Students' Perspectives on the Nature of Mathematics

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This paper reports on one small component of a much larger study that explored the perspectives of students towards mathematics learning. Students were asked "What do you think maths is all about?" Some students responded in terms of mathematical content. Others commented on learning in general, or on problem-solving in particular. Some students talked about the usefulness of mathematics for everyday life. An overwhelming number of students answered the question by talking about the importance of mathematics for the future.

The nature of mathematics has been the focus of much writing over the last few decades (e.g., Begg, 1994, 2005; Dossey, 1992; Fuson, Kalchman & Bransford, 2005; Ocean, 2005; Presmeg, 2002; Winter, 2001). Dossey (1992) argues that different conceptions of mathematics influence the ways in which society views mathematics. This can influence the teaching of mathematics, and communicate subtle messages to children about the nature of mathematics that "affect the way they grow to view mathematics and its role in their world" (p. 42). Similarly, Presmeg (2002) has argued that beliefs about the nature of mathematics either enable or constrain "the bridging process between everyday practices and school mathematics" (p. 295).

Different dichotomies have been used to highlight the contrasting ways in which mathematics is viewed. For example, Dossey (1992) has distinguished between external conceptions of mathematics held by those who believe that mathematics is a fixed body of knowledge that is presented to students, and internal conceptions that view mathematics as personally constructed, internal knowledge. Begg (1994, 2005) has contrasted mathematical content (knowledge and procedures) with mathematical processes (reasoning, problem solving, communicating, and making connections). Winter (2001) has written about a tension between a mechanistic view of mathematics (as in the development of skills and knowledge), and mathematics as a means towards fostering citizenship and responsibility within society (as in the development of personal, spiritual, moral, social, and cultural dimensions).

A distinction has been made between mathematical activity carried out for its own sake, and mathematical activity that is useful for something else (Huckstep, 2000). In order to distinguish between the aims and purposes of mathematics education, Huckstep asks: "What are we trying to do in mathematics education?" and "What are we trying to do it for?". This particular dichotomy is closely related to the debate about what is mathematics and what is numeracy (Hogan, 2002; Stoessiger, 2002). Definitions of numeracy emphasize the practical or everyday uses of mathematics in contexts such as homes, workplaces, and communities (Stoessiger, 2002). Writers who argue that mathematics is valuable for its own sake often write about the beauty and aesthetics of mathematics, and the sheer enjoyment of doing mathematics (e.g, Holton, 1993; Winter, 2001).

The current curriculum document for schools in New Zealand gives consideration to the nature of mathematics, and states that

> Mathematics makes use of specific language and skills to model, analyse, and interpret the world… [It] involves creativity and imagination in the discovery of patterns of shape and number, the perceiving of relationships, the making of models, the interpretation of data, and the communication of emerging ideas and concepts" ((Ministry of Education, 1992, p. 7).

Although children spend a lot of time doing mathematics, we know little about how they view the mathematics they do. A few studies have explored this issue, but all were with children at the Year 5/6 level (Grootenboer, 2003; Howard & Perry, 2005; Masingila, 2002). Howard and Perry (2005) held conversational interviews with Aboriginal children living in a remote rural community in New South Wales to explore their beliefs about learning mathematics. Most examples of the children's responses seemed to reflect an external conception of mathematics, with the children positioning themselves as passive recipients of the teacher's wisdom and superior knowledge. According to Howard and Perry, these children did not seem to be aware of their own mathematical competencies, strategies and problem-solving abilities in mathematics. Instead they emphasized the importance of watching and listening to the teacher. Grootenboer (2003) investigated the views and feelings of New Zealand children on the nature and purpose of mathematics and how they saw themselves as learners of mathematics. The children's responses indicated a rather narrow conception of mathematics, limited mostly to number concepts and arithmetic.

In recent years, many writers have drawn attention to the importance of talking with and listening to students, in order to appreciate their unique perspectives (e.g., Fielding, Fuller, & Loose, 1999; Rudduck & Flutter, 2000; Young-Loveridge, 2005; Young-Loveridge & Taylor, 2005; young-Loveridge, Taylor & Hawera, 2005). The present paper reports on one aspect of this study, which set out to look at children's views of mathematics and mathematics learning.

Method

Participants

The participants in this study were approximately 400 students from years 2 to 8 (six- to thirteen-year-olds) attending six primary schools and six intermediate schools in two major urban centres.

Procedure

Schools were asked to nominate students from across a range of mathematics levels. The students were interviewed individually in a quiet place away from the classroom. Students were told initially that the interviewer was interested in finding out more about "how kids learn maths and how their teachers can help them" and "what kids themselves think about learning maths." The interviews were audio-taped and later transcribed. A content analysis of the tape transcripts is being completed to identify common themes and ideas. Coding categories are being constructed for further data analysis.

This paper focuses on students' responses to the question: "What do you think maths all about?" At the time of writing, the transcribing was not quite finished, so the following results are based on our initial impressions from having conducted the interviews and on a preliminary analysis of transcripts from two of the schools.

