an external force is required to *change* motion—not to sustain constant motion (McCloskey, 1983a).

Student Misconceptions of Motion

Students develop their "theories of motion" by generalising the ideas they acquire by observation of the behaviour of specific objects in everyday situations (Keeports, 2000; McCloskey, 1983b). The research literature shows that students develop many misconceptions, and that these resemble the historical theories just described.

Objects Launched by Firing

In a study reported by McCloskey (1983a), college students were asked to draw the path of a metal ball pushed along the top of a cliff at high speed so that it went over the edge. More than a third of the high school and college students interviewed thought that the ball would travel as predicted by the impetus theory of Albert of Saxony and more that 5% of the students followed Ibn-Sina's ideas.

A common idea is the so-called *more of A, more of B* theory (Minstrell, 1991; Stavy & Tirosh, 1996; Tirosh & Stavy, 1999). Students believe that a projectile launched at a high speed will accelerate or will travel for longer than an object travelling at a lower speed. An increase in speed (*more of A*) produces an increase in the distance or time of flight (*more of B*) (Jimoyiannis & Komis, 2001). There appears to be no historical anologue to this belief.

Objects Dropped From a Moving Carrier

Students may be just as confused when they think about objects released from a moving carrier. Many believe, as did Jean Buridan, that these projectiles do not possess forward motion when released and so have no impetus. Consequently, dropped objects move backwards or fall straight down (Millar & Kragh, 1994). Some students think that the speed of the carrier is important and, therefore, consider the motion of an object dropped from a person walking as different from that of an object dropped from a plane.

The misconception with objects dropped from a plane may come from films taken from the bomb bay of a plane, in which the bomb appears to drop straight down. The students do not realise that the plane and the bomb initially have the same horizontal velocity.

The Concept of Force

A sound concept of force is necessary in order to understand Galileo's method of decomposing motion into horizontal and vertical components. Many students feel that the concept of force is easy to learn because its meaning seems obvious from everyday experience (Schecker & Niedderer, 1996). Students' ideas regarding force include:

- If an object is not moving, then there can be no force acting on it.
- If an object is moving, then there must be a force in the direction of motion (Tao & Gunstone, 1999).
- Force as a kind of fuel or energy that sustains the motion but at the same time is consumed by the motion itself (Tao & Gunstone, 1999; Vosniadou, 1994). This is the impetus notion of John Philoponus.
- An increase in force will produce an increase in speed (*more of A, more of B*).

Students are also unsure about the nature of gravity—just as Galileo had problems with the idea. Some beliefs include:

- If an object is on the ground then gravity is not acting on it, because it has already fallen to the ground.
- Gravity is the result of air pressure.
- Gravity is a property of the object itself.
- Those objects that fall have more gravity than stationary objects, or gravity is not exerted upon stationary objects (Thagard, 1992; Vosniadou, 1994).

The Present Study

The study reported in this paper set out to answer four questions:

1. Do New South Wales students show the same misconceptions as have been reported in the research literature?
2. Do their misconceptions consistently fall into the historical categories?
3. Is there a relationship between the students' conceptions of the motion of fired and dropped objects?
4. How do their conceptions of force affect their conceptions of motion?

It is part of a wider study of the learning and teaching of projectile motion (Prescott, 2004).

Method

Interviews

A semi-structured interview was designed to assess senior students' conceptions of projectile motion. In the 15 to 20-minute interview, several projectile situations were described and a variety of questions posed. Students answered in writing or by drawing, and were then asked to explain their answers. The questions were so arranged that the same general concepts were investigated in different contexts in non-consecutive questions.

Eight questions asked about objects launched by firing, including the situation of rolling off a cliff. Three questions asked about the relative position of objects dropped and fired simultaneously, and five questions asked about the relative time of flight of such objects. Three questions asked about objects dropped from various moving carriers—a plane, a walker and a conveyor belt—and another asked about a flare fired vertically from a moving snowmobile. One question asked students directly about the forces on a stone that was thrown vertically upwards.

For a copy of the interview schedule, an indication of the sources of the questions, and a description of the pilot-testing, see Prescott (2004).

Participants

Two schools agreed to participate in the study. They were both independent girls' schools in the Sydney metropolitan area, and were predominantly non-selective. Forty-seven Year 12 students were interviewed. The students had not met the topic of projectile motion in mathematics, but 17 were also doing physics and had studied it in Year 11.

Results

Objects Launched by Firing

When asked to choose the correct trajectory for a ball rolling off a track, 85% of students correctly chose the parabolic path. However, further questioning revealed that many students held misconceptions about fired objects.

