Structuring the Talk Towards Mathematical Inquiry

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In the current mathematics education reform efforts teachers are challenged to develop discourse communities where students learn to construct and evaluate arguments used in mathematical reasoning. The challenge for teachers is to know what actions to take to establish such discourse. In this study I investigated the actions a teacher took to establish the discourse of inquiry and challenge within a community of diverse learners. I report on the way in which the teacher shifted back and forth among a range of roles, as a facilitator of the discourse, as a participant in the discourse and as a commentator about the discourse.

The past decade has seen significant shifts in the way teaching and learning of mathematics is conceptualised in New Zealand and internationally. In New Zealand changes advocated, include an increased focus on students communicating mathematical reasoning, "developing arguments and thinking flexibly" (Ministry of Education, 1992, p. 27). The American policy document 'Principles and Standards for School Mathematics' takes a similar line stating that if students are to learn to "construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential" (National Council of Teachers of Mathematics, 2000, p. 18). For teachers however, constructing such a learning environment has many hurdles, not the least being the teachers' own lack of experience of learning in such environments (Huferd-Ackles, Fuson, & Sherin, 2004). Moreover, traditional beliefs about the non-contentious nature of mathematics and a view of mathematical discourse as non-adversarial (Weingrad, 1998) are challenged. The research reported in this paper examines how one teacher engaged in a collaborative research project, purposely changed her practice, and that of her students. The focus of the paper is on the interactional strategies used by the teacher. I examine how those strategies were used to gradually shift the discourse toward all students assuming responsibility to engage in productive mathematical inquiry and argumentation.

Argumentation has drawn increased attention of educators and researchers in recent years for a range of reasons. These include recognition of its "importance for solving differences and reaching consensus, as well as its central role in thinking and scientific language" (Rojas-Drummond & Zapata, 2004). Rojas-Drummond and Zapata maintain that argumentation is a powerful reasoning tool in that it "allows individuals to deny, criticise and justify concepts and facts as well as find opposing views and generate a new perspective in social interaction or in self-deliberation" (p. 543). But it is not only listeners who benefit from explanations of mathematical reasoning—the explainers also gain opportunities to review and reconstruct their mathematical thinking. During the course of their presentations, explainers have the opportunity to re-look at problems and build stronger arguments (Whitenack & Yackel, 2002).

In classrooms where the use of discourse extends to justification and argumentation, both cognitive demand and student participation in conceptual reasoning is increased (Forman, Larreamendy-Joerns, & Brown, 1998; Wood & McNeal, 2003). However, reasoning at complex cognitive levels, and working through a communal argumentative process, is not something many younger students are able to achieve easily without explicit

adult mediation (Mercer, 2002). Teachers as the more expert members of the community take a significant role in orchestrating respectful exchange of ideas, within extended conversations which involve inquiry and challenge (Mercer, 2000; Weingrad, 1998).

The seminal work of Lampert (2001) provides examples of interactional strategies of Lampert herself facilitating student engagement in productive discourse. Lampert, as teacher-researcher, structured her lessons to resemble mathematical arguments in which all participants engaged in disciplined public construction and evaluation of reasoning. An initial focus was placed on structuring the discourse norms. These norms were continually revised and reconceptualised so that over time the press for explanatory justification was increased.

Developing student autonomy and competence in mathematics includes ensuring that students gain access to both the mathematical content discussions and the social discourse process (when and how to explain, question, agree, disagree or challenge). In Lampert's (2001) study, the teacher's role as a skilful listener was evident as Lampert shifted back and forth between mathematical content discussions and talking about the social discourse process. Lampert explicitly taught 'politeness' strategies, the essential qualities or norms for ways students might disagree. By affirming the importance of disagreement and inducting students into ways to question and disagree, students were positioned to articulate their reasoning, accept opposing views, and negotiate at cognitively advanced levels. Similarly, White (2003) illustrated how two teachers working with diverse groups of students, through careful listening to student thinking and the structuring of student interaction, facilitated a productive discourse community in which students engaged in rich analytical mathematical conversations. Commonalities between the aforementioned studies include the ways in which the teachers actively engaged in understanding, interpreting, and advancing student reasoning. Both studies showed the importance of teachers building on student contributions to the discourse, in ways which indicate that the contributions are valued.