Findings

Students responded to the question about the nature of maths in a number of ways. A notable group of students were unable to give any response at all. Those who did respond seemed to interpret the question in a variety of different ways. Some children appeared to interpret the question in terms of their immediate mathematics learning in the classroom. Other children interpreted the question in terms of the purpose of mathematics for them in the "here-and-now." Another group also interpreted the question with respect to the purpose of mathematics, but considered this in terms of their long-term futures. The final group of students seemed to have really thought about the nature of mathematics and commented on the intrinsic value of learning mathematics.

The responses of those who did answer the question have been organised according to a number of major themes, and these are explored below. [Note: numbers in brackets refer to the identity of individual students. Identifying codes beginning with N are for children in Years 2-4, C and D are Years 5/6, and G Years 7/8].

Mathematical Content

One group of students referred to particular aspects of mathematical content in their explanations of what mathematics is about. Many spoke about aspects of number and/or operations. A few mentioned geometry and statistics. One child stated that

> maths is not just about numbers; maths is something that you can make really fun, especially with geometry and symmetry, because you can draw shapes and draw characters that you like [C26]

> I would say maths … has a lot of different strands like geometry and stuff, where you work with shapes and there's hard sums and easy sums and shortcuts and such things [G40]

One year 4 student mentioned patterns, and explained how various operations and domains are interconnected.

> Patterns … because plus is minus and plus is times and times is division and division is fractions and fractions is decimals and decimals is percentages and it goes on and on [N32]

Some children's responses reflected the difficulty they experienced in trying to say what mathematics is.

> I know you use maths for everything in normal day life, but I'm not sure what it's about… I'd just say it's about numbers and working numbers together and taking them away to work out stuff [G39]

> Maths is like, you write down, you've got all these numbers and you've got all these maths symbols so you've got numbers from 1 to 10, you have to try and squash them together, so like for example, 1 plus 9 equals 10 [N2]

> Just memorising numbers, learning how to divide, subtract and stuff, 'cause if we didn't have the numbers then it would be totally different, you wouldn't be able to count things so you wouldn't be able to know how much you'd need for stuff, you'd put the wrong amount, there wouldn't be an amount [D11]

Processes

A substantial group of students spoke about processes. These were further subdivided according to whether the focus was on general cognitive processes such as learning and thinking, or on mathematical processes such as problem-solving specifically.

Learning

Some children commented that the nature of mathematics was about learning. These responses tended to be extremely brief and did not explain how mathematics was akin to learning. Instead they tended to focus on a justification for being involved with mathematics. The following are just a few examples of responses that referred to learning.

> Just learning, for when you get older [G48]
>
> Learning and education and finance, stuff like that [N9]
>
> Learning and helping you get brainier [N21]

Thinking

Quite a number of children commented that mathematics was about thinking or using their brain.

> Well it's kind of like a challenge for your brain and stuff [N12]
>
> It's about learning, helps kids think [N14]
>
> Using your brain and thinking [N35]

Problem Solving

A small group of children said that mathematics was about problem solving, but gave little or no explanation for their views.

> I think maths is about problem solving [G45]
>
> It helps you so you can get smarter, and when you're in a problem or something [D19]

Effort and persistence was mentioned by one child who responded that mathematics is about:

> trying your best on your work, like don't give up on your work and just do scribble, and just give it a try if it's too hard [N24]

The Utility of Mathematics

Quite a large number of children talked about the usefulness of mathematics. This group was further sub-divided into those that considered mathematics in terms of its immediate utility (in the "here-and-now"), and those who were more concerned about their long-term futures.

Everyday Life Here and Now

Some children focused on the usefulness of mathematics for their everyday lives and many of these referred to needing maths so as to be able to work with money.

> Maths is like something you use every day. You need to learn it because it can help you in life, 'cause you use it like every day when you're doing stuff. Like money and stuff, you calculate your money [G49]
>
> Trying to learn them for when you're older, for when hard questions come and stuff. Like paying bills and stuff, or loans and stuff [G37]
>
> If you need some money out of your wallet you might be able to use maths and equations, or if you work at a bank or at a dairy, maths would help you out, how much change you get [N36]
>
> So when you grow up, instead of, when you go shopping, you know it straight away instead of

> going like that and using your fingers [D9]

Life in the Future

Quite a number of students chose to respond to the question about the nature of mathematics by talking about how worthwhile it was for the future.

Some students' comments about the future were in relation to higher levels in the school system.

> I think maths is teaching me more so I can move on to the senior school and I can be ready to learn even harder maths questions [N1]

> Learning and teaching about maths, like if you go to senior schools, they tell you about maths, and at senior schools, it's more harder and it's better [N19]

Children in all age groups commented on the importance of maths for the future in terms of getting a job. Below are the responses of two Year 3 and 4 students:

> Learning your maths so you get better when you are older for your job because you need maths [N6]

> I think it is about learning new… like if you want to teach other kids when you are an adult, when you want to be a teacher, you have to learn from your last teacher that taught you [N18]

Year 5 and 6 children gave slightly more sophisticated explanations, including comments about handling money.

