Education for Early Mathematical Literacy: More Than Maths Know-How

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Helping young students develop mathematical literacy involves promoting their development of mathematical knowledge, but also their ability to adapt and apply mathematical knowledge to a range of tasks. Extensive research has been directed towards the development of large-scale programs that have successfully promoted young children's acquisition of early mathematical knowledge. Less research has explored the application of mathematic knowledge to tasks which cross disciplinary boundaries. This paper argues for further research of this type, to support the development of approaches that promote comprehensive early mathematical literacy.

What is the purpose of early mathematics education? Until recently, mathematics curricula have largely presented mathematics education as being about the production of disciplinary knowledge. That is, as about assisting students to acquire the repertoire of knowledges, skills and practices considered the essential contents of the discipline of mathematics. Mason and Spence (1999) described this type of education as fundamentally concerned with teaching students to know *about* mathematics. This view of mathematics education has been particularly influential during the first three years of school, deemed foundational years, and assigned the task of supporting students to acquire a basic mathematical 'tool kit' in preparation for later participation in sophisticated mathematical tasks such as reasoning and problem solving (Anderson, 2003).

In recent times, mathematics educators have placed increasing emphasis on the importance of "knowing-to" (Mason & Spence, 1999, p. 135). Instead of equating mathematics education with teaching students about mathematics, mathematics educators have come to view mathematics education as also involving teaching students to know how and when to use their mathematical knowledge (e.g., National Council of Teachers of Mathematics, 2000; Steen, 1997). It is the combination of knowing about mathematics and knowing when, where and how to apply this knowledge that is mathematical literacy: the ability to apply and use mathematical knowledges, skills and practices, in flexible and adaptive ways (Willoughby, 2000).

Considerable research attention has been directed towards identifying effective ways of assisting young children to develop mathematical knowledge, skills and practices. In Australia, three large-scale research programs have each developed a learning and assessment framework describing how children develop important mathematical constructs. These programs have also designed associated programs of professional development to enhance early childhood teachers' mathematical content and pedagogical knowledge. *Count Me In Too* (New South Wales Department of Education and Training, 1998), the *Early Numeracy Research Project* (Clarke, 2000; Clarke, Gervasoni & Sullivan, 2000), and *First Steps* in Western Australia (Treacy & Willis, 2003; Willis, 2000, 2002) have each demonstrated considerable success in enhancing young students' knowledge about mathematics (see Bobis, Clarke, Clarke, Thomas, Wright & Young-Loveridge, 2005). Each of these programs has the proven ability to equip students with mathematical tools. However, these programs do not actively promote or assess students' ability to apply these tools to the solution of real life problems, or work that crosses disciplinary boundaries. This paper argues that further research should be directed towards developing knowledge about ways in which young students might learn to apply mathematical learning to the solution of problems across a range of contexts, and proposes a program of research that could contribute to such knowledge. This argument begins with a discussion of why mathematics education should be concerned with promoting students' abilities to solve problems which cannot be contained within disciplinary boundaries, followed by a discussion about whether or not it might be appropriate to involve young children in problem solving activities of this type.

Problem solving across disciplinary boundaries

A person cannot be considered mathematically literate if he/she is unable to apply mathematical learning to the solution of real life problems. The solution of real life problems increasingly requires individuals to work not only within disciplines, but across and between those disciplines. For example, in the world of work, the rapidly changing demands of the global economy have caused organisations to re-orient themselves towards "continuous innovation" (Rifkin, 2000, p. 49), and "mutual, spontaneous learning" (Hargreaves, 2003, p. 17). This environment has created a need to apply and adapt knowledge production to the solution of an expanding range of urgent and unpredictable problems (Homer-Dixon, 2000). Certain of these problems resist solution from within the boundaries of traditional disciplines, and instead require individuals to bring a range of practical and theoretical understandings to bear as they work towards problem solution (Klein, 2004). Sometimes, problem solution can be achieved via multidisciplinary activity, wherein knowledges drawn from several disciplines are applied separately to the problem task (Bruce, Lyall, Tait & Williams, 2004). Other problems may demand interdisciplinary activity, wherein distinct forms of disciplinary knowledge must be negotiated and integrated to develop a solution. Even more complex problems may demand a solution the shape of which is "beyond that of any single contributing discipline: (Gibbons, Limoges, Nowotny, Schwartzman, Scott & Trow, 1994). Solution of such problems requires transdisciplinary activity, wherein all forms of disciplinary knowledge are treated as part of a unified conceptual frame from which new knowledge is articulated (Ramadier, 2004). Individuals engaged in this type of problem solving must contribute, combine, adapt and innovate upon practical and disciplinary knowledge to produce new knowledge which is both context and problem-specific (Klein, 2004, p. 517). To be prepared to engage in these different types of problem solving, students need to develop comprehensive and portable mathematical literacy that they can adapt and apply to a range of tasks. This is mathematical 'know-to' in action, an important component of mathematics education that seeks to equip students for full and active participation in the contemporary knowledge society.

