Catering for Individual Differences: Lessons Learnt from the *Australian Mathematics Competition*

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A large, national, mathematics data base, the Australian Mathematics Competition [AMC], is used to investigate the appropriateness of curriculum content for different groups of students. The cross-sectional comparisons afforded by the data enable characteristics of mathematical problems found easy and difficult by "average" and "able" groups of students at different grade levels to be identified. Gender differences in performance on the papers are also explored.

Mathematics is well known to be a critical filter for many educational and career opportunities and is widely regarded to be a key component of the school curriculum. Yet problems with mathematics and science studies at school and university level have been highlighted by diverse groups, including teacher and parent organizations, business and professional bodies (see e.g., Committee for the Review of Teaching and Teacher Education, 2003; Dobson, 2003). Politicians, too, have turned their attention to student performance in mathematics and science. For example, within days of taking up her appointment as federal minister for education, minister Julie Bishop indicated that she "was concerned about the level of support in some schools in subjects … (including) maths" (Topsfield, 2006, p 9) and argued that more curriculum support for teachers was needed. Boosting "the number of students studying maths and sciences at school and university" was listed as a high priority by the Federal education opposition spokeswoman, Jenny Macklin, who further noted that "the number of students studying maths in Year 12 had declined by 4420 since 2001 alone" (Topsfield, 2006, p. 9). Is inappropriate curriculum content — work that is too easy or too difficult - responsible for the drift away from mathematics and the physical sciences? Is there evidence that, as argued by Gross, "in every class of 30 students there would be at least two or three who could work about three years beyond their age" (cited by Ferrari, 2006, p. 1)? Questions such as these motivated the investigation described in this paper. Specifically, data from the Australian Mathematics Competition for the Westpac Awards [AMC] - a large, national, mathematics data base — are used to explore the suitability of curriculum content for secondary school students with different mathematical abilities. Given the continuing interest within and beyond the mathematics education research community of possible gender differences in mathematics learning outcomes, the data are reported overall and separately by gender.

Previous research

Curriculum Issues

Much has been written about students' reasons for continuing with mathematics once the subject is no longer compulsory. Pragmatic grounds were given at the beginning of this paper: mathematics serves as a gate keeper to many courses and careers, and this is emphasized in various Department of Education, Science & Training publications (see e.g., Web sites such as jobguide.dest.gov.au; dest.gov.au/sectors/careerdevelopment; myfuture.edu.au).

In their study of talented teenagers, Csikszentmihalyi, Rathunde, and Whalen (1993) found that immediate and long term extrinsic rewards, enjoyment from working with mathematics, and functional support from school and teachers were pivotal for the development of mathematical talent and student persistence with mathematics. Capable mathematics students thrived on appropriately challenging activities and materials. Working with students in a selective accelerated grade 9 class in a school in the metropolitan area of Melbourne, Landvogt, Leder and Abbott (2001) similarly reported that students valued being given more challenging work in their mathematics (and English) classes. Yet Barrington and Brown (2005) have argued that "(t)eachers are well aware that if students perceive a subject to be too difficult, they will avoid it in favour of one they believe they can cope with (p. 21). Determining appropriate curriculum content involves balancing students' needs and capabilities.

Writing about the educational environment in the United States, Willoughby (2000) noted the increasing trend for state and local communities to specify the mathematics curriculum content to be covered at each grade level in schools under their jurisdiction. Barrington and Brown (2005) described a similar situation in Australia. "Pre-tertiary-entrance mathematics subjects across the Australian States and Territories vary enormously, with differences in philosophy, mathematical content and assessment so great no two States' Year 12 mathematics subjects could be described as equivalent" (p. 4). Nevertheless, inspection of curriculum documents reveals much overlap in the broad goals found across countries and settings. Invariably students are expected to master the basic and higher order thinking skills of mathematics and to travel a journey that allows them to cope with increasingly difficult mathematics problems. Different theoretical frameworks and terminology have been used to portray increasingly sophisticated levels of student thinking — among them Bloom's (1956) taxonomy of educational objectives and the Structure of Learning Outcomes or SOLO taxonomy (e.g., Biggs & Collis, 1982). These and similar taxonomies can also track expected progressions embedded in curriculum guidelines specified in different educational settings.