Mathematical discourses are acquired and learnt through authentic engagement. When students engage in mathematical discourse they are using a social language which displays a particular socially situated identity appropriate to the mathematical situation (Gee & Clinton, 2000). However, without explicit discussion of the structure of the discourse—how it works, its norms and rules—some students may not be able to participate fully in the rich conversations of their mathematics classrooms. In this paper, the examination of how one teacher changed and established the discourse patterns of her classroom also provides evidence of how she reconstructed her expectations and obligations for student participation in mathematical discourse.

The theoretical standpoint of this study is drawn from a sociocultural perspective which supports a view of mathematical teaching and learning as inherently social and founded on active participation in communicative reasoning processes (Lerman, 2001). From this perspective, ways of knowing and doing are mutually constituted. Students successively gain greater levels of legitimate participation through guided socialisation into discursive interaction because, as Lave and Wenger (1991) explain, "learning as legitimate peripheral participation means that learning is not merely a condition for membership, but is itself an evolving form of membership" (p. 53).

Research Design

The study reports on one teacher case study from a teaching experiment involving three teachers. The study was conducted in a small urban New Zealand primary school Year Five-Six class. The teacher was of a New Zealand Maori ethnic grouping and the majority of the students were of New Zealand Maori or Pacific Nations ethnic groupings with many of whom spoke English as their second language. Based on the results from the New Zealand Numeracy Project Assessment tool (Ministry of Education, 2004) the 10 and 11 year old group were achieving at significantly lower numeracy levels than comparable students of similar age grouping in New Zealand schools at the beginning of the study.

Teaching experiment design (Cobb, 2000) was used in order to focus teacher and researcher attention on the social process of mathematical discourse, while remaining mindful of the mathematical product of the discourse. Taking cognisance of the two characteristics of teaching experiment design research—the iterative cycles of analysis, and an improved process or product—a hypothetical communication and participation trajectory was used to map the progression of the mathematical discourse and to focus subsequent participation and communication goals. For example, after Moana (pseudonym for the teacher) had taught a fractional number unit and before she returned to teaching a whole number unit, the types of questions and interactions anticipated to scaffold a further shift toward mathematical inquiry and argumentation were considered and mapped out.

Data collection over the one year included three teacher interviews, classroom artefacts, field notes, twice weekly video captured observations of lessons, written and recorded teacher reflective statements and teacher recorded reflective analysis of video excerpts. On-going data collection and analysis maintained a focus on the developing mathematical discourse, and supported iterative cycles and revision of the communication and participation strategies. Data analysis occurred chronologically using a grounded approach in which codes, categories, patterns and themes were created. Through use of a constant comparative method which involved interplay between the data and theory, trustworthiness was verified or refuted.

Results and Discussion

During the research year Moana's use of discourse alternated between being a participant in the discourse, acting as a model for the discourse, and being a commentator on how students were talking using the mathematical discourse. In order to structure the discourse towards inquiry and challenge, Moana simultaneously employed talk as a tool to describe and to model the ways in which the students should engage in the discourse of inquiry and argumentation. Additionally she used discourse as a tool to provide models of student use of the developing mathematical discourse.

Explaining and questioning reasoning

At the beginning of the research Moana's focus was toward developing expectations of active and collective engagement of all students in the mathematical discourse:

> I want you to talk to each other before you even touch the sticks. Lots of talking and listening. I might ask you what someone in your group said, not you, so you need to discuss things please and make sense of what someone else says. Listen carefully to each other. I want you to discuss what is happening in your patterns of two…and how many you have.