Asked to predict the motion of two balls, one rolled off a cliff and the other dropped simultaneously from the same height, most students answered incorrectly. The most common incorrect answer, given by 40% of the students, was that the dropped ball was travelling a shorter path so it would reach the ground first. The other incorrect students guessed or thought "fast objects get there first". However, even when the students gave a correct answer, their reasoning often revealed misconceptions:

> But then X and Y may be at the same spot because X might have more speed than Y does, because Y's just been let go and one's rolling on. I think it might be the same height.

In the similar question, about the trajectory of a bullet fired from a gun and another bullet dropped simultaneously, most students (71%) incorrectly decided that the dropped bullet would hit the ground before the fired bullet.

Asked to compare two bullets fired horizontally at different velocities, the most common incorrect answer (38%) was that the faster bullet would be in the air longer. The following quote indicates that some students included a force other than gravity:

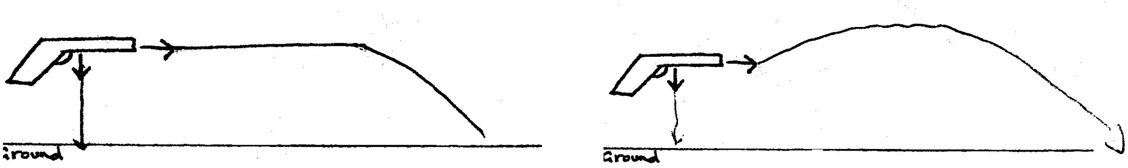
> I don't know how to explain it. If it's going at a slower speed the gravity will be acting on it faster so it will go down faster, but at the faster speed the gravity will act like the force acting on it. So it will take more time for it to hit the ground.

Students thought quite differently about the motion of two balls rolling off a cliff at different speeds: only 15% thought that the faster object would be in the air longer, whereas 43% thought that it would reach the ground first. One student explained her thinking as follows:

> It's probably different from this, but in normal life faster things get there first.

Students' drawings of trajectories indicated some misconceptions very clearly. For example, they often indicated they thought the force from a gun wears out after a time, at which point the path of the bullet changes markedly (see Figure 1a). A physics student drew the diagram in Figure 1b and used trigonometry to describe the motion:

> This one's got a force which is pushing it up in like a projectile. It goes up and then like it doesn't have enough force when it reaches the velocity cos0, so it starts getting pulled down because of gravity. It's still got the component pushing it that way.



(a) (b)

Figure 1. Examples of trajectories drawn by students.

When asked to predict the horizontal distance a cannonball would travel in the 2nd second, given that it travelled 10 metres horizontally in the first second, the most common incorrect answer (60%) was that the cannonball would travel less than 10 metres in the 2nd second.

Objects Dropped From a Moving Carrier

There was a lack of consistency in students' answers to questions about objects dropped from a moving carrier. The percentage correct ranged from 25% (snowmobile) to 30% (plane) to 34% (walker) and 38% (conveyor belt).

The most common incorrect answer for the path of a ball dropped from a moving carrier was that the ball would drop straight down (conveyor 30%, walker 49%, plane 30% and snowmobile 53%):

> The ball goes straight down because once the conveyor belt releases the ball it's not carried along by the conveyor belt's forward motion any more. So it's just gravity acting on it so it just goes straight down.

Some students were aware that the horizontal motion is the same for the snowmobile and the flare, but were unsure about the implications. The following student made the sketches shown in Figure 2 and explained:

> You can really see from the diagram and because the motion is that way, it's going straight up so, relative to the snowmobile, it will land behind. … Except I'm not sure in reality if that would be the case because the flare has the motion of the snowmobile when it's launched and I'm not really sure how to factor that in so I'm going to go with [straight up and down].

While many other students did not draw a diagram, their explanations were very similar to the first part of this student's explanation.

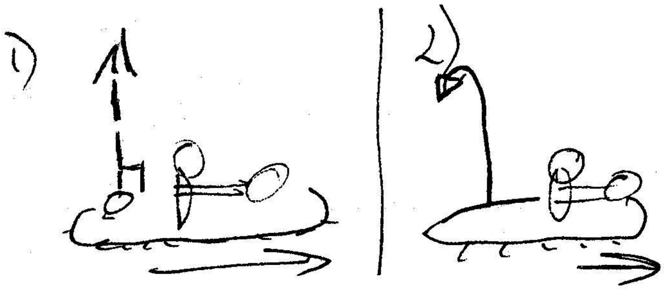


Figure 2. One student's drawing for the path of a flare from a moving snowmobile.

