Classroom Arrangements That Benefit Students

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Recent reform efforts in New Zealand prescribe classroom grouping as a key pedagogical strategy, yet current understandings of how and for whom grouping makes a positive difference is by no means conclusive. This paper unpacks what the literature says about grouping and how it influences students' active participation in a classroom community. The research evidence reveals that quality grouping arrangements are built on knowledge of the different purposes of, and roles within, the particular social arrangements established, and demand constant monitoring for inclusiveness and effectiveness for the classroom community.

Introduction

Mathematics is a powerful social entity. Arguably the most international of all curriculum subjects, mathematics plays a key role in shaping how individuals deal with the various spheres of private, social and civil life. Yet we know that for many students mathematics is a series of hurdles and challenges. Today, just as in past decades, many students do not succeed with mathematics, are disaffected by it and continually confront obstacles to engage with the subject. The challenge for researchers is to understand how teachers might break the pattern. In this paper we explore one aspect drawn from effective teaching research findings that might begin to make a difference to all learners. We do so, acknowledging the complexity of teaching and, in particular, the interrelatedness of processes and people that, taken together, contribute to the advancement of students' mathematical understanding. The discussion is drawn from a larger synthesis of the literature on Pedagogical Approaches that Facilitate Learning for Diverse Students in Pàngarau/Mathematics (Anthony & Walshaw, in press).

Our exploration looks at the way teachers organise students for participation and learning within their classroom communities. In particular, we explore how classroom grouping, as an infrastructural element, influences students' active participation in a classroom community. Classroom grouping has been prescribed as a key pedagogical strategy in the New Zealand Numeracy Development Project (NDP) (Ministry of Education, 2005). The NDP prescribes a teaching model that encourages both whole class and small group teaching as a means of developing numerical understanding. Suggested group arrangements include grouping students, of varying levels of mathematical development within the class, according to same strategy stages. Specifically, teachers are requested to "either cross-group…between classes or compromise[e] by putting together students from close strategy stages to reduce the number of teaching groups" (Ministry of Education, 2005, p. 3). Through interaction with others within these teaching groups the intent is that students will gradually develop the skills of and dispositions towards mathematically accepted ways of thinking and reasoning.

Notwithstanding the intent, our understanding of how and for whom grouping makes a positive difference is by no means conclusive. We attempt to address this issue by, first, offering a theoretical basis for thinking about classroom organisational arrangements. We draw on the notion of 'communities of practice' to do this. We then look at the evidence of the sort of grouping arrangements that contribute to positive outcomes for diverse students. In particular, we investigate how students' engagement with mathematics is enhanced through equitable classroom communities that allow a space both for individual thinking and for collaborative mathematical explorations. In doing this we provide a systematic and credible evidence base about approaches to classroom organisational arrangements that enhance students' active participation. The evidence base is not intended to be read as a prescription of what teachers should do to teach mathematics. Rather, the synthesis is intended to stimulate reflection amongst mathematics education researchers and to generate productive critique of participation arrangements current within the discipline.

Communities of Learning

Community is the cornerstone for developing a sense of belonging (Goos, 2004) that is essential to students' active participation with mathematics. This claim falls naturally from theoretical framings that take as their central tenet the idea that knowledge evolves with community and culture. Mathematics knowledge, in this framing, is socially constructed and it is this understanding that has framed the particular attention we have given to classroom organisational arrangements. In exploring what consequences an educational environment has on the social and academic outcomes for students, our commitment is to the complex web of relationships around which knowledge production and exchange revolve.

Our conceptual framework draws on Vygotskian ideas and the work of post-Vygotskian activity theorists such as Davydov and Radzikhovskii (1985). This body of work proposes a close relationship between social processes and conceptual development, endeavouring to "unify culture, cognition, affect, goals, and needs" (Lerman, 2000, p. 37). It forms the basis of Lave and Wenger's (1991) well-known social practice theory in which the notions of 'community of practice' and 'the connectedness of knowing' are central features.