Problem Solving and Early Mathematics Education

Problem solving is central to mathematical literacy at all ages (D'Ambrosio, 2003; Lambdin, 2003; National Council of Teachers of Mathematics, 2000). In line with the changing demands of contemporary society, a comprehensive mathematics education acknowledges that, in addition to acquiring a repertoire of mathematical knowledge, "[students] also need to learn how to use their existing knowledge when they are confronted by new problems in novel contexts. There is little point being 'numerate' if they cannot apply what they know" (Hughes, Desforges, Mitchell & Carré, 2000, p. 2).

Queensland's recently published new mathematics curriculum expounds this view when it states that, in addition to knowing about and knowing how to do mathematics, students must know when and where to use their mathematical knowledge (Queensland Studies Authority, 2004).

Young children should not be excluded from opportunities to apply mathematical learning. Current research evidence demonstrates the positive effects of helping students learn to apply mathematical knowledge in disciplinary settings. The mathematical learning of even very young children can be enhanced by engaging them in problem posing (Lowrie, 2000), problem solving (English, 1997, 2004), and mathematical investigations (Diezmann, Watters & English, 2002). However, as the following discussion of existing research will show, it is still not known how participation in extra-disciplinary activity might impact the mathematical literacy development of children in the early years of school, despite strong advocacy for such approaches in literature from the field of early childhood education (e.g., see Katz & Chad, 2000).

Existing Research

Mathematics educators have made a number of attempts to investigate the impact student involvement in activities requiring the application of mathematical learning within and across disciplinary boundaries, with various models of curriculum integration proposed, implemented and evaluated (Budgen, Wallace, Rennie & Malone, 2003). In the United States, a range of "integrated mathematics" programs was developed in response to the NCTM's (1989; 2000) identification of making mathematical connections as one of four process standards spanning the mathematics curriculum. Many of these programs, such as *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel & Phillips, 1996), *Mathematics in Context* (National Center for Research in Mathematical Sciences Education and Freudenthal Institute, 1997-1998), *Investigations* (TERC, 1998) and *Everyday Mathematics* (Everyday Learning Corporation, 1996), have focused only on supporting students to make connections between knowledge in various mathematical domains, rather than with knowledge in disciplinary fields external to mathematics. Others have taken a broader interdisciplinary approach, such as *Maths Trailblazers* (Wagreich, Goldberg & Staff, 1997) and *Investigations* (Mokros, Russell & Economopoulos, 1995), by investigating connections with other disciplinary fields.

Research undertaken to evaluate these programs has tended to focus on the effects of participation on students' performance on traditional standardised tests, which may not represent an authentic measure of mathematical literacy, but simply evidence students' ability to 'do mathematics'. For example, in their discussion of the *Everyday Mathematics* program, Carroll and Isaacs (2003) described how the program had impacted students' mental computation, number sense, geometrical knowledge and multidigit computation. When discussing *Maths Trailblazers*, Carter and associates (2003) reported students' performance on a test of mathematics basic skills as evidence of the program's effectiveness. In Mokros' (2003) discussion of the *Investigations* project, she did describe how the program had impacted students' proportional reasoning related to mathematics, but focused on computational and word problems, rather than complex real-life problems. No mention was made in any of these evaluations of the various programs' effects on students' ability to apply mathematical reasoning to complex real-life problems.

