Mathematics Teachers and Working Mathematically: Responses to Curriculum Change

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As part of the moves to reform mathematics teaching in New South Wales, the Years 7-10 Mathematics Syllabus emphasises *working mathematically*. This paper presents the results of interviews conducted with 39 teachers to examine how they interpret the aims of working mathematically and the extent to which they are implementing working mathematically in their classrooms. The results indicate that while a small number of teachers have embraced the reforms, most have a limited conception of what working mathematically means and have not made any substantial change in their practice. The possible reasons for this situation are discussed.

Calls to reform the teaching of school mathematics are long established. In the United States, the National Council of Teachers of Mathematics [NCTM] produced the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) which recognised the need to develop new curricula and innovative forms of mathematics instruction. Whereas traditional mathematics teaching has tended towards procedural learning based largely on drill and practice, the NCTM agenda encourages models of teaching and learning that provide opportunities for students to engage in meaningful problem solving activities. The aim of the reform movement is to develop students' conceptual understanding and improve their capacity to reason, communicate and reflect on their own learning experiences.

The NCTM standards document was followed soon after by a similar report from the Australian Education Council [AEC], the National Statement on Mathematics for Australian Schools (AEC, 1991). The National Statement identified three domains of mathematical thinking: *Attitudes and Appreciations*, *Mathematical Inquiry*, and *Choosing and Using Mathematics*. Each domain is not bound to particular content areas but broadly describes how mathematical knowledge is developed, applied and communicated. The domains include processes such as observing and generalising patterns, problem solving and mathematical modelling, communicating, and the justification of insights.

The syllabus documents in all Australian states and territories now include outcomes that are consistent with the ideas of mathematical thinking described in the National Statement. In New South Wales, the notion of *working mathematically* has been developed by the Board of Studies [BOS] in the Years 7–10 Mathematics Syllabus (BOSNSW, 2002). The Working Mathematically strand of the syllabus incorporates five interrelated processes: questioning, applying strategies, communicating, reasoning and reflecting. The syllabus describes working mathematically as providing "opportunities for students to engage in genuine mathematical activity and to develop the skills to become flexible and creative users of mathematics" (BOSNSW, 2002, p. 45) and the document includes numerous working mathematically teaching ideas for every topic.

Teachers are "encouraged to extend this list [of teaching ideas] by creating their own working mathematically experiences for students to engage with each of the five processes" (BOSNSW, 2002, p. 44). To this end, the syllabus incorporates a *continuum of learning* which recognises that because students learn at different rates and in different ways, some

may not achieve all of the outcomes at a particular stage. The continuum framework is therefore designed so that rather than feeing pressured to cover the entire syllabus content, teachers might take the opportunity to include a greater number of working mathematically activities in their lessons. In addition, the syllabus encourages the use of alternative assessment tasks to complement pencil-and-paper tests, outcomes-based recording and reporting of students' progress, and greater use of ICT resources such as spreadsheets and graphics calculators.

Theoretical Framework

Research has identified a number of significant factors that determine how successfully teachers are able to adopt the kind of reform agenda described here. The role of teachers' beliefs is critical in their attitude to the kinds of changes proposed in many new syllabus documents. Beliefs about the nature of mathematics and about how best to teach mathematics are closely connected (Cooney & Shealy, 1997) and often deeply entrenched (Wilson & Goldenberg, 1998). Therefore any transition from traditional instruction practices to a more constructivist approach must be accompanied by adjustments in the beliefs and understandings that underpin those practices (Goldsmith & Schifter, 1997; Thompson, 1992).

Teachers' beliefs not only impact on their classroom instruction but are also an important element in their interpretation of the reform movement itself. Woodbury (2000) studied four mathematics teachers who were trying to improve their practice in accord with the NCTM standards. Each teacher completed surveys, undertook interviews with the researcher and was observed extensively in the classroom over a one year period. Woodbury found that while the teachers in her study were aware of the aims of the reform movement, the extent to which they adopted them was commensurate with their perceptions of what needed to be changed in their own practice. Unless the teachers were dissatisfied with their present teaching methods, they did not see any need to change them and paid little attention to the reform messages. In fact, the teachers only attended to those aspects of the reform agenda that related directly to the specific areas of their practice with which they were unhappy. As Woodbury describes, the NCTM standards appeared to offer solutions to problems that some of these teachers did not recognise as their own and so they followed a somewhat incomplete version of the standards. A similar phenomenon has been noted in the Australian context, particularly among more experienced teachers who are likely to resist what they see as pedagogical fads and maintain a strong adherence to a more traditional, tried and true teaching methodology (Perry, Howard, & Tracey, 1999).

