Visual Perturbances in Digital Pedagogical Media

Nigel Calder
University of Waikato
<n.calder@waikato.ac.nz>

Several studies have investigated how the formation of informal conjectures, and the dialogue they evoke, might influence young children's learning trajectories, and enhance their mathematical thinking. In a digital environment, the visual output and its distinctive qualities can lead to interpretation and response of a particular nature. In this paper the notion of *visual perturbance* is explored, and situated within the data obtained, when ten-year-old children engaged in number investigations in a spreadsheet environment.

When learners engage in mathematical investigation, they interpret the task, their responses to it, and the output of their deliberations through the lens of their fore-conceptions; their emerging mathematical discourse in that perceived area. Social and cultural experiences always condition our situation (Gallagher, 1992), and thus the perspective from which our interpretations are made. Learners enter such engagement with fore-conceptions of the mathematics, and the pedagogical medium through which it is encountered. Their understandings are filtered by means of a variety of cultural forms (Cole, 1996), with particular pedagogical media seen as cultural forms that model different ways of knowing (Povey, 1997). The engagement with the task likewise alters the learner's conceptualisation, which then allows the learner to re-engage with the task from a fresh perspective. This cyclical process of interpretation, engagement, reflection, and re-interpretation continues until some resolution occurs.

This echoes of Borba and Villareal's notion of humans-with-media (2005), where they see understanding emerging from an iterative process of re-engagements of collectives of learners, media and environmental aspects, with the mathematical phenomena. Some models of human behaviour likewise incorporate mind, mediating tools and tasks with societal and community influences, for example, activity theory (Engestrom, 1999). Other researchers emphasise the eminence of mental schemes, which develop in social interaction (e.g., Keiren & Drijvers, 2006). In essence the mathematical task, the pedagogical medium, the fore-conceptions of the learners, and the dialogue evoked are inextricably linked. It is from their relationship with the learner that understanding emerges. This understanding is their interpretation of the situation through those various filters.

When learners investigate in a digital environment, some input, borne of the students' engagement with, or reflection on the task, is entered. The subsequent output is produced visually, almost instantaneously (Calder, 2004) and can initiate dialogue and reflection, perhaps internally for the student working individually. This will lead to a repositioning of their perspective, even if only slight, and they re-engage with the task. They either reconcile their interpretation of the task with their present understanding (i.e., find a solution) or they engage in an iterative process, oscillating between the task and their emerging understanding. This allows for a type of learning trajectory that can occur in various media (Gallagher, 1992), but is evident in many learning situations that involve a digital pedagogical medium (Borba & Villareal, 2005).

There are, however, opportunities or constraints associated with the process. This paper is concerned with one aspect that might be perceived as a constraint, *visual perturbances*, but which can offer opportunities for enhanced mathematical understanding. When the students' fore-conceptions suggest an output that is different to that produced, a tension arises. There is a gap between the expected and the actual visual output. It is this visual perturbation that can either evoke, or alternatively scaffold, further reflection that might lead to the reshaping of the learners' perspectives: their emerging understanding. It shifts their conceptual position from the space they occupied prior to that engagement. The learner's reaction, if it emerges as a conceptual tension, is what I am defining as a *visual perturbance*. It is the tension for the learners between what their fore-conceptions indicated and the actual visual output the pedagogical medium produced.

As learners re-engage with the task, informal mathematical conjectures often have their speculative beginnings (Calder, Brown, Hanley, & Darby, 2006). Other researchers have noted that the development of mathematical conjecture and reasoning can be derived from intuitive beginnings (Bergqvist, 2005; Dreyfus, 1999; Jones, 2000). This intuitive, emerging mathematical reasoning can be of a visual nature. In both algebraic and geometric contexts learners have used visual reasoning to underpin the approach taken to conjecturing and generalisation (Calder, 2004; Hershkowitz, 1998). Meanwhile, Lin (2005) claims that generating and refuting conjectures is an effective learning strategy, whereas argumentation can be used constructively for the emergence of new mathematical conceptualisation (Yackel, 2002). Visual perturbances, and the dialogue they evoke, can generate informal conjectures and mathematical reasoning as the learners negotiate their interpretation of the unexpected situation. Research into students' learning in a computer algebra system environment (CAS), likewise revealed that probably the most valuable learning occurred when the CAS techniques provided a conflict with the students' expectations (Keiran & Drijvers, 2006). If the visual perturbance induced by investigating in a digital medium meant the learner framed their informal conjectures in a particular way, it is reasonable to assume that their understanding will likewise emerge from a different perspective.