In addition to these programs, which claim to promote connectedness but generally remain firmly bounded within the discipline of mathematics, a range of international studies has explored how mathematics, science and technology learning can be integrated within technology-based tasks (e.g., Bruce et al., 2004; Budgen et al., 2003; Sinn, Walthour & Haren, 1993; Wicklein & Schell, 1995). Studies conducted in Australian contexts have also investigated the effects of integrating mathematics, science and technology (Nason & Woodruff, 2003; Norton, 2004; Venville, Rennie & Wallace, 2004; Venville, Wallace, Rennie & Malone, 1998, 2000). These studies describe attempts to enhance students' understandings of mathematics by involving them in multidisciplinary and interdisciplinary activity, but examine the mathematical literacy development of middle or secondary school students, rather than those in the early years of school. As yet, there is little research addressing the question of whether participation in problem solving which crosses disciplinary boundaries enhances or constrains young students' development of mathematical literacy. This may be due to a particular view of the early years, a view described by Lesh and Doerr (2003) as one which frames the early years as years best spent learning the prerequisite knowledge, skills and practices for later application to mathematical problem solving.

Research Opportunities

In Australia, an opportunity to conduct research on how the mathematical literacy development of young children might be impacted by participation in tasks that require them to work both within and across disciplinary boundaries is presented by the trial of the *New Basics Project* (Education Queensland, 2000). The *New Basics* is a program of school reform which seeks to support students to learn the knowledge, skills and capacities needed for participation in future life roles by promoting consolidated disciplinary learning and developing students' capacities for transdisciplinary activity. Rather than attempting to reform early mathematics education in isolation from other disciplines, the *New Basics Project* attempts to reconceptualise the entire school curriculum, and includes all levels of compulsory schooling.

Some research conducted during the trial of the New Basics Project has examined the effects of engaging students in transdisciplinary learning activities on their mathematical learning (Lerman, 2004; Renshaw, 2004; Zevenbergen, 2004). To date, this research has focused on the mathematical learning of students at upper primary level, rather than in the early years of school. Conducting similar research with groups of young children might reveal the impact involving these children in transdisciplinary activity has on their mathematical literacy development.

Examining the practices of teachers and students in early years classrooms involved in the New Basics Project would offer an opportunity to explore how mathematical literacy development is impacted by student participation in an educational program which includes transdisciplinary learning tasks. A number of cases could be investigated, with each case comprising an early years class situated within a school participating in the New Basics Trial. By investigating three cases, such a study would have the potential to investigate practices across the span of years designated the early years of schooling. Teachers and students who work in these classes could be recruited as research participants. Such a study would collect multiple sources of evidence (Yin, 2003) to describe how learning and teaching of mathematical literacy occurs in each class.

Teacher knowledge could be investigated by engaging teachers in semi-structured individual interviews, during which teachers might engage in concept mapping activities (Novak, 1991, 2004) designed to reveal quantitative and qualitative aspects of their knowledge of mathematics content, mathematics pedagogy, and student cognition as related to mathematics. Teacher participants could also be involved in a focus group interview (Morgan, 2004), during which they would discuss shared understandings of how the New Basics model impacts upon the learning and teaching of mathematical literacy in their classrooms. Learning and teaching practices which occur in each class could be investigated by conducting classroom observations, during which video footage of learning and teaching episodes will be captured (Hall, 2000). Student learning outcomes could be investigated by analysing existing data related to a selected group of students' performance on a range of mathematics assessment instruments. In addition, data could be gathered which describes students' application of mathematical knowledge during transdisciplinary tasks.

The purpose of a study of the learning and teaching practices related to early mathematical literacy development in New Basics classrooms would be to contribute knowledge to the field about how students' mathematical literacy development might be enhanced or otherwise by involving them in activities which transcend disciplinary boundaries. Such a study might build upon the extensive work already done which describes ways to support students to learn more about mathematics, to suggest ways that mathematics educators might assist students to know more about how to use mathematical knowledge across a range of settings.

References

Anderson, J. (2003). Teachers' choice of tasks: A window into beliefs about the role of problem solving in learning mathematics. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 72-79). Sydney: MERGA.

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., & Young-Loveridge, J. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57.

Bruce, A., Lyall, C., Tait, J., & Williams, R. (2004). Interdisciplinary integration in Europe: The case of the Fifth Framework programme. *Futures, 36*(4), 457-470.

Budgen, F., Wallace, J., Rennie, L., & Malone, J. (2003). Rocket racers and submarines: Two Australian case studies of curriculum integration in practice. In S. McGraw (Ed.), *Integrated mathematics: Choices and challenges* (pp. 211-276). Reston, VA: NCTM.