Teachers who wish to adopt reform-oriented pedagogies must fundamentally reconceive their roles and adopt new approaches to student learning, a significant challenge for many (Clarke, 1997). This can be problematic because most teachers are themselves the successful products of traditional pedagogies so they either fail to see any need for change or, even if they are committed to reforming their practice, they have virtually no personal experiences on which to draw of what a reform classroom might actually look like (Lloyd, 1999). The provision of new curriculum materials is not of itself sufficient to bring about change in teaching methods (Norton, McRobbie, & Cooper, 2002) because these materials are always viewed by teachers through the prism of their beliefs (Wilson & Lloyd, 2000).

It is therefore vital that teachers are provided with examples of reform-oriented

teaching (Manouchehri & Goodman, 2000). Goldsmith and Schifter (1997) suggest that teachers should observe students working on problem solving exercises and exploring mathematical concepts in new ways. In doing so, Goldsmith and Schifter argue that teachers may begin to reconsider their own ideas about how individuals best learn mathematics. More importantly, teachers might also benefit from their own first-hand experiences as learners by participating in investigations and other innovative activities. If teachers are then given time to think about the learning processes in which they have engaged, such reflection has the potential to bring about changes in beliefs about the teaching and learning of mathematics (Wilson & Cooney, 2002).

Teachers' content and pedagogical knowledge exert a strong influence on how they teach (Brown & Borko, 1992). So, even when teachers begin to recognise the need to reform their practice and take tentative steps to implement change, they must acquire new knowledge and skills. Teachers need to learn how to guide and support students' mathematical investigations, how to create a classroom climate that fosters mathematical inquiry and risk-taking, and how to encourage students to reason and reflect on their work. In short, if students are to become sense-makers rather than rote learners of mathematics, then many teachers will need to undergo a significant pedagogical shift.

Manouchehri and Goodman (1998) studied 66 middle school mathematics teachers over a two-year period as they used reform-oriented curriculum materials. Manouchehri and Goodman reported that although some teachers listened to students' explanation and arguments and sought to use these as the basis for further instruction, they did not always know how to build on students' answers and assist students in the process of generalising their results. The teachers also had difficulty in dealing with multiple solution methods offered by students in the classroom and did not capitalise on the students' alternative strategies as well as they might have done.

Teaching that is consistent with the National Statement (AEC, 1991) and other reform documents also requires extensive knowledge of students' thinking and how they learn mathematics on the part of teachers (Schifter, 1998). In a reform classroom, teaching is seen as an act of facilitating problem-solving and investigative thinking while learning is viewed as a cycle of hypothesising, exploring, testing out alternative approaches and reflecting. In such a learning environment, there is a shift in intellectual authority from the teacher and the textbook to all members of the class. As a result, there is a greater imperative for teachers to gain insights into students' current thinking and how it is structured. This knowledge is essential if teachers are to design tasks that are accessible to students and provide sufficient challenges for them. In short, if instructional practices are to change, then many teachers will need to undergo a significant epistemological shift and reconceptualise their views on the process of learning mathematics.

Spillane and Zeuli (1999) observed instruction in the classrooms of 25 teachers and noted that while all of teachers were familiar with the NCTM standards and reflected at least some aspects of the reforms in their instruction, many still tended to use a predominantly teacher-centred and procedural approach. For example, while students worked together in small groups on problem-solving tasks, the teacher still emphasised procedural approaches, and the quest for a single correct answer dominated the classroom interactions. The teachers were essentially trying to graft new learning styles onto traditional teaching practices, resulting in a somewhat ineffective hybrid. Spillane and Zeuli conclude that "while these teachers had behavioural moves that went against the grain of

traditional practice, they had not challenged the epistemological regularities that are the mainstays of traditional practice" (p. 20).

The importance of the social context of teachers' work should not be overlooked since it can have a strong influence on their classroom actions (Perry, Howard, & Tracey, 1999). Lave and Wenger (1991) developed the notion that learning always occurs in a *community of practice* which is defined as a group of individuals who share common understandings and have a sense of common purpose. For Lave and Wegner, learning is the process of moving from peripheral to fuller forms of participation in one's community of practice. This learning occurs as individuals within the community share their knowledge and experiences in collaboration with each other and with outside experts. The mix of novices and experts is a useful ingredient in ensuring a steady flow of ideas and information among the members to promote greater learning for all. In schools, this might involve teachers working together to develop supportive staffroom environments where inquiry and discussion about the reform of current practice are encouraged. It might also include fostering teachers' exploration of and experimentation with new forms of teaching. Such a feeling of collegial support and encouragement among teachers may act in a similar way to the classrooms of inquiry promoted by reform curricula: just as students learn mathematics through a process of activity and social construction of knowledge, teachers too can apply these principles in the staffroom as they strive to reform their instructional practices.

There are clearly many factors that might contribute to teachers' willingness and ability to adopt the kind of approach delineated in the Working Mathematically strand of the new syllabus. This paper explores teachers' knowledge of the principles of working mathematically and the extent to which they have embraced them in their classrooms.