The Concept of Force

When asked about the forces on a stone that had been thrown in the air, only 33% knew that the only force on the stone would be gravity. Most students thought that the only force on the stone was from the hand when the stone was thrown into the air. One student thought that there would be two forces, the initial impetus and gravity:

> I would have thought that you've got the force from the person throwing it up, but you've also got gravity, so that'd be lots of force there because it's on its way up still. So it's still got lots of energy to keep going, but there's also the force that makes it come down in the end.

Lack of understanding of gravity as a force was revealed in students' responses to other questions. The following is a typical explanation of why the path of a bullet changes markedly at one point (see Figure 1a):

> I suppose [the bullet changes direction] if there's wind or air resistance, but after a while the force pushing it out from here, pushing it out from the gun, will wear it out and it won't be able to travel in the straight line. So it ... so gravity or something like that will push it down to the ground.

One student compared the motion of a fired and a dropped bullet as follows:

> Well the one that's going from the actual barrel of the gun will go further and that means it will drop later because there's a force acting on it that will push it further. This one's being dropped so there's no force propelling it to the ground.

Students often explained that gravity only began to influence projectiles when they reached their highest point, or that gravity was different at different points of the trajectory.

The students also had frequent difficulties predicting the influence of air resistance.

Discussion

The results of this study indicate that the notion of impetus put forward in the 6th century is still alive and well in the 21st. There was clear indication that many students believe that a fired or thrown object is given an impetus which maintains its motion but is gradually used up: In each of the fired projectile questions, 32% - 50% of students gave answers based on this idea. There was also clear evidence that many students believe that dropped objects do not receive any impetus from a moving carrier: In each of the dropped object questions, 30% - 53% of responses indicated this misconception.

Most students treated dropping and firing as different situations and not as different examples of projectile motion. However, students who believed that a fired object gains impetus when fired tended also to believe that objects dropped from a moving carrier gained no impetus and so dropped straight down. Apart from this association, and contrary to the findings of McCloskey and others, the students had no set framework for predicting the motion of a projectile. Instead, they seemed to have mini-theories for each specific situation (Millar & Kragh, 1994). Nearly every student used a mixture of Aristotelian, impetus and Newtonian theories, applying different conceptions in different situations (Halloun & Hestenes, 1985a, b). Indeed, some students realised that their ideas contradicted earlier answers in the interview and wrote “to be consistent I will say …” or “I know this contradicts what I said before but …”.

Most students also indicated misconceptions about gravity: all but three students at some point gave a response which was inconsistent with the Newtonian conception that gravity is (in the situations discussed) a constant force acting vertically downwards. In fact, one could claim that it was students’ inadequate concept of force that lay at the basis of all the misconceptions found in this study.

Students frequently mentioned that they were trying to imagine what was happening in each problem situation. In other words, they were looking for attributes within each context that would help them answer the questions rather than applying general principles that would work for all situations. It is clear that most students did not recognise the underlying similarities between the different projectile situations, a prerequisite for the abstraction of general principles (Mitchelmore & White, 2004). These similarities are, of course, not obvious, and it took many centuries and the genius of Galileo and Newton to identify them. But without the general principles that Newton enunciated, it is not possible to meaningfully apply such techniques as finding equations of motion by considering separately the horizontal and vertical forces on a moving body.

A closer examination of the responses given by the physics students in the present sample indicated that they were only slightly less susceptible to misconceptions about projectile motion that those who had not previously studied projectile motion. This result suggests that the teaching of Newton’s Laws they had experienced the previous year had probably not led to any marked change in their conceptualisation of projectile motion.

Implications

Given that the students in this study came from two academically distinguished schools, it may be inferred that a great proportion of Australian Year 12 students hold misconceptions about projectile motion that are likely to seriously affect their mathematical study of this topic (Gunstone & White, 1981). It is therefore incumbent upon educators to seek a way of eliminating, or at least reducing the effect of these misconceptions.

Another part of the wider study (Prescott, 2004) investigated the teaching of projectile motion and is reported in a separate paper (Prescott & Mitchelmore, 2005). Three findings are relevant here: Firstly, as for the teaching of physics noted above, traditional mathematics teaching seems to have little or no effect on students’ misconceptions. Secondly, teachers themselves seem to hold many of the same misconceptions about projectile motion that their students do. Thirdly, it does seem to be possible to teach projectile motion in Year 12 mathematics classes in such a way as to reduce students’ misconceptions—but that eliminating them altogether would require a greater investment of time than can be made in the Year 12 syllabus. One solution may be greater coordination between the science and mathematics curricula.

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