Carroll, W. M., & Isaacs, A. (2003). Achievement of students using the University of Chicago School Mathematics Project's Everyday Mathematics. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based mathematics curricula: What are they? What do students learn?* (pp. 79-108). Mahwah, NJ: Lawrence Erlbaum.

Carter, A., Beissinger, J. S., Cirulis, A., Gartzman, M., Kelson, C. R., & Wagreich, P. (2003). Student learning and achievement with *Math Trailblazers*. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based mathematics curricula: What are they? What do students learn?* (pp. 45-78). Mahwah, NJ: Lawrence Erlbaum.

Clarke, D. (2000). The Early Numeracy Research Project: Some insights from an exciting first year. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd Annual Conference of the Mathematics Research Group of Australasia, pp. 180-187). Sydney: MERGA.

Clarke, D., Gervasoni, A., & Sullivan, P. (2000, December). *The Early Numeracy Research Project: Developing a framework for describing early numeracy learning*. Paper presented at the 2000 Annual Conference of the Australian Association of Research in Education, Retrieved January 28, 2005 from http://www.aare.edu.au/00pap/cla00024.htm.

D'Ambrosio, U. (2003). Teaching mathematics through problem-solving: A historical perspective. In F. K. Lester (Ed.), *Teaching mathematics through problem-solving* (pp. 37-50). Reston, VA: NCTM.

Diezmann, C. M., Watters, J. J., & English, L. D. (2002). Teacher behaviours that influence young children's understanding. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics* (pp. 289-296). East Anglia, UK: International Group for the Psychology of Mathematics.

Education Queensland. (2000). *New Basics Project technical paper*. Retrieved January 28, 2005, from http://education.qld.gov.au/corporate/newbasics/docs/nbftech.doc

English, L. D. (1997). Promoting a problem-posing classroom. *Teaching Children Mathematics, 4*(3), 172-179.

English, L. D. (2004). Mathematical and analogical reasoning in early childhood. In L. D. English (Ed.), *Mathematical and analogical reasoning of young learners* (pp. 1-22). Mahwah, NJ: Lawrence Erlbaum Associates.

Everyday Learning Corporation. (1996). *Everyday Mathematics: Student achievement studies*. Chicago, IL: Author.

Gibbons, M., Limoges, C., Nowotny, H., Schwartzman, S., Scott, P., & Trow, M. (1994). *The new production of knowledge*. London: Sage Publications.

Hall, R. H. (2000). Videorecording as theory. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 647-664). Mahwah, NJ: Lawrence Erlbaum.

Hargreaves, A. (2003). *Teaching in the knowledge society: Education in the age of uncertainty*. New York: Teachers College Press.

Homer-Dixon, T. (2000). *The ingenuity gap: Can we solve the problems of the future?* Toronto: Alfred A. Knopf.

Hughes, M., Desforges, C., Mitchell, C., & Carré, C. (2000). *Numeracy and beyond: Applying mathematics in the primary school*. Buckingham, PA: Open University Press.

Katz, L., & Chad, S. G. (2000). *Engaging children's minds : The project approach* (2nd ed.). Standford, CT: Ablex.

Klein, J. T. (2004). Prospects for transdisciplinarity. *Futures, 36*(4), 515-526.

Lambdin, D. V. (2003). Benefits of teaching through problem solving. In F. K. Lester (Ed.), *Teaching mathematics through problem-solving* (pp. 3-13). Reston, VA: NCTM.

Lappan, G., Fey, J. T., Fitzgerald, W., Friel, S. N., & Phillips, E. D. (1996). *A guide to the Connected Mathematics curriclum: Getting to know Connected Mathematics*. Palo Alto, CA: Dale Seymour.

Lerman, S. (2004). Researching numeracy teaching and learning with ICT: Facing the problems of innovation. In I. J. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 618-621). Sydney: MERGA.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematical problem-solving* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum Associates.

Lowrie, T. (2000). Grade 1 children in problem-posing contexts: Problem solving prior to completing the task. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd Annual Conference of the Mathematics Research Group of Australasia, pp. 385-392). Sydney: MERGA.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38*(1-3), 135-161.

Mokros, J. (2003). Learning to reason numerically: The impact of *Investigations*. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based mathematics curricula: What are they? What do students learn?* (pp. 109-131). Mahwah, NJ: Lawrence Erlbaum.