Investigating Change

A two-page questionnaire was sent to a random sample of mathematics teachers in approximately 480 secondary schools in New South Wales. The questionnaire was designed to determine the teachers' views on the Years 7–10 Mathematics Syllabus (BOSNSW, 2002) and to gather data about how they were implementing the syllabus with their classes. It was also used to recruit participants who wished to discuss their responses in more detail since teachers could indicate their willingness to be interviewed by including their contact details in the returned questionnaire.

There were 193 questionnaires returned and 63 teachers provided their contact details. Of these, 18 teachers did not subsequently provide the required signed consent forms from their school principals, 4 teachers decided to withdraw from the project and 2 teachers were not able to be contacted. The final number of interviewees was therefore 39, including 21 head teachers. The group comprised a reasonable distribution of teachers in government and non-government schools across metropolitan and rural districts. The median secondary mathematics teaching experience of the participants was 15 years.

Each teacher was interviewed individually for approximately 30 minutes. The interview was semi-structured and covered topics such as the teacher's attitude to and familiarity with the broad aims of the new syllabus and the Working Mathematically strand in particular, how each teacher was dealing with practical concerns such as preparing programs and registers, and what changes, if any, the focus on alternative assessment, the continuum of learning and working mathematically had brought to the teacher's classroom practice. All of the interviews were audio-taped and transcribed for later analysis of

recurring themes. This paper focuses particularly on the issues concerned with working mathematically raised during the teacher interviews.

Teachers' Responses

Familiarity with the Working Mathematically syllabus strand varied greatly among the teachers as did their self-reports about the changes that had occurred in their classroom practice. The majority of teachers had not embraced the reforms and appeared to have an incomplete understanding of what working mathematically actually meant or what it might look like in a classroom setting. Others were aware of working mathematically and could describe at least some of its main features, but claimed they were not yet ready to implement it in their teaching practice. Another small group of teachers reported that they were making a conscious effort to incorporate working mathematically activities whenever they could. These groups largely confirm those reported by Zevenbergen (2005) who identified *conservatives*, who rejected curriculum reforms as reducing the intellectual rigour of the subject, *pragmatists*, who were generally supportive of the reforms but had some reservations about practical implications such as increased workloads, and *contemporaries*, who welcomed the reforms.

Barriers to Change

Most teachers were content with their current teaching practices and saw no need to change them. They regarded mathematical knowledge as immutable and so too the way it should be taught. As one teacher remarked, "Maths is maths and Pythagoras was doing it, so why should we change?" These teachers felt that there would always be a need for what they described as a "chalk and talk" approach because they regarded it as the best way for students to learn mathematics. The use of this teacher-centred approach was born out of a desire to maintain a particular kind of classroom environment where students sat quietly and worked on their own. The teachers saw this kind of solitary practice as beneficial because students could "get into a rhythm and that often brings the understanding because suddenly it clicks".

Teachers' perceptions of their students' preferred learning styles also influenced their pedagogy and many teachers commented that students expected a certain type of teaching when they entered a mathematics classroom.

> It's very easy to fall into the habit of chalk and talk, but the kids expect that, they want it, they see other subjects as hands-on but not maths which is "tell me how to do it" … One reason kids like maths, or dislike maths, is they come in and they know exactly what to expect. They're going to work from the textbook and get homework every night and some kids really like that.

When some of these teachers first attempted to incorporate working mathematically activities with their classes, they met a good deal of resistance from students who saw little value in tasks that did neither conformed to their usual lesson structures nor appeared to have any direct bearing on their formal assessment for the subject. As a result, the students were unlikely to persevere in their search for a solution and the teachers were reluctant to try working mathematically again.

> As soon as you try to get them to do something, they don't see it as real maths. So if you're doing group work or an activity that's more practical they don't necessarily see the point of it because they see maths as doing questions and answers from the book. I also find with kids if they don't get the answer within say 30 seconds they say it's too hard so they don't have that perseverance with maths. To them it's either right or wrong, I can do it, I can't do it.

The teachers in this group did not convey clear ideas about the meaning of working mathematically and believed that showing students real-world examples or asking them to explain how they arrived at a particular answer would be sufficient to meet the syllabus requirements. As a result, they had not recognised that working mathematically might involve a more student-centred pedagogical style focused on conceptual understanding rather than chalk and talk. There was a strong feeling among the teachers that students needed to have a good understanding of "the basics" before they could profitably attempt any working mathematically tasks, particularly in lower-ability classes where learning was seen as the mastery of a set of steps.

> In my low-ability Year 8 class, I'm constantly putting up steps on the board for them because by remembering those steps they are in the same process of understanding why are we doing step one, why are we doing step two, and they can learn that structure and they at least feel a sense of accomplishment. If a child is not mathematically minded you can't give them an investigation to discover it for themselves because they don't know where to start or which pathway to take.