Method

This paper reports on an aspect emerging from the data of an ongoing study into how spreadsheets, as a pedagogical medium, might influence learning trajectories and filter understanding in problem solving processes. This part of the study involved a group of ten-year-old students, attending five primary schools, drawn from a wide range of socio-economic backgrounds. There were four students from each school, who had been identified as being mathematically talented through a combination of problem solving assessments and teacher reference: eleven boys and nine girls. Their discussions were audio recorded and transcribed, each group was interviewed after they had completed their investigation, and their onscreen output was printed out. For this paper, the transcripts and printouts, together with informal observation and discussion formed the data that were analysed. The data were coded for NVIVO analysis, and then analysed for emerging patterns.

Results and Discussion

The data in this study illustrated the notion of visual perturbance. We examine some of the episodes in the data that illustrate different types of visual perturbance and ways in which they influenced the students' interpretation and learning trajectories. It is interesting to note that they do not necessarily emerge discretely, but that an episode can illustrate several types of visual perturbance in an interrelated manner.

Episode 1

This relates to an activity set in a scenario that allowed the children to explore different ways that they could get a pocket money allowance. This particular dialogue and output relates to investigating one possible option: receiving one cent the first week, and then doubling each week, that is, two cents the second week and so on. The children initially began to enter the counting number sequence into the spreadsheet.

1
2
3

Mike, using his current understandings in number operation, immediately had a conflict between what he saw, and what his more global perspective was telling him it should be. This created the visual perturbance, one that prompted re-engagement of an exploratory nature.

Mike: Hey, there's a bit of a twist, look, third week he gets 4 cents. We'll have to change it.

His mathematical fore-conceptions and understanding of the situation allowed him to predict with confidence the outcome of 4 cents for the third week, yet the screen displays 3. Hence he recognised the tension and articulated the need to reconcile this. This facilitated the process by which the output is produced. It also suggested a process of re-negotiation of what the task was about: their interpretation of the task rather than the engagement in its investigation. His partner Jay started to enter input into cell A2.

Mike: No, no, no we'll have to be in C (column C of the spreadsheet).

This was another visual perturbance, but of a different nature. It seemed to be primarily due to his present understandings of the structure and processes of the spreadsheet environment, rather than his mathematical fore-conceptions. Thus, they were addressing a technical or formatting aspect associated with their investigation. Mike was also perhaps looking to show in some way the relationship between the counting sequence, in this case illustrating the number of weeks, and the amount of money received each week. The pedagogical medium through which he engaged the mathematical phenomena was beginning to structure his approach to the task and his thinking. It was this informal indication of a relationship, and the possibility of a pattern to the amount of money received, that was the beginning of the mathematical thinking, however.

Jay entered 1 into cell C1 to represent the cent for the first week. He began to enter a formula into C2, which he simultaneously verbalised:

Jay: = C1 + 1 + 0

The output in C column was now:

1
2

Mike suggested the next entry:

Mike: = C2 + 2

The output was now:
1
2
4

Jay: Goes up by two. We have to double each week.

He pondered on the input to the next cell (Cell C4).

Jay: = C3 +

He considered which number to add to C3 to continue the doubling pattern. Mike meantime, addressed the same output, but his fore-conceptions were different, so his thinking was too. His interpretation of the question, the spreadsheet, and his mathematical understanding of the processes involved also influenced his thinking.

Mike: According to this it doubles each week.
Jay: How do you make it double?
Mike: Times by two, and star is times.

Mike took over the keyboard and entered =C3*2 into cell C4 then filled down in the cells below.

Jay: Look at the amount of cash you get on double though.
Mike: That's the biggest one.
Jay: See that huge amount of cash.

The spreadsheet has enabled them to process the large amounts of data quickly with the particular medium shaping their investigation in a distinct, structured manner. Their surprise with the difference between what they expected from option 2, and the size of the actual output is illustrative of a visual perturbance. Throughout the process, the visual perturbances, the difference between what their existing understanding suggested and the actual output, influenced their decisions, and hence their learning trajectory. Their mathematical reflection was a function of their interaction with the task filtered by the pedagogical medium through which it was encountered, and their prevailing mathematical discourse. As their perspective was also repositioned through each interaction, the spreadsheet environment has also influenced this aspect.