Gender Differences and Mathematics Learning

Gender differences in mathematics learning have attracted much attention, from researchers, practitioners, and policy makers. In recent years evidence of gender differences in performance has become more equivocal, with females at times reported, and perceived, as outperforming males (Cox, Leder, & Forgasz, 2004; Forgasz & Leder, 2000). Indeed, at times it has been argued that intervention programs aimed at improving mathematics learning for females have been so successful that males, as a group, should now be perceived as disadvantaged: in terms of educational participation, adjustment to schooling, and achievement in most subjects - including mathematics (Department of Education, Science and Training, 2003; Freeman, 2004). Exploration of gender differences in mathematics achievement is addressed in this paper by comparing the average performance of females and males on the AMC papers as well as the performance of the best students, i.e., the top 2% of performers on these papers.

The Study

Three separate AMC papers are set for secondary school students: the Junior, Intermediate and Senior papers for students in grades 7 and 8, grades 9 and 10, and grades 11 and 12 respectively. Each of these contains unique questions as well as some which appear on at least two papers. Thus there are questions which are attempted by students at four different grade levels. Drawing on these items it is possible to identify:

- Problems on which the best students, defined here as those in the top 2%, already reach maximum performance in grade 7 but other students, on average, show increasing mastery with increasing grade level,
- Items on which the performance of all students — those of the group as a whole and those in the top 2% - improves with grade level, and
- Questions on which the performance of only those in the top 2% increases with grade level.

Hence, through the cross-sectional comparisons afforded by the data base, characteristics of mathematical problems found easy and difficult by different groups of students in different grade levels can be identified.

As already foreshadowed, the AMC data are also used to explore whether:

- There are gender differences in the performance of those who enter the AMC, overall and for those who comprise the top 2% of students.

A brief description of the AMC is given next to gauge its relevance to the curriculum content in the various Australian states.

The Australian Mathematics Competition

The first AMC was conducted in 1978, with 60,000 students from 700 schools — including about 900 students from 15 New Zealand schools - entering the competition. By 2004 over 340,000 students from more than 2450 schools throughout Australia participated in the AMC, i.e., about one-quarter of Australia's secondary school students. Students from an additional 40 countries (approximately) also took part.

There are now five separate papers in the AMC. Collectively these cover the school years from grade 3 to grade 12. Only the data for students in grades 7 to 12 are relevant for this paper.

The AMC's organisers aim to give "average" students a sense of achievement as well as rewarding outstanding performance. The top 50% of entrants at each participating grade level (grades 7 to 12), and within each state, gain special recognition. Medals are awarded to the highest performers (.01% of entrants) and the top approximately 0.3 % of students within their geographic region and grade level are awarded a Prize. Those in the top 2% of their grade level and geographic region and who have not received another award are awarded a High Distinction, Distinctions are given to students in the top 15% of their grade level and geographic region and who have not received a higher award, and Credits to students in the top 50% of their grade level and geographic region and who have not received a higher award. A Participation Certificate is given to students who have participated in the AMC but have not received a higher award.

Over time the scoring system has been modified so that in recent years, including for the 2005 AMC papers, incorrect answers have not attracted a penalty.

Throughout the Competition's existence, the AMC papers have been devised by a committee drawn from experienced teachers and university academics to measure achievement in mathematics. The Junior, Intermediate, and Senior papers each contain 30 questions, 25 of which are multiple-choice. The questions are graded and comprise a balance of arithmetic, algebra, and geometry problems with which students should have classroom experience, and problems of a general nature which require a synthesis of mathematical skills for their solution.

> Much of our paper is familiar to the average student, and all the mathematics is checked for meeting the various syllabi in Australia and New Zealand. We also try to challenge students of all standards. The earliest questions are accessible to all students, while the questions get progressively harder. (Taylor, 1993, p. vi)

The consistently high student participation rates are clear testimony to the perceived trustworthiness of the AMC and the value assigned by schools, teachers, and parents to the competition.

Method

Selecting the Data

As noted before, each year there are a number of questions which appear on more than one paper. Seven items were common to the 2005 Junior and Intermediate AMC papers; 11 items appeared on both the Intermediate and Senior papers. Out of these, one question appeared on all three papers and could thus be attempted by students at all six grade levels. Details are shown in Table 1.

Results

The total group

Two further sets of information are shown in Table 1. For each grade, the percentages of students who obtained the correct answer on the common questions are provided as well as the percentage of females among each such group.

