Language Appropriate for the New Zealand Numeracy Project

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The Numeracy Project suggests a language model based on teachers listening and questioning students' justifications and explanations. This paper presents the analysis of the discourse in one classroom lesson and compares it to the language features identified in the Numeracy Project. The interview and lesson analysed here display features that appear to be consistent with the aims of the project. There is also evidence to suggest that students used some characteristics of this discourse with one another when working on problems together. The discourse in a second classroom provides a contrast. The Numeracy Project places little emphasis on using correct mathematical terms and presenting complete evidence of the forms that guide more advanced mathematics. It is suggested that educators become aware of these weaknesses and place an emphasis on these aspects of mathematical discourse in addition to those aspects promoted by the Numeracy Project to increase the likelihood of the students developing mathematical language and thinking.

The essence of the New Zealand Numeracy Development Professional Project, according to its guide information, is "improving student performance in mathematics through improving the professional capability of teachers" (inside front cover, Ministry of Education, 2004). To do this the Project makes explicit and implicit recommendations for classroom discourse. It models a question format in which emphasis is placed on students explaining their thinking while solving a problem. They are expected to do this with one another as well as with their teacher. In doing this, the audience changes from the traditional classroom focus of the teacher, to that of convincing peers and of students convincing themselves. These features are similar to those promoted in American reform mathematics and emphasised in research on socio-mathematical norms (e.g. Forman, Larreamendy-Joerns, Stein & Browns, 1998; Yackel, Cobb, Wood, & Merkel, 1990) Classroom discourse has several components that can be distinguished. Although this paper separates some of these components, this is for the purpose of analysis only. Dialogue or conversation is an integrated whole, particularly when it involves expectations for each party's contribution. In this paper we concentrate on the following aspects of discourse: vocabulary, dialogue between teachers and students, dialogue between peers, and the structure of phrases.

This study analyses the discourse of a teacher who had participated in Numeracy Project training. Some comparisons are made to a teacher from a similar school who had also participated in Numeracy training. They turned out to be markedly different in their use of language, both in assessing their students and in their classroom discussions. This contrast helped draw our attention to differences between teachers in the same project. The focus teacher's classroom language had similar features to those of the Numeracy Project interviews. This analysis relates to a particularly interesting part of a larger study that will follow this teacher and the discourse in her class over two years.

The discourse sample came from a series of probability lessons, a curriculum strand only recently covered by the Numeracy Project. By 2006, all teachers will have had access to the Numeracy Project so its potential for influence is great. This study first identifies some of its language features and then considers whether these are present in the discourse of class lessons.

Background

Language is regarded as important in the New Zealand mathematics curriculum. This curriculum (Ministry of Education, 1992) describes one of its aims as providing opportunities for students to “develop the skills and confidence to use their own language, and the language of mathematics, to express mathematical ideas” (p.23). Pimm (1987) supports the idea of the mastery of mathematical language imbuing the communicator with power as he sees communication as part of being a mathematician. If mastery is important, then we must know how to identify, and then how to teach, mathematical language. Khisty and Chval (2002) found teachers are often unaware of when it is appropriate to use mathematical language and when to use everyday language.

The mathematics register may not receive the attention that it deserves in New Zealand classrooms. Some studies of mathematical register have concentrated on overall classroom discourse (e.g. Khisty & Chval, 2002; Pimm, 1987). Other researchers concentrate on specific terms (e.g. Cowan, 1991) or linguistic forms (e.g. Presmeg, 1997). There is also a considerable amount of literature on mathematical discourse and learners of English as an additional language (e.g. Hofstetter, 2003; Moschkovich, 1999), work which is particularly relevant to the students in this study.

The goal of mathematics learning is to think mathematically, including reasoning and questioning, and not just to get the right answers (Burns, 1985; Cobb & Bauersfeld, 1995; Hufferd-Ackles, Fuson, & Shein, 2004). Focussing on the explanation and justification of a student’s method rather than their answer encourages mathematical thinking (Steffe, Cobb, & von Glasersfeld, 1988; Yackel, Cobb, Wood, & Merkel, 1990). Hufferd-Ackles et al. (2002) believe asking students to explain their methods leads to the students offering explanations as a normal part of their answer. Children need to decide on the reasonableness of their answers, justify their methods and verbalise them and then to reflect on their thinking (Forman, Larreamendy-Joerns, Stein, & Browns, 1998; Krummheuer, 1995). Radical constructivists believe that knowledge only exists in the shared constructions of the learners and knowledgeable others (Steffe et al., 1988; Yackel, et al., 1990) so the students need to be the ones who voice the abstracted mathematical meaning. This theory supports the teachers as questioners and guides rather than ‘answer givers’.