Episode 2

The next scenario illustrated a different type of visual perturbance. Tension evoked from the variance between the expected and actual output was evident, but in this situation the visual perturbance arose when the actual output was beyond the scope of the children's current conceptualisation. This involved the scientific form of very large numbers. The students sought teacher intervention, for reconciliation of their mathematical fore-conceptions with the output.

This episode related to a traditional Grand Vizier problem with the doubling of grains of rice for each consecutive square of a chessboard, and investigating how long this might feed the world for. This investigation was initiated after the children had already had some experience of using the spreadsheet. They were less tentative regarding the operational aspects of using them, for example, they were more comfortable generating formulas, and had an expectation of what output they might get based on some accumulated experience.

Ana: It goes 1,2,4,8,16 … , so its doubling
Lucy: =A1 times 2.
Ana : Is that fill down.
Lucy: Go down to 64.
Ana: Right go to fill, then down.

They made an initial interpretation of the problem, and immediately saw a way the spreadsheet would help them explore the problem. However, there was some unexpected output in a visual form they could not recognise.

Lucy: What the …
Ana: Eh…
Lucy: What you…
Ana: 9.22337 E+18.

The unexpected outcome produced a significant perturbation as they attempted to reconcile it with their existing understanding. This was a visual perturbance that was associated with an idea or area they had no previous conceptual cognition of, that is, scientific notation. They quickly decided it was beyond their conceptual scope and sought the teacher's input. The teacher gave some explanation about scientific form related to place value. They made sense of this within their current conceptualisation.

Lucy: So that would be the decimal space up 18 numbers.

They wrote it out on paper to get a picture of it within their current frame:
9223370000000000000
They re-engaged with the activity from their repositioned perspective.

Lucy: We have to add it all up.
Ana : Wow it's big.
Lucy: = A1+A2+A3 …
Ana : Takes a long time, because its 64.

Lucy was using a simple adding notation with the spreadsheet, to sum the column of spreadsheet cells A1, A2, A3 etc. Ana realised, and articulated, that there were 64 cells from A1 to A64, so it would take a long time to enter them individually. They acknowledged the scope of this particular task, and intuitively felt the medium offered possibilities for a more efficient approach. They reflected on prior knowledge and earlier experiences, and negotiated a way to undertake their decided trajectory more easily.

Lucy: Sum.
Ana : = sum (A1:A64).
Lucy: 1.84467E19.

Ana: How long will that feed?
Lucy: 1.84467E19 divided by 2000.

The sum of the values in cells A1 to A64 was 1.84467 x 10^{19} that is, 18446700000000000000. There was no reaction to the scientific form of the output at all this time, and they were almost seamlessly moving into the next phase of their investigation with the newly reconciled concept. Their prevailing discourse in this area had been repositioned through the reconciliation of their fore-conceptions with the unexpected output. This reconciliation and subsequent repositioning was initiated by the visual perturbance they encountered as a result of investigating in this particular pedagogical medium.

Episode 3

The next two scenarios related to an activity investigating the pattern formed by the 101 times table.

The two students had entered the counting numbers into column A and were exploring the pattern formed when multiplying by 101 in column B:

1
2
3 etc.

Awhi: =A2 * 101. Enter.
Ben: 202.

Contemplating the output produced from their unique conceptual perspective, they postulated an informal, rudimentary conjecture through prediction.

Awhi: Now let us try this again with three. Ok, what number do you think that will equal? 302?
Ben: No, 3003. They copy the formula down to produce the output below.
1 101
2 202
3 303 etc.

Ben: (continues) 303.

The actual output was different to the output they expected. This created a visual perturbance, which in this case was easily reconciled with their present understanding. The visual perturbance had caused a reshaping of their prediction that allowed them to reposition their conceptualisation. It also initiated the beginnings of a conjecture or informal generalisation.

Awhi: If you go by 3, it goes 3 times 100 and zero and 3 times 1; 303.

They then explored a range of two and three digit numbers, before extending the investigation beyond the constraints of the task.

Awhi: Oh try 1919.
Ben: I just have to move that little number there, 1919.