> A community of practice is a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. (Lave & Wenger, 1991, p. 98)

Theories of interrelationships revolve around the idea that individual and collective knowledge emerge and evolve within the dynamics of the spaces people share and within which they participate (Walshaw, 2004). Lave (1988) stresses both the individual and social components of the relation when she says: "[d]eveloping an identity as a member of a community and becoming knowledgeably skilful are part of the same process, with the former motivating, shaping and giving meanings to the latter, which is subsumed" (p. 65). For Wenger (1998), participation in a community of practice is "a complex process that combines doing, talking, thinking, feeling, and belonging" (p. 56). Participation enables the practices, values, conventions, and beliefs characteristic of the wider communities of mathematicians to be enacted progressively and gradually appropriated by diverse students.

Our conceptual framework offers a focus for exploring pedagogical arrangements that support or delimit student participation in classroom communities. In the discussion which follows we provide an overview of what the research says about whole class, peer grouped, or individual arrangements that operate to advance or impede students' mathematical understanding.

Whole Class Participation

Yackel and Cobb (1996) have found that cognitive development begins with a taken-as-shared sense of the expectations and obligations of mathematical participation. Routines, and rituals that characterise the "socially developed and patterned ways" (Scribner & Cole, 1981, p. 236) of the classroom play an important part in how students perceive and learn mathematics. Establishing participation processes and responsibilities for class discussion is an important pedagogical strategy that has been shown to assist in the development of mathematical thinking. Yackel and Cobb have found that classroom teachers who facilitate student participation, elicit contributions from them, and who invite students to listen to one another, to respect one another and themselves, to accept different viewpoints, and who engage in an exchange of thinking and perspectives with students by differentiating between the mathematical integrity embedded within those ideas, exemplify the hallmarks of sound pedagogical practice.

Whole class discussions, however, do not necessarily offer equitable learning opportunities for students. From their research Planas and Gorgorió (2004) illustrate the ways in which teachers regulate participation by creating inconsistent rites across student groups. The study was undertaken in a Spanish secondary mathematics classroom with a high percentage of immigrant students. Unlike local students, immigrant students were not permitted to participate actively in mathematical argumentation and hence did not have the personal experience of how participation could help clarify and modify thinking. The researchers observed the teacher's 'subtle systematic refusal' of immigrants' attempts to explain and justify their strategies for solving problems. Like the students in lower streamed classes in the studies undertaken by Boaler, Wiliam, and Brown (2000) and Zevenbergen (2005), immigrant students in the Planas and Gorgorió study were confronted with a protracted curriculum. The reduced social obligations and lesser cognitive demands placed on these students had the effect of excluding them from full engagement in mathematics and hence constrained their development of a productive mathematical disposition.

Some students, more than others, appear to thrive in whole class discussions. In their respective research, Baxter, Woodward, and Olson (2001) and Ball (1993) have found that highly articulate students tend to dominate classroom discussions and tend to offer valuable insights to the mathematical conversation. Typically, low academic achievers remain passive; when they do participate visibly their contributions are comparatively weaker and their ideas are sometimes muddled. Quality teaching ensures that participation in classroom discussion is safe for all students — that the norms of student participation and contribution are equitable.

Small Group Participation

Many researchers have shown that small group work can provide the context for social and cognitive engagement. Advocates of grouping claim the organisational practice provides students with the opportunity to articulate thinking and understanding without every classroom eye and mind on what is being said. Artzt and Yaloz-Femia (1999) have shown that collaborative activity within a small supportive environment allows students to exchange ideas but also test those ideas critically. Through groups, students learned to make conjectures as well as learn to engage in argumentation and validation. In their meta-analysis of the effects small group learning in mathematics Springer, Stanne, and Donovan (1997) found that such processes have significant and positive effects on undergraduates.

Helme and Clarke (2001) found in their secondary school classroom study that peer interactions, more than teacher-student interactions, provided greater opportunity for students to engage in high-level cognitive activity. These researchers stress the important role the teacher plays in establishing social rules governing participation. They point out that the nature of those rules, and the way in which those rules are established and monitored, serve to structure the way cognitive engagement is taken up and expressed. Thornton, Langrall, and Jones (1997) also provide empirical evidence from a small study that classrooms organised effectively for group work can provide a rich forum for diverse students to develop their mathematical thinking. They cite a study by Borasi, Kort, Leonard, and Stone (1993) in which a student who had a severe motor disability in writing, in addition to a 'numerical' disability, learned from his peers about how to share ideas and articulate his thinking.