> Learning so you can handle with money so then you can grow up and get a job, you'll have to know how to sort out money. There's this kid called Mac in my class — he says that maths is stupid and he doesn't need maths to be a mechanic and stuff… I say that you need maths for every job when you grow up because it has maths. You need maths to sort out the money [D1]

One articulate year 6 student gave a response to the question showing considerable reflection on the importance of mathematics to him personally.

> Well for me … the main thing in school for me because most jobs you go to basically every job involves a bit of maths, quite a lot of maths actually, and so by learning maths, sometimes I don't enjoy it but I know that's like a good thing to learn and so it's sort of like a goal setter for life, if you know it, it just helps you to become more independent in a way 'cause you're not relying on the teacher a lot … if you don't do maths, you won't really get a good job, so… it's sort of a thing that sets you up for life really [C2]

Not surprisingly, Year 7 and 8 students expressed the most sophisticated ideas, commenting on their future roles as adults, jobs, getting on in life, and other activities reflecting independence and autonomy.

> Future jobs, you need to use it a lot. You can't just go through school without maths, you need to know how it works to see, like statistics with graphs and stuff, you need to be able to read them and understand them to see other things [G32]

> Figuring out and adding for lifestyle for when you're an adult… If you're a person at the shop, giving the person back their change, figure out how much they get back [G55]

> Maths is in every average day in everything you do, and I think maths is just helping you for the long run. And when you'll need to use it, and it's also a good general knowledge thing, just to know what to do, 'cause it's everywhere, maths [G43]

Enjoyment

A small but notable group of students considered the nature of mathematics to be about having fun. A year 4 student commented that mathematics was about

> Having fun and trying to get your numbers and answers right, and just try and learn quicker and easier [N20]

Further analysis of the transcripts will show just how prevalent this view is.

Non-Responders

Quite a large number of children appeared to have no view at all about the nature of mathematics. These children said things like

> I've never thought about it [J17]
>
> I have no idea [J29]
>
> I'm not completely sure [J36]

Although we do not yet have all the interview transcripts, our sense is that about a quarter of the children were categorised as Non-Responders. It will be interesting to see whether the Non-Responders are distinctive in terms of gender, ethnicity, or mathematical ability.

Discussion

It was evident from the responses analysed so far that it was difficult for some children to talk about the nature of mathematics. These findings suggest that children do mathematics without much thought or opportunity to discuss what it actually might be.

Many children who offered ideas about the nature of mathematics referred to aspects of the number domain. This is consistent with Grootenboer's (2003) finding that children's views of mathematics tended to revolve around number concepts and arithmetic. Like Fuson et al. (2005), we found that many of these students' responses reflected the view that mathematics is about computation. This is not altogether surprising, given that the Numeracy Development Project currently being implemented in schools emphasizes number and mental computation, particularly in the early years of school.

We were interested that a large number of students chose to comment on the usefulness of maths. We found, like Masingila (2002), that these students' perceptions of what mathematics is were linked to how they thought they used it. We were intrigued to find that many students talked about the usefulness of maths for their futures. This is consistent with much of the rhetoric about the importance of mathematics for the "knowledge society" (see Commonwealth of Australia, 2000; Ministry of Education, 2001; National Council of Teachers of Mathematics, 2000). Masingila's 11-year-olds did not appear to refer to the future when talking about the nature of mathematics. It is not clear whether this simply reflects the fact that data was collected several years ago before future-focused discourse became the norm, or whether it reflects the fact that US students have different concerns from those in New Zealand.

Despite the 1992 curriculum document devoting a strand of the mathematics curriculum to mathematical processes such as problem-solving, developing logic and reasoning, and communicating mathematical ideas, we noted that few children talked about the nature of mathematics in this way, although some children spoke about general cognitive processes such as learning and thinking. We wonder if this reflects the relatively narrow views of mathematics held by many people, which in turn impacts on children's views.

Mathematicians such as Holton (1993) have written about the pleasure people get from

doing mathematics for its own sake. It was heartening to see that some students saw mathematics as being about having fun. It seemed to be the younger children who described mathematics as an enjoyable pursuit. In our further analysis of the data, we will be interested to see whether older children also perceive mathematics as fun, or whether there is an age-related decline in students' enjoyment of mathematics.

We were interested in those children who did not appear to have a view about the nature of mathematics (the Non-Responders). As Presmeg (2002) has pointed out, beliefs about the nature of mathematics are important because they can either help or hinder the making of links between school mathematics and everyday practices. If children don't have a view about what mathematics is, then that may make it difficult for them capitalise on the mathematics they encounter at home and in other out-of-school settings.

Much of the data seems to indicate that children do perceive mathematics in dichotomous ways. Some children considered that mathematics was an external body of "stuff to be learned". Others suggested that they needed to make sense of the mathematics, in order to make connections between related mathematical ideas. Many children were aware of the significance of the mathematics in society, but for others, a more mechanistic view was noted.

Given that mathematics is part of the core curriculum, we think it could be important for teachers to help children engage with ideas about the nature of mathematics. In preparation for such discussions, teachers might benefit from examining their own beliefs about what mathematics is, and reflecting on the subtle messages they might convey to students about the nature of mathematics.

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