Practical concerns like a lack of time for lesson preparation, difficulties in maintaining order in the classroom when students were using concrete materials or working in small groups, and the need to prepare students for examinations were often raised by teachers as significant barriers to incorporating any working mathematically activities. When teachers feel pressured, "the first thing that goes is working mathematically" and the tendency towards more traditional teaching methods grows because "that's a more efficient way of getting through the content".

The Path to Reform

Six teachers strongly supported the aims of the new syllabus and reported that they had begun to implement working mathematically with their classes. These teachers articulated a more informed understanding of working mathematically and described it in language that was consistent with the syllabus document.

> It's the five processes I've been talking about, that it's all in the kids' reasons and their discussions. It's thinking; they're not just doing the work, there's something happening. They're discovering and maybe even predicting and checking their hypothesis and seeing if it works.

Although many of these teachers were trained in a transmissionist tradition where the teacher was the ultimate classroom authority, they now conceived a model of mathematics instruction that was aligned more closely to the principles of working mathematically.

> I was brought up in the system of I'm the teacher, they're the students and never the twain shall meet, and you were imparting knowledge down. But now I think that we actually never teach kids anything, we help them discover for themselves and so that's my role now. And I'm discovering more and more that if kids are working as a group and helping to explain things to each other, it grounds them more in their understanding because they have to communicate that to their peers.

A number of common themes emerged from the interview data as possible sources for the development of this constructivist pedagogy. Some teachers recounted their own experiences, either as students themselves or during more recent professional development sessions, when they sat and listened for much of the time. In reflecting on these encounters, the teachers started to wonder what it might be like for their own students in lessons where they were essentially passive recipients of knowledge. Other teachers reported how they empathised with the looks of incomprehension they often encountered from students who, after churning out pages of repetitive exercises, were stymied by even the slightest variation in the next example. The teachers' dissatisfaction with such an outcome caused

them to re-think what they were doing and look for an alternative approach.

The teachers in this group also noted the importance of sharing their success stories of working mathematically with supportive and interested colleagues.

> You come in from your lesson, "Oh we did such and such" and if it went well and you like it, someone else will say, "Can I have that sheet or idea?" or whatever. So the best way to get people doing anything a bit differently is not by putting it in the program, but by actually talking about it amongst staff in an informal basis. If we have a professional development day everyone sits there and nods but we can get quite robust discussion at recess.

These discussions helped to create an environment where teachers could learn from each other and benefit from the knowledge and experience of their peers. However, one head teacher observed that such collegiality can extend only so far in the evolution to a more productive pedagogy before external guidance is needed.

> We've probably gone as far as we can with individual teachers and us as a faculty discussing some of the aspects of the syllabus. I suppose we need to see some concrete modelling of how you do it now … we're about as far as we can take it, we're stretching our imagination and our ideas, we're using all the resources that are available to us … but maybe through a consultancy level we can come up with something that will help us extend that.

Implications and Further Research

The path to reform of mathematics teaching is slow and replete with challenges. The results of the present study suggest that teachers are more likely to embark upon the journey if they are encouraged to reflect dispassionately on their own learning experiences and those which they provide for their students. Those who discover the results of these reflections to be unsatisfactory are probably better placed to reconsider their current practice and look for alternative approaches. However, this task usually requires a significant reconfiguring of a teacher's beliefs about the nature of mathematical knowledge and pedagogy. It also requires new insights into students' cognitive processes and how they structure mathematical concepts and accommodate new ideas. In addition, on-going professional development and support that provides practical examples of working mathematically tasks for teachers to implement and evaluate is essential.

The interviews demonstrate that the legitimate concerns of teachers who are resistant to change must be addressed. Teachers need to hear from more experienced reform-oriented practitioners that working mathematically is a legitimate approach to mathematics teaching and learning that neither relies upon nor precludes the development of basic skills and can be undertaken within a reasonable timeframe. Teachers will also benefit from the lessons learned by those who are already working mathematically about how to deal with the resistance from students they are likely to encounter, particularly in their first attempts to use more open-ended tasks with their classes.

> I always say to beginning teachers if they haven't done this style of question before, please don't walk in and think it's going to happen. You need to model, you need to show the students what you want. Otherwise, if all they've ever done is closed questions they're never going to know what you want them to do. So you've got to do the teaching that goes with that and you build it up over a period of time.

The evidence gathered here offers a useful starting point for investigating how secondary teachers deal with the demands of working mathematically. However, it is necessary to confirm these self-report findings by other means and the next step in the current project is to conduct lesson observations of those teachers who indicated that they have started to implement working mathematically in their classrooms. The aim of this

further research is to document how successfully the teachers achieve the syllabus outcomes for working mathematically and to collect examples of best practice for dissemination among the wider mathematics education community.

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