Baxter, Woodward, Voorhies, and Wong (2002) explored student group processes over a 7-month intervention period. Of the 28 students in the classroom, two were categorised as at risk and one other received the assistance of a teacher aide. The intervention was focused on the academic development of these low achievers. The target students participated in different mixed ability groupings during small group discussions. The teacher aide's key responsibility during these discussions was to provide support for the target students to actively participate in group discussions. Specifically, she ensured that they understood the problem, and where necessary, adapted the level of difficulty for them. She made sure that they listened to the contributions of others, that they offered their own contributions, and that they could articulate the group's strategy for solving the problem. Baxter et al. report that the exposure to a wide range of ideas, strategies and solution pathways from their more academically able peers supported rich social-emotional as well as cognitive outcomes for the target students than would have been possible in a remedial classroom setting.

Doyle (1983) provides evidence that quality teaching also pays attention to the important fact that students' willingness to contribute in the public arena of the classroom, is influenced not only by the nature of the community established; it is also "affected by a students' ability to function in social situations and interpret the flow of events in a discussion. For some students the social skills needed for classroom lessons are not necessarily fostered at home or other nonschool settings" (Doyle, p. 180).

Peers serve as an important resource, away from the demands of whole class communication, for developing mathematical thinking. White (2003) found that limited-English-speaking students were more inclined to share their thinking with a friend rather than with the whole class. Through peer discussion students clarified their thinking about the nature of task demands and how those demands could be met. The teacher noted: "A lot of time they won't share something with the whole group. But they will share it with somebody sitting next to them, or they can sometimes get ideas from other kids who are sitting next to them" (p. 42). Holton and Thomas (2001) have named two-way peer tutoring as "reciprocal scaffolding" (p. 99). They noted that successful interaction required competence and experience in asking appropriate questions of themselves and each other. Goos (2004) documents the way in which one student views his interactions with peers in his senior mathematics class as an enriching learning experience:

> Adam helps me … see things in different ways. Because, like, if you have two people who think differently and you both work on the same problem you both see different areas of it, and so it helps a lot more. More than having twice the brain, it's like having *ten* times the brain, having two people working on a problem. (p. 278)

Gifted students, as well as low attainers, benefit from collaboration with peers. From a study with six mathematically gifted students Diezmann and Watters (2001) provide evidence that small homogeneous group collaboration significantly enhanced knowledge construction. Group participation also developed students' sense of self-efficacy. In particular, collaborative work that was focused on solving challenging tasks produced a higher level of cognitive engagement than that produced by independent work. The supportive group provided a forum for the giving and taking of critical feedback and building upon others' strategies and solutions. From their investigation Diezmann and Watters claim that the positive effects of homogenous groupings for gifted students outweigh those offered through heterogeneous arrangements.

However, not all groups work effectively. Peter-Koop (2002) provides evidence of students' refusal to interact with others. In a study that explored group processes within young students' (third and fourth grade) classrooms, the researcher demonstrated the difficulty for students to engage with a new line of thought within the distractions of group discussion. A New Zealand study undertaken by Higgins (1997) revealed that young students' group work (New Entrant to J2) is not as effective as their teachers believed it to be. Higgins showed that student explanations appeared to be constrained by the group process. Higgins (2000) demonstrated that teachers are often unclear about their role during student group work. However when the mathematical intent of the group activity was articulated at the beginning of the activity and again during the feedback episode, and when student contributions were evaluated in terms of that intent, students appeared to engage more actively with the mathematics.

Outcomes are not always positive for individual group members. Summers (2006) has reported that students' motivation in mathematics declined over time in groups that valued the academic goals of group work. Sixth grade students in the research became performance avoidant as a function of their group membership. The dynamics of collaborative work has also been studied extensively by Barnes (2005). In her investigation into classroom group processes at the senior secondary school level, Barnes found that both poor social relationships and poor communication within groups contributed to limited student mathematical engagement in an activity. Barnes analysed video data to explore precisely who introduced ideas, the response of others, who controlled, sustained, or impeded the discussion. She provides evidence that two students were frequently interrupted and their efforts ignored by others during group work. These 'outsiders' were assigned their position by others who did not recognise their rights to explain, question or challenge. Barnes reports that these students learned less, and although the video data revealed that they offered the group distinctive mathematical insight, these students tended to lose confidence in their mathematical ability. Barnes suggests that a pedagogical practice that includes all students in group work regularly reinforces the norms of careful and courteous listening.