To realise her immediate goal of building a supportive mathematics community Moana frequently used the words ‘we’ and ‘us’ when asking questions during discussions. The inclusion of herself in the discourse indicated that she considered herself a participant of the community with similar obligations to the students:

> Can you show us what four groups of four look like because we want to think of other ways than adding on or skip counting don’t we?

At the same time, Moana scaffolded the students to use questions which elicited further explanatory information:

> Think about what he is doing. Think of the questions you need to ask, the ‘what’ questions.

Moana’s participation in discussions enabled her to model ways to ask questions which would elicit further information. In questioning her students Moana indicated that their responses provided her with more information as a listener while acknowledging that the explainer understood the reasoning which supported their explanation. An example of this occurred when a student recorded 24/8 next to a drawing of three cakes divided into eighths. When questioned by another student only a brief explanation is provided:

> Hone: What are you doing?
>
> Anaru: Twenty four eighths.

Moana then interceded and stressing the word ‘I’ and ‘we’, said:

> But I am not sure…we know what you mean? Can you explain it?

However when Anaru frowned and shook her head in response, Moana shifted back to a facilitating role. Rather than continuing to probe Anaru’s thinking, she drew on Hemi (another member of the explaining group) to continue the explanation. At this point she assumed the role of teacher to ask clarifying questions:

> Hemi: (points at the symbols 24/8 which Anaru had recorded next to the drawing): I can. Twenty four eighths because there are eight in each cake and there’s eight slices in each cake and it all adds up to twenty four.
>
> Moana: Twenty four what? What does that bottom number mean?
>
> Hemi: That means how much slices in each cake.
>
> Moana: Okay what does twenty four represent?
>
> Hemi: It means how much altogether.
>
> Moana: Altogether. Yeah twenty four bits, slices and they are all eighths.

In accepting Anaru’s refusal to continue, Moana provided Anaru and other students with a message that this was a supportive environment, where they were safe to take—or not to take—risks and that it was not discourteous not to comply with her request to continue. Engaging in the discourse of inquiry was a new experience for many of these students and so it was not unusual for a student to begin an explanation, or start to question, then hesitate and stop and say “oh I forgot”. Within this supportive environment the students knew they could withdraw to rethink and reshape their thoughts without losing ‘face’.

Weingrad (1998) maintains that intellectual risk-taking requires an environment which is respectful of student reasoning. As novices in an environment of inquiry and challenge,

explaining and answering questions about the mathematical thinking potentially posed considerable risks to the student's self esteem. To encourage student confidence Moana regularly described or affirmed incidences of risk-taking she observed:

> Is there anyone else who can model another equivalent fraction? Good, Rona, for taking a risk like this, just go ahead and construct another fraction the same.

Guidance was also provided in ways students could 'politely' question and challenge an explanation given by another student. Moana balanced a need for students to gain immediate clarification of sections of an explanation, with a need to maintain the explainer's confidence. In the early stage of the research, the students would indicate that they wanted to speak by putting up their hands and Moana would act as a conduit. However, toward the end of the research period she recorded a reflective statement that indicated that she saw need for changes to the communication and participation structure:

> The students are starting to engage in debate and risk taking. I am setting the scene to try and get this to happen more and more. Opportunities here exist for children to ask and dispute and so I need to let the children guide their own questioning and discussion more.

Moana recognised that within this safe learning environment the students were able to increasingly engage in the discourse of inquiry and debate. She recorded reflectively:

> A real need to move, shift from the surface questions or practising how to…to take a good look up close and personal, shift thinking to challenging, justifying, validating, creating other possibilities.

At a midpoint in the research year, discussion between Moana and the researcher, the use of video observation reviews and Moana's reflective statements culminated in an increased press in the hypothetical trajectory toward justification of reasoning.