Using argument-based discourse encourages participation (Forman, Larreamendy-Jones, Stein, & Browns, 1998; Krummheuer, 1995). To assign roles in the discussion, the teacher may use revoicing. This technique involves either direct repetition of the student’s answer or expanding it and/or rephrasing it in another register (Forman et al., 1998).

Several studies have found that children mirror the teacher’s language (Fullerton, 1995; Khisty & Chval, 2002; Raiker, 2002). This means that the language that the teacher uses is an important factor in determining the quality of language the children speak.

The Study

Participants

The teacher who was the focus of this study is a New Zealand European. All but one of the students in her year 5 and 6 class (age 9 and 10) were of Pacific Island descent. Some of these students were born in New Zealand and others had recently arrived, having been schooled in the language of their home country. Their command of English was an issue. The teacher used for comparison taught a similar group of students.

Method

Each teacher was videotaped while giving four or five individual assessment interviews and while teaching a whole group. After the whole group session, small groups of students were videotaped while carrying out their assigned mathematical tasks. By chance, both teachers were teaching a unit from the statistics and probability strand of the curriculum. In both cases, the second author observed classes before videotaping and spoke informally with the students to allow them to be familiar with her and obtain their consent to be videotaped.

Analysis

Digital videotapes were transferred to DVDs and transcribed. Intensive analysis was done on similar sections from each teacher to see if the language features identified in the Numeracy Project material were present in the samples. Patterns were checked with the full transcript to see whether or not they were representative.

Results and Discussion

From the data available we can identify characteristics of the teacher's questioning, her listening - by measuring the time she waits, her response to students' answers, and the focus of the dialogue including whether it is on answers or the thinking process. These were first searched for in the interviews to provide evidence that the teacher had adopted them. This set the framework for the analysis of the mathematical discourse in the one class lesson transcribed, and in one conversation between students while working on a set of problems.

The Teachers' Questioning, Wait Time, and Responses to Students

The interview sets the model for how the teacher is to ask questions and expectations for responses. It is divided into two sections. In the knowledge section, these closed questions require one answer. For the strategy questions, they are relatively open questions that request a child to explain his or her thinking. The focus here is only on the strategy questioning.

This teacher's questioning was exactly as prescribed in the interview script, although the teacher had memorised it and used a conversational tone. Her questioning made it clear that she was interested in how students thought rather than in particular answers.

She waited for long periods for students to answer. Pimm (1987) notes that silence after a question gives students time to think. Several of the teacher's wait times were over 30 seconds and one was 48 seconds. An example was:

Teacher: At the car factory they need 4 wheels to make each car. How many cars could they make with 72 wheels?
Student: [after 41 seconds] Not sure.
Teacher: Not sure. You don't want to just give it a try?
Student: [after 48 seconds] Oh, I lost my count
Teacher: OK. Do you want to tell me how you were working it out so far.
Student: I was using my four times table and 4 wheels is one car, 8 wheels is 2 cars.
Teacher: Working it out that way.

This passage provides evidence of the teacher's expectations of the student. The student's responsibility was to do some thinking and be able to explain that thinking rather than just come up with an immediate and accurate answer.

The teacher's response indicates that she appreciates the way that he was working out the answer and that his explanation of his thinking was adequate for her to score the strategy used for this item. The students appear to recognise this focus on thinking. One student evaluates the teacher's question, congratulating her on having a "good question" before he answered.

Another indication of the importance that this teacher placed on the student doing the thinking and explaining was the ratio of words that she used in comparison to those used by the student. This ratio was 3:2. In comparison, the ratio of words used by the other teacher observed to that of her students was 3:1. In one of the other teacher's interviews, a child spoke 6 words during four minutes and five seconds of an interview question; the teacher spoke 405 words. That teacher had very few periods of silence. If she thought that the student was not going to succeed she reworded the question, presented materials to help the student work the problem out, and sometimes talked over the student in her eagerness to have the student succeed. This characteristic of teachers, to have their students succeed because the teacher knows the answer, has been called "teacher lust" by Maddern and Court (1989). It is a characteristic that all teachers need to be aware of and control if they want their students to do the thinking.