The common items, it can be seen from Table 1, cluster into three groups:

- Those on which there is virtually no change (less than 10%) over increasing grade level in the performance of students, i.e., items J9/I3, I14/S13, I17/S14, J24/I18, I22/S18, I23/S22, J25/I24, I26/S26, I29/S28, and J30/I30/S30. With the exception of J9/I3, which most students could already handle in grade 7, these items proved consistently difficult: at most one-third at each grade level solved them correctly,
- Those on which students showed some improvement (up to 20%), with increasing grade level, i.e., items I8/S5, J12I/9, I10/S8, I21/S16, and
- Those on which students showed a steady and substantial improvement (20% or more) with increasing grade, i.e., items I7/S3, J13/I11, and J14/I6. At least 40% of the groups answered items in the latter two sets of questions correctly.

The percentage of females who entered the AMC at each grade level is also shown in Table 1 (see the last row). Males consistently outperformed females, with item 17 on the Intermediate paper the only exception.

Table 1

Students' Performance on Common Items, by Grade Level and gender[4].

Division & Item			Grade: % correct total group and (F as % of correct respondents in that group)					
J[1]	I[2]	S[3]	7	8	9	10	11	12
9	3		89 (47)[4]	92 (48)	95 (49)	96 (49)		
	7	3			37 (45)	47(44)	64 (42)	71 (40)
	8	5			32 (50)	36 (49)	42 (46)	45 (42)
12	9		52 (46)	57 (46)	61 (46)	66 (46)		
	10	8			25 (43)	29 (43)	37 (41)	40 (37)
13	11		31 (45)	37 (46)	47 (48)	55 (47)		
	14	13			19 (49)	18 (48)	19 (43)	21 (40)
	17	14			16 (52)	16 (50)	18 (45)	20 (40)
14	6		34 (45)	40 (46)	47 (46)	53 (46)		
24	18		30 (45)	31 (46)	32 (47)	37 (46)		
	21	16			29 (48)	33 (45)	38 (42)	42 (40)
	22	18			24 (49)	25 (49)	26 (46)	25 (43)
	23	22			33 (49)	33 (48)	31 (45)	33 (43)
25	24		28 (47)	27 (49)	26 (49)	26 (49)		
	26	26			3 (49)	3 (47)	2 (45)	2 (43)
	29	28			5 (48)	5 (41)	4 (45)	6 (34)
30	30	30	3 (47)	3 (48)	2 (48)	3 (46)	2 (44)	3 (40)
(F as % of Grade entrants)			(48.5)[5]	(49.2)	(50.1)	(49.5)	(46.2)	(44.0)

[1]Junior paper for grades 7 and 8.

[2]Intermadiate paper for grades 9 and 10.

[3]Senior paper for grades 11 and 12.

[4]89 % of the group gave a correct response; of those 47% were female (shown in brackets).

[5]Females comprised 48.5% of grade 7 students who entered the AMC.

The top 2% of students

Data for students in the top 2% of entrants are shown in Table 2. The proportions of students with correct answers are shown separately by gender, with data for the males shown in brackets. For each grade level, the proportion of females among those in the top 2% is reported in the last row.

From the data in Table 2 it can be seen that items again cluster into groups:

- Those on which there is virtually no change (less than 10%) over increasing grade level in the performance of students:
  (1) some of these (e.g., J9/I3, I7/S3, J12/I9 and to a lesser extent J13/I11, J14/I6, J24/I18, and I21/S16) most students in the top 2% could already solve correctly in grade 7,
  (2) others, e.g., I14/S13, I22/S18, I23/S22, and J25/I24, were solved correctly by about half the group, broadly independent of grade level,
  (3) several items (I26/S26, I29/S28, and J30/I30/S30) remained very difficult for all students.
- Those on which students improved with increasing grade level, e.g., I8/S5, I10/S8, and I17/S14.