The following output was produced:

193819

Interestingly, they seemed to disregard this output and form a prediction based on their fore-conceptions.

Awhi: Now make that 1818, and see if its 1818 (the output).
Ben: Oh look, eighteen 3, 6, eighteen.

There was a visual perturbance, which made them re-engage in the activity, reflect on the output, and attempt to reconcile it with their current perspective. It caused them to reshape their emerging conjecture.

Awhi: Before it was 193619-write that number down somewhere (183618) and then we'll try 1919 again.
Ben: Yeh see nineteen, 3, 8, nineteen. Oh that's an eight.
Awhi: What's the pattern for two digits? It puts the number down first then doubles the number. This is four digits. It puts the number down first then doubles, and then repeats the number.

The visual perturbance made them reflect on their original conjecture and reposition their perspective on the initial, intuitive generalisation. It stimulated their mathematical thinking, as they reconciled the difference between what they expected and the actual output, and rationalised it as a new generalisation. This new generalisation was couched in visual terms.

Episode 4

The next episode was part of the same investigation, but with a different pair of children, as they began to explore what happens to decimals. Ant predicted that if they multiplied 1.4 by 101, they would get 14.14.

Bev: I get it, cos if you go 14 you'll get fourteen, fourteen.
Ant: We'll just make sure.

They entered 1.4, expecting to get 14.14 as the output.

Bev: 141.4, it should be 1, 4 (after the decimal point, that is 14.14).

This created a visual perturbance. They began to rationalise this gap between the expected output (14.14) and the actual output (141.4). This visual perturbance caused a reshaping of their conjecture or informal generalisation. In doing so they drew on their current understandings of decimals and multiplication, but also had to amend that position to reconcile the visual perturbance the pedagogical medium has evoked. Again they used a visual lens to do so.

Ant: We're doing decimals so its 141.4.
Bev: So it puts down the decimal (point) with the first number then it puts the 1 on, then it puts in the point single number whatever.
Ant: It takes away the decimal to make the number a teen. Fourteen.
Bev: 141.
Ant: Yeah. It takes away the decimal (14 – my insertions) and then it adds a one to the end (141), and

then it puts the decimal in with the four (141.4).

Bev recognised that this as more of a visual description of this particular case rather than a generalisation. There was still a tension with her existing understanding.

Bev: No it doesn't, not always, maybe. It might depend which number it is.
Ant: Try 21 or 2.1. See what that does.

According to Ant's conjecture from earlier they would be expecting to take away the decimal point (21), add a one to the end (211), and then re-insert the decimal point and the one (211.1). However the output was 212.1, which created another visual perturbance to be reconciled.

Bev: No it doesn't.
Ant: Two, where's point? One two point one.
Bev: Oh yeah, so its like, the first number equals…

They tried to formulate a more generalised conjecture. Bev proffered a definition that they negotiated the meaning of, then situated within their emerging conjecture.

Ant: Takes away the decimal and puts that number down then puts the first number behind the second number. Aw, how are we going to write this?
Bev: It doubles the first numbers.
Ant: Takes away the decimal, doubles the first number, then puts the decimal back in.
Bev: How does it get here?

They then entered 2.4 and made predictions regarding the output in light of their newer conjecture.

Ant: Twenty-four, twenty-four with the decimal in here.
Bev: It will be doubled; twenty-four, twenty-four but the last number has a point in it, a decimal.

Their predictions were confirmed, and they negotiated the final form of their generalisation. They were still generalising in visual rather than procedural terms, and Bev suggested a name for their theory, double number decimals, that they both had a shared sense of understanding of. This mutual comprehension had emerged through the process: the investigative trajectory they have negotiated their way through. The investigative trajectory was directly influenced by the pedagogical medium through which they engaged the mathematical activity. More specifically, the questions evoked, the path they took, and the conjectures they formed and tested were fashioned by visual perturbances: the tension arising in their prevailing discourse by the difference between the expected and actual output. The process should not necessarily stop just there, however. An intervention, perhaps in the form of a teacher's scaffolding question, might initiate the investigation of why this visual pattern occurs.

Conclusions

Each of the above episodes illustrated how the learning trajectory, was influenced by the learners' encounter with some unexpected visual output as they engaged in tasks in this particular domain, through the pedagogical medium of the spreadsheet. The perturbation, and the dialogue that ensued as the learners reconciled their existing perspective with this unexpected output, seemed to create opportunities for the re-positioning of their existing understanding, as they negotiated possible solutions to the situations.