A teacher who adopts the pedagogy of the Numeracy Project will have some of the same questioning and response techniques in her class teaching, although class teaching will also have some instances of instruction when necessary. The students working in groups should also adopt some aspects of the same discourse, in that they should be interested in each other's thinking and ask for it to be explained when necessary. They also need to be able to evaluate their own answers. In the portion of the class that was led by this teacher, she used a similar pattern of acknowledging but not immediately evaluating students' responses. She asked for other students' responses and then asked them to evaluate. The following transcript comes from the introduction to a probability lesson about playing cards, in which students were asked: "Can you tell me, using likely, unlikely, and impossible that she would pick a card that would be less than ten."

Teacher: What do you think Chris?
Student 1: Unlikely
Teacher: Unlikely. OK what do you think?
Student 2: Likely
Student 3: Likely
Student 4: Likely
Student 5: Likely
Teacher: Is there anyway we can prove this?

The balance of teacher and student talk in a classroom is a good index of whose job it is to do the thinking. The ratio of teacher to student talk is usually much higher in the period in which the teacher is working with the whole class. The pattern of the discourse is usually that of Teacher, Student 1, Teacher, Student 2, Teacher, Student 3, etc. and can be pictured as a star. The pattern of the discourse is traditionally that of teacher's initiation, student's response, and teacher's evaluation (IRE) (see Cazden, 2001, for example). This pattern assumes that teachers are asking questions that they know the answers to and that the students' task is to find the answer that the teacher has in mind. Frequently, the teacher does the vast majority of the talking and, presumably, of the thinking. This teacher rarely used this pattern of discourse in the lesson analysed. The ratio of teacher's words to students' words in this instruction period was 5:1, with many of the teacher's words being ones that showed that she was listening, like okay or yes. She often revoiced the students' answers. This is also a technique evident in the lesson scripts of the Numeracy Project. Revoicing provides a second opportunity for students to hear a good model of speaking

(Khisty & Chval, 2002). O'Connor and Michaels (1996, cited in Forman, Larreamendy-Joerns, Stein, & Browns, 1988) believe that it may also help students "see themselves and each other as legitimate participants in the activity of making, analysing, and evaluating claims, hypotheses, and predictions." (p. 78) The teacher's discourse is the same regardless of ability. This high expectation of quality thinking means the students are not restricted by the "discourse of the less able" as Brown, Eade, and Wilson (1999) phrase it.

Her students asked questions of one another in the whole class session, sometimes spontaneously and sometimes when prompted. Most of the teacher's follow-up questions are for justifications. She does give explanations when she believes they are needed. Only at the end of the extended dialogue does she evaluate the students' work.

Children rely on this pattern. They seem to be anticipating the teacher's explanation question and so do not verbally provide the teacher with an explanation or justification until she asks them. The teacher does not model a complete mathematical statement and she does not ask the students to combine the elements of the statement into one complete sentence either. This also seems to be an aspect that should be fostered in the Numeracy Project. There is very little research on students' use of mathematical statements, yet using mathematical statements as part of language is often cited as one of the goals of mathematical thinking (Pimm, 1987). Kristy and Chval (2002) found that pushing students to use complete sentences was rewarded by students using such statements in later lessons.

Student-to-student Discourse

The real test of how well students understand this pattern of discourse is whether or not they use it among themselves. The teacher gave explicit instruction on how to structure their discourse. She asked the children to question implicitly and evaluate each other's thinking and to play the role of the teacher: "…Before you write it down I want you to justify it to your partner. So if you say there's eight queens you partner needs to say, 'How do you know that there's eight queens?'" Forman, Larreamendy-Joerns, Stein, and Browns (1988) and Sierpinska (1999) suggest that there needs to be shared understanding of how the discourse is structured in the classroom. Delpit (1988) found some of the rules about how to talk are left implicit and some students may be excluded.

There are some examples of the modelled and suggested discourse pattern in the student-to-student dialogues captured on tape. More examples would be needed to claim that students had adopted the discourse that emphasised how the answers were obtained rather than the correct answer. However, analysis of one provides a framework for further analysis of such dialogues. The discussion analysed involved a discussion of the mathematical equation for the probability of drawing a red card from a pack. This discussion included claims, challenges, counter claims, and explanations.

Student 2	I think I know how to work it out. And ten (places a ten on top of another ten)
Student 1	So see there's 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 22, 26 so there's 26 packs
Student 2	No. cause it's 2…
Student 1	26 cards
Student 2	No wait 2 x 13 and that's 26. The two stands for there's two suits and there's 13 altogether. Thirteen in the red cards. See
Student 1	Yip and that equals 26 … 36
Student 2	26
Student 1	I mean 26.
Student 2	No we've got to make times thirteen to make 13
Student 1	Thirteen is an odd number you can't divide 13. (unintelligible) Only an even number like 12 or 18
Student 2	yeah, but we …

Student 1 See 13 x 2 equals 26 and 26 is an even number.
Student 2 Oh yeah… that is right
Student 1 And then the two the thirteen in it
Student 2 And there's 26
Student 1 That two suits and thirteen, thirteen …thirteen, thirteen is
Student 2 no it's thirteen altogether in red.
Student 1/ Student 2 Thirteen altogether in red. (Children say it in unison as they write it)

This passage has some of the characteristics of joint problem solving between peers. Although not equal in the number of their concessions and requests for agreements and explanations, they are listening to each other and are actively engaged in making sense of the mathematics of their task.