Table 2
Performance of Top 2% of Students' on Common Items, by Grade Level and gender

Division & Item			Grade: %F correct and (%M correct)					
J[1]	I[2]	S[3]	7 F (M)	8 F (M)	9 F (M)	10 F (M)	11 F(M)	12 F(M)
9	3		99 (100)	100 (100)	100 (100)	100 (100)		
	7	3			93 (93)	92 (95)	98 (97)	96 (99)
	8	5			72 (77)	85 (79)	87 (89)	89 (94)
12	9		99 (98)	98 (98)	95 (97)	97 (97)		
	10	8			71 (76)	78 (80)	76 (84)	84 (83)
13	11		84 (88)	88 (90)	89 (92)	90 (94)		
	14	13			41 (40)	46 (46)	39 (53)	44 (62)
	17	14			38 (37)	39 (47)	52 (65)	70 (73)
14	6		84 (89)	89 (87)	83 (85)	82 (86)		
24	18		88 (87)	92 (91)	85 (85)	86 (88)		
	21	16			79 (81)	80 (87)	80 (83)	84 (86)
	22	18			49 (51)	50 (51)	46 (49)	48 (39)
	23	22			57 (58)	62 (62)	59 (53)	57 (58)
25	24		53 (50)	57 (53)	59 (55)	61 (59)		
	26	26			8 (9)	11 (12)	7 (10)	10 (13)
	29	28			28 (25)	20 (27)	7 (20)	23 (20)
30	30	30	15 (17)	21 (20)	12 (12)	19 (17)	6 (8)	6 (8)
%F in top 2%			32	25	24	19	19	24

It can further be seen that:

- Males outnumbered females in the top group,
- For many of the items, but not all (e.g., J24/I18 and J25/I24), the percentage of males in the top group with the correct answer was slightly higher than the corresponding percentage of females.

Item Descriptions

Selected examples illustrating the different item categories and using the terminology of Biggs and Collis' (1982) SOLO taxonomy are shown in Table 3. Items such as J13/I11 and J24/I18 confirm Gross's assertion, quoted earlier in the paper, that "in every class of 30 students there would be at least two or three who could work about three years beyond their age".

Table 3
Selected Items Illustrative of Different Categories

Item		Comment	SOLO category
J9/I3	A lesson finished at 10.10 am. If the duration of the lesson was 55 minutes, it started at: (A) 9.15 am; (B) 9.45 am; (C) 9am; (D) 8.45 am; (E) 8.30am	Almost all students solved this correctly already in grade 7	Readily solved by applying the appropriate algorithm (SOLO taxonomy: uni-structural)
J13/I11	Seven consecutive integers are listed. The sum of the smallest is 33. What is the sum of the largest three? (A) 39; (B) 37; (C) 42; (D) 48; (E) 45	"Average" students improved with increasing grade level; little change for the top 2% - most in grade 7 solved this correctly	Sequential multi-step problem (SOLO taxonomy: multi-structural)
J25/I24	A 3 x 3 square is divided into nine 1 x 1 unit squares. Different integers from 1 to 9 are written into these unit squares. Consider the pairs of numbers in the squares sharing a common edge. What is the largest number of pairs where one number is a factor of the other number? (A) 7; (B) 8; (C) 9; (D) 10; (E) 12	Little change in performance with increasing grade level. "Average" students continued to find this very difficult; about half of the top 2% solved this correctly, irrespective of grade level	Considerable integration or synthesis of information required (SOLO taxonomy: relational)

Concluding comments

Gender differences in performance are still evident on the AMC papers. Small differences in performance in favour of males were found consistently for the group as a whole. Differences in the proportions of female and male entrants who comprised the top 2% were substantial, with males outnumbering females at least 3:1 at each grade level. For students in the top 2%, males typically - but not invariably - solved common items correctly more often than their female peers. Nevertheless, reference in this paper to continuing gender differences should not be allowed to obscure the large overlap in the performance of males and females, overall and in the top 2%.

Interrogation of the cross-sectional AMC data enabled the performance of students in four grade levels to be compared and problems found easy and difficult by different groups of students to be highlighted. The types of problems identified as more suitable for "average" students and for the top 2% were not unanticipated. For the former but not the latter group, students showed improved performance with increasing grade level on sequential multi structural items. Relational items (Biggs & Collis, 1982) proved suitably challenging, yet within reach for those in the top group, but quite difficult for the cohort overall. The AMC data base is clearly a useful and timely resource for those planning instructional mathematics materials and wishing to cater for individual differences. With careful planning a balance should be achievable between the requirements of those who argue with Csikszentmihalyi et al. (1993) that good mathematics students thrive and enjoy working on challenging materials, and those who are mindful of Barrington and Brown's (2005) warning that students avoid subjects they find dauntingly difficult. Whether more careful matching of students' capabilities and the prescribed mathematics curriculum will indeed boost "the number of students studying maths and sciences at school and university" (Topsfield, 2006, p. 9) requires qualitative and more fine grained information to supplement that provided through exploration of the data base.

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