Justifying and challenging reasoning

When focusing on changes to student participation norms, the teacher recognised the need to allow students to contribute to the management of the flow of discussion. Moana placed koosh balls in the discussion circle. Students used these to indicate that they wanted to question or challenge mathematical reasoning. This slowed the discussion down, made the questioner more accountable, but all the while it protected the self esteem of the explainer. This is illustrated when Hone explains an addition strategy for 236, 219, 221, and 214:

> Hone: We added 236 and 214 which equals 450.
>
> Aroha: (picks up a koosh ball and points at 219 and 221) Why didn't you add those two?
>
> Hone: Because it tidies up these numbers because there is a six and four which equals ten.
>
> Aroha: But why didn't you use the other ones?
>
> Moana: (stepping in to press for further clarification and justification) But why didn't you just add the 236 and 219?
>
> Hone: Because then it will be harder…because it won't…like not equal…it won't like be tidied up.

While providing a justification or convincing others was a key feature of the communication and participation trajectory, learning to disagree and challenge were also important skills that Moana wanted to foster. Through reflection and discussion with the researcher Moana was aware that when the focus in classrooms is toward teaching students

to work together there is potential that they interpret this as always needing to be in agreement (Mercer, 2000). To support her students to accept disagreement and challenge as learning tools she explicitly and on many occasions asked the students if they agreed, or disagreed, with conjectures. This required the students to not only take a position, but also to be prepared to explain their reasoning:

> So do you think that's just a little bit quicker than your skip counting? Who agrees that that is a quicker way? Maybe you can say why. Who disagrees? You can disagree, people.

Observing a child nod his head indicating his disagreement Moana paused:

> Okay Manu just talk about it, what other ways can you use?

This action indicated to the students that both agreement and disagreement are important in mathematical talk but that either position requires further explanatory justification.

Moana also spent time socially scaffolding the students in ways to engage in mathematical talk when working in small discussion groups. She reiterated the need for collaborative support, but also emphasised that sense-making evolved from questioning and challenge:

> If you do not understand something ask somebody in your group because you know people everybody understands more when you work together with your talking and looking out for everybody in your group. That's your whanau [family]. You make sure they understand as much as you so Te Paaki asks Ioane and Ioane asks Anaru right? I do not want to see those looks of absolute confusion on peoples' faces. I want to see faces where lots of talking, questioning and looking at the thinking, challenging the thinking has taken place.

An effective way that Moana illustrated student engagement in the discourse of inquiry and challenge followed from the students' small group work. Moana would use models of student actions to show how they had worked together. She also affirmed specific individuals who had actively supported group members to challenge thinking or to promote argumentation:

> There's lots of really interesting korero [talk] going on. I really spent most of my time with this group because they were having problems and arguments and Wiremu was really good…you were really good in that position Wiremu, you were helping your group and you weren't giving out the answers and that's really good but you were really pushing them to think. Yes you had everyone talking about and discussing how they were going to sort out the ideas. You were challenging and other people were following your lead so the arguing was really kapai [good].

In order to establish the discourse of inquiry Moana deliberately stepped in and out of being participant in the discourse, to being a commentator about the discourse. For example, after group work was completed she refocused discussion toward use of questioning as a means to prompt revision and re-evaluation of the reasoning:

> Moana: When we were working before in our groups we were making people think by posing questions, getting them to rethink and revoice their answers okay? And how were we doing that?
>
> Matangi: Rethink your mind.

Moana accepted Matangi's statement, revoicing it by extending it into a question:

> Moana: Good boy. Rethink your mind about what?

Moana then held up her hand to indicate that no one else was to speak and a lengthy pause ensued. Through this action she indicated that she valued the contribution and if the student wanted to he had the right to time, to further clarify his meaning. The lengthy pause allowed Matangi space to reshape what he meant. He responded in the form of a

question which modeled what he intended in his first statement.