Mokros, J., Russell, S. J., & Economopoulos, K. (1995). *Beyond arithmetic: Changing matehmatics in the elementary classroom*. Palo Alto: Dale Seymour.

Morgan, D. L. (2004). Focus groups. In S. Nagy Hesse-Biber & P. Leavy (Eds.), *Approaches to qualitative research: A theory on research and practice* (pp. 263-285). New York: Oxford University Press.

Nason, R., & Woodruff, E. (2003). Fostering authentic, sustained, and progressive mathematical knowledge-building activity in computer collaborative learning (CSCL) communities. *Journal of Computers in Mathematics and Science Teaching, 22*(4), 345-363.

National Center for Research in Mathematical Sciences Education and Freudenthal Institute. (1997-1998). *Mathematics in Context*. Chicago, IL: Encycolopaedia Britannica.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Retrieved October 4, 2004, from http://standards.nctm.org/document/chapter1/index.htm

New South Wales Department of Education and Training. (1998). *Count me in too: Professional development package*. Ryde, NSW: Professional Support and Curriculum Directorate.

Norton, S. (2004). Using Lego construction to develop ratio understanding. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 414-421). Sydney: MERGA.

Novak, J. D. (1991). Clarify with concept maps: A tool for students and teachers alike. *The Science Teacher, 58*(1), 45-49.

Novak, J. D. (2004). Reflections on a half-century of thinking in science education and research: Implications from a twelve-year longitudinal study of children's learning. *Canadian Journal of Science, Mathematics and Technology Education, 4*(1), 23-42.

Queensland Studies Authority. (2004). *Mathematics: Years 1 to 10 syllabus*. Brisbane, QLD: Author.

Ramadier, T. (2004). Transdisciplinarity and its challenges: The case of urban studies. *Futures, 36*(4), 423-439.

Renshaw, P. (2004). ICT-Numeracy practices in a school serving a disadvantaged community. In I. J. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010* (pp. 622-625). Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia. Townsville, Australia.

Rifkin, J. (2000). *The age of access: The new culture of hypercapitalism where all of life is a paid-for experience*. New York: Vintage Books.

Sinn, J., Walthour, S., & Haren, D. (1993). Technology based mathematics and science applications: One approach. *The Technology Teacher, 53*(1), 29-36.

Steen, L. A. (1997). Preface: The new literacy. In L. A. Steen (Ed.), *Why numbers count: Quantitative literacy for tomorrow's America* (pp. xv-xxviii). New York: College Entrance Examination Board.

TERC. (1998). *Investigations in number, space and data*. Palo Alto, CA: Dale Seymour.

Treacy, K., & Willis, S. (2003). A model of early number development. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (Proceedings of the 26th Annual Conference of the Mathematics Education Group of Australasia, pp. 674-681). Sydney: MERGA.

Venville, G., Rennie, L., & Wallace, J. (2004). Decision making and sources of knowledge: How students tackle integrated tasks in science, technology and mathematics. *Research in Science Education, 34*(2), 115-135.

Venville, G., Wallace, J., Rennie, L., & Malone, J. (1998). The integration of science, mathematics, and technology in a discipline-based culture. *School Science and Mathematics, 98*(6), 294-302.

Venville, G., Wallace, J., Rennie, L., & Malone, J. (2000). Bridging the boundaries of comparmentalised knowledge: Student learning in an integrated environment. *Research in Science and Technological Education, 18*(1), 23-35.

Wagreich, P., Goldberg, H., & Staff, T. P. (1997). *Maths Trailblazers: A mathematical journey using science and the language arts*. Dubuque, IA: Kendall/Hunt.

Wicklein, R. C., & Schell, J. W. (1995). Case studies of multidisciplinary approaches to integrating mathematics, science and technology education. *Journal of Technology Education, 6*(2), 59-76.

Willis, S. (2000, 15-17 October). *Strengthening numeracy: Reducing risk.* Paper presented at the Improving Numeracy Learning: Research Conference 2000, Melbourne, Australia.

Willis, S. (2002). Crossing borders: Learning to count. *The Australian Educational Researcher, 29*(2), 115-129.

Willoughby, S. S. (2000). Perspectives on mathematics education. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 1-15). Reston, VA: NCTM.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage Publications Inc.

Zevenbergen, R. (2004). Using ICTs to support numeracy learning: The way of the future? In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 618-624). Sydney: MERGA.