The engagement with the task, and with the medium, often evoked dialogue. This was an inherent part of the negotiation of understanding. When the students' fore-conceptions suggested an output that differed to that produced, a tension arose. This output, in visual form, initiated the learners' reactions, reflections and subsequent re-engagement with the task. The learners posed and tested informal conjectures, and negotiated a common interpretation through dialogue. This facilitated mathematical thinking, and they developed new understanding.

The data in this study illustrated the notion of a visual perturbance. Within this notion there seemed to be several manifestations or variations.

1. When the visual perturbance led to a change in prediction. It caused an unsettling and repositioning of the prevailing discourse, but the re-engagement was of an exploratory nature.
2. When the visual perturbance caused a reshaping of the conjecture or generalisation. This was similar to that above, but the re-engagement was more reflective and global in nature as compared to a specific example. This was more often accompanied by a significant amount of dialogue and negotiation of meaning.
3. When the visual perturbance made them re-negotiate their sense making of the task itself. This was not a distinct process from the investigative trajectory, but interwoven, with each influencing the other.
4. When the visual perturbance was associated with an idea or area they had no previous conceptual cognition of. The tension this evoked often led them towards seeking further intervention, frequently in the form of teacher led scaffolding.
5. When the visual perturbance led them to further investigate and reconcile their understanding of a technical or formatting aspect associated with their exploration. This was often also symbiotically linked to the conceptual exploration, but sometimes in unexpected ways. For instance, the rethinking of their approach to formatting an actual formula due to a visual perturbance was a structural aspect, but they were simultaneously re-engaging with a mathematical process while negotiating their understanding of the format, for example, in this case some form of algebraic thinking.

These episodes illustrated that the particular pedagogical medium of the spreadsheet, at times induced a particular approach to mathematical investigation. This occurred through the tension that arose from the learners' engagements with the task, when the actual output differed from that which their fore-conceptions led them to expect. This output being in visual form, led to the term visual perturbances, and it appeared this was a particular characteristic of the learning trajectory when using spreadsheets. It may be that this is a generic characteristic of learning trajectories in digital media. Certainly the literature suggested that with CAS software, unexpected outcomes that arose while engaging with algebraic tasks through that medium, influenced the learning trajectories and provided rich opportunities for learning (Kieren & Drijvers, 2006). It appears to be an area that would benefit from further investigation.

References

Bergqvist, T. (2005). How students verify conjectures: Teachers' expectations. *Journal of Mathematics Teacher Education*, *8*, 171-191.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the re-organisation of mathematical thinking: Information and communication technologies, modelling, experimentation and visualisation.* New York: Springer.

Calder, N. S. (2004). Spreadsheets with 8 year olds: Easy to visualise? *Teachers and Curriculum, 1*, 125-143.

Calder, N. S., Brown, T., Hanley, U., & Darby, S. (2006). Forming conjectures within a spreadsheet environment. *Mathematics Education Research Journal, 18*(3), 100-116.

Cole, M. (1996). *Cultural psychology: a once and future discipline.* Cambridge, MA: Harvard.

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics, 38*, 85-109.

Engestrom, Y. (1999). Activity theory and individual and social transformation. In Y. Engestrom, R. Miettinen, & R.-L. Punamaki (Eds.), *Perspectives on Activity Theory* (pp. 19-38). Cambridge: Cambridge University Press.

Gallagher, S. (1992). *Hermeneutics and education.* New York: State University of New York Press.

Hershkowitz, R. (1998). About reasoning in geometry. In C. Mammana & V. Villani (Eds.), *Perspectives on the Teaching of Geometry for the 21st Century* (pp. 29-37). Dortrecht: Kluwer.

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*(1-2), 55-85.

Keiren, C., & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning, 11*, 205-263.

Lin, F. (2005). Modelling students' learning in argumentation and mathematics proof. In H. Chick & J.L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 1, pp. 3-18). Melbourne: PME.

Povey, H. (1997). Beginning mathematics teachers' ways of knowing: The link with working for emancipatory change. *Curriculum Studies, 5*(3), 329-343.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behaviour, 21*(4), 423-440.