Individual Thinking Time

Other studies provide evidence of the benefits for some students that can be gained through independent learning approaches. For example, McMahon (2000) reported on successful individualised teaching in a Mathematics Recovery Programme targeting Year 2 students. The two teachers involved had undertaken a year long programme to develop purposeful instructional resources and strategies for their individualised teaching sessions. They focused the level appropriately, encouraged independent learning, and provided the students time to reflect on their thinking and methods. The success of the intervention depended crucially on the teachers' sophisticated craftsmanship that involved both anticipating and supporting students' responses. The researchers report that the students increased their confidence in their ability and their mathematical understanding.

Thomas (1994) explored what students actually say when they are engaged in classroom talk. In analysing the classroom talk of 46 New Zealand primary school students during mathematics lessons, Thomas reported that "in the many hours of recorded and transcribed talk there were few instances of the children engaged in talk which could be directly linked with learning in the sense of a child obviously understanding something as a result of their talk with another child" (p.ii). Research undertaken by Sfard and Kieran (2001) lends support to these findings. At the same time, the research underscores the importance of the role of the teacher in instructing students about effective group processes. The researchers report on two students, Gur and Ari, who were set a task by the teacher and expected to work together towards producing a solution to the task. Classroom observations led the researchers to believe that the students were working together but further scrutiny of their working together revealed otherwise:

> While having a close look at a pair of students working together, we realised that the merits of learning-by-talking cannot be taken for granted. Our analyses compel us to conclude that if Gur did make any real progress, it was not *thanks* to his collaboration with Ari but rather *in spite* of it, and if this collaboration did, in the end, spur Gur's development, it was probably mainly in an indirect way, by providing him with an incentive to learn. Our experiment has shown that the interaction between the two boys was unhelpful to either of them. The present study, therefore, does not lend support to the common belief that working together can always be trusted to have a synergetic quality. It is not necessarily true that two people who join forces can do more than the sum of what each one of them can do alone. (p. 70)

Sfard and Kieran showed how articulating mathematical thinking to oneself can have beneficial effects for the individual. These researchers conceptualise 'talking to oneself' as a form of communication and record from their research how an invisible and inaudible discourse to self created mathematical thinking. They make the point that "interaction with others, with its numerous demands on one's attention, can often be counterproductive. Indeed, it is very difficult to keep a well-focused conversation going when also trying to solve problems and be creative about them" (p. 70).

Conclusion

This paper has explored the sort of classroom arrangements that contribute to the growth of students' mathematical identities and competencies. It responds to an overriding concern amongst educators in providing equitable access to opportunities that will develop in students a mathematical disposition and enhance life chances. However, although there is agreement about this overarching goal, there has not been shared understanding about how classroom organisational practice might contribute to equitable access. Researchers have long known that teachers and teaching are different from one classroom to another. Hence, determining the characteristics of grouping arrangements that make a significant difference for learners in every classroom setting is a highly complex task. However from this synthesis of research on group work a number of beneficial practices appear to hold good across people and classrooms.

What the synthesis has shown is that quality teaching provides a space for both the individual, partnerships, small groups and whole class arrangements. It has demonstrated that all such organisational structure *can* provide the context for social and cognitive engagement. It has revealed that inclusive classroom practices and organisation that govern patterns of participation, focused on mathematical ways of being and doing, are key resources for effective mathematics teaching. Effective teachers use a range of organizational processes to enhance students' thinking and to engage them more fully in the creation of mathematical knowledge. The synthesis has shown that within the classroom all students need some time alone to think and work quietly away from the demands of a group. This line of research has also revealed that reliance on classroom grouping by ability may have a detrimental effect on the development of a mathematical disposition and on students' sense of their own mathematical identity.

Teachers who make a difference to all learners work at establishing a web of relationships within the classroom community. They do this to encourage active participation, taking into account the different purposes of, and roles within, the particular social arrangements they establish for their students. Organisational structures are established with a view towards the potential of those arrangements in developing students' mathematical competencies and identities and in providing other positive outcomes for students in particular contexts. But, more significantly, over and above establishing participation structures, the effective teacher constantly monitors, reflects upon, and makes necessary changes to, those arrangements on the basis of their inclusiveness and effectiveness for the classroom community.

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