> Matangi: Are you sure that's right?
>
> Moana: Are you sure that's right? Sure? That's a good way of getting a person to rethink, to go back and have a bit of a think. Think some more about what they have just said and why. Kind of listen to themselves when they are thinking.

In accord with the hypothetical trajectory established in partnership with the researcher, Moana shifted the focus towards a press for students to provide explanatory justification of their reasoning. She included in the press an expectation for them to be more precise with their questioning and a requirement to challenge other students' reasoning where appropriate. To realise these expectations Moana provided the students with explicit scaffolds:

> I want you people asking questions. After or throughout the explanation ask questions. Why did you come to that decision? Or why did you use those numbers? Or can you convince me that this one works?

As a result of these question prompts, the explainers engaged in justification of their reasoning:

> Aroha: (records 43, 23, 13, 3 and then 3 x 4 = 12) I am adding forty three, twenty three, thirteen and three so three times four equals twelve.
>
> Kea: Why are you trying to do that with those numbers? Where did you get the four?
>
> Aroha: (points at the 3 digit on the four numbers) These threes, the four threes.
>
> Donald: (a second explaining group member provides justification) All she is doing is like making it shorter by like doing four times three.
>
> Hone: (the third member of the explaining group adds) Because there are only the tens left.
>
> Donald: Three times four equals twelve and she got that off all the threes like the forty three, twenty three, thirteen and three. So she is just like adding the threes all up and that equals twelve.

Progress along the proposed communication and participation trajectory was evidenced in Moana's shift toward an expectation of explanatory justification. For Moana, a noticeable shift (both observed and reported in self-reflections) had occurred at mid-point in the research study. A significant indicator was evident in the nature of her questioning. Questions shifted towards asking students to make comparisons and analyse the reasoning. The students would confidently agree or disagree as illustrated by Anaru in the following episode in which Anaru analyses the explanation of another group and provides convincing reasoning to support the conjecture that the explanations were similar:

> Moana: Did they use the same strategy as Anaru?
>
> Faalinga: I don't think so.
>
> Moana: You don't think so? So they haven't used the same strategy?
>
> Moana: (Danny is nodding his head in agreement. The teacher turns to him) Argue with them Danny.
>
> Danny: They have used the same sort of strategy because they have both added the tens and the twenties and the forties and they have both added the threes together.
>
> Moana: Well done you guys. They all added the threes together. You prove that you did the same

thing Anaru, you show what you did with the threes.

Anaru: (Draws four sets of threes as sets which she then represents as an array of three multiplied by four and four multiplied by three) Three plus three plus three plus three or four times three or three times four.

Conclusions and implications

Within the teaching design experiment successive reviewing of the hypothetical communication and participation trajectory was used to scaffold the teachers' enactment of an inquiry discourse, in her classroom. Over the year, Moana was increasingly able to support her group of diverse students' engagement in productive mathematical inquiry and argumentation. As Lampert (2001) illustrated, expectations and obligations for communicating and participating in mathematical reasoning were formulated and reformulated as the teacher shifted the mathematical discourse from explaining and questioning to justifying and challenging reasoning.

The episodes provided in this paper illustrate the demands for the teacher to engage in a number of pedagogical actions in order to develop and maintain student participation in the discourse community. These paralleled those described by Lampert (2001). They include the teacher as a participant in the discourse, a facilitator of the discourse, and a commentator about the discourse. The teacher actions which focused on simultaneously engaging students in discussion of mathematical content and discussion on how to engage in mathematical discussion deepened student participation in intellectual risk taking and argumentation.

The findings of this research support the argument of Weingrad (1998) and White (2003), illustrating that diverse students to engage in productive discourse require a consistent and continual focus on the enactment of a respectful classroom climate in which student contribution is valued and built on.

Effecting change to students' engagement with mathematical reasoning was a lengthy process. Further research is needed to examine other factors which are important in enacting, maintaining and sustaining a respectful climate in which diverse learners are able to confidently engage in productive discourse.

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