Audience

In the focus class, the students use self-talk in the whole class section of the lesson. While the teacher herself does not think aloud in this lesson, she gives the students 'air-time'. "Give him time to think," she instructs a few eager students at one point. The child mumbles his answer to himself then he faces the class and explains his thinking to them. Twice in the lesson, the children faced the class to explain their answers. The student takes the role of the teacher.

The students in the focus class entered into discussion with each other about the activities as shown in the dialogue in the previous section. While the students were on task in the other class, we were not able to capture any student-to-student discussion. They worked in pairs but did not discuss their thinking.

Use of Mathematical Vocabulary

Some of the students in the focus class did have difficulty with terms used in mathematics. The second author observed two classes in which students were struggling to distinguish the meaning of "likely" and "unlikely", a difficulty that the teacher attended to. However, the teacher also used the colloquial language used by the children rather than mathematical vocabulary, substituting "sum" for equation and "timesing" for multiplying. In another instance, she repeats a child's answer which uses the term "small numbers" instead of revoicing using a mathematical term, for example, "numbers less than 10." One student had difficulty remembering the term for ½, jokingly calling it a "twoth" and a "second." All of these language uses are understandable. A teacher wants to be understood and it is easiest to use the common language of students, such as "timesing".

Khisty and Chval (2002) found that a classroom rich with mathematical language nurtured mathematical thinking, as the children had the words to express themselves. The teacher in their report took a class of students with English as an additional language from being two-years behind grade level in achievement to being two-years above grade-level. One of the key success factors identified was that the teacher introduced mathematical terms early, often using revoicing of the student's answers with the correct terms, and expected her students to use them. Brown and Renshaw (2004) call this a replacement pattern of discourse. Walls (2004) suggest that there is very limited emphasis in the Numeracy Project on introducing and using correct mathematical terms or the mathematics register in general. The advice given in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) may compound the difficulty as it instructs teachers to use everyday language with their students before introducing mathematical language. Use of children's language versus the use of mathematical language is an additional issue in classrooms where students do not have a firm grasp of English.

One recommendation that could be made on the basis of this analysis of a teacher's and Pasifika students' language is that there is a place in the Numeracy Project for emphasis on using mathematical terms that will enable students to master more complex mathematics, rather than relying on the students' everyday language.

Conclusions

The discourse samples discussed in this paper were from everyday lessons overlaid with all the constraints of classroom teaching. They provide a snapshot: an in-depth study of a small sample. Neither of the researchers knew the dynamics of the classrooms in depth or the reasons for which the teachers chose their method of instruction. Due to unforseen factors, it was not possible to assess students' progress so we do not know if their discourse could be related to their expected progress.

We are therefore comparing the discourse of the students to the findings of other studies and to theory. This comparison suggests that listening to what students understand, and letting them work out their own methods and understandings of concepts in a peer-dominated discourse, promotes mathematical thinking. This is likely to occur in this class. We believe that the mathematics in this class and in the Numeracy Project in general could be strengthened by strong focus on the importance of terms and discourse patterns which lead to understanding higher mathematics. In this class, everyday language dominated with limited suggestion of the need to use mathematical vocabulary. Students were not encouraged to speak whole mathematical statements, and received only minimal help in learning the mathematics register. We postulate that emphasising the language of mathematics would be particularly valuable for these students from a non-English speaking background. They would benefit from teachers modelling complete statements and encouraging students to verbalise them. Additional emphasis on the mathematics register and full mathematical phrases would strengthen their model of mathematical language.

It is inappropriate to make too many generalisations from two samples, but the evidence we have suggests that some language features of the Numeracy Project have been adopted by the teacher focused upon in teaching probability. She used questioning to identify what the students understand. This may show that the Numeracy Project has influenced the teaching of other curriculum strands.

If several lessons at various points throughout the year were analysed, a fuller picture of the students' progress could be obtained. It would then be possible to identify the language features they enter the class with. It might also be possible to see which of the teacher's language features were adopted by the students at various times, and how the teacher fosters this development.

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