Students' Tendency to Conjoin Terms: An Inhibition to their Development of Algebra

Judith Falle
University of New England
<judith.falle@une.edu.au>

When students' responses to a test of introductory algebra items were Rasch modelled, three distinct "ability" clusters occurred. The question then arose as to the mathematical thinking that could characterise each of these groups. Data from the test revealed that the tendency to conjoin terms inappropriately occurred with different frequencies in each of the three groups. Interview data and error analyses provided further insight into the students' thinking that resulted in these types of errors. Implications for classroom practice are considered.

Many students find the demands of shifting their thinking from arithmetic to algebra challenging and, perhaps, frustrating in its strangeness. This is evident from the errors made by students, and the underlying misconceptions held by the students. Many of these misconceptions arise from students' arithmetic experiences that they (with a certain logic) generalise to their new experiences of algebra (MacGregor & Stacey, 1997). These errors seem to persist across the grades, despite increased exposure to algebra. If these errors can be understood as resulting from students' incorrect generalisation from previous (arithmetic) learning rather than as being symptomatic of cognitive immaturity (MacGregor & Stacey, 1994), then they may be addressed, once identified, by appropriate teaching methods (Tirosh, Even, & Robinson, 1998; Hall, n.d.; Tall, 1994).

One type of error made by students beginning algebra is that which arises from students' tendency to conjoin terms inappropriately (i.e., $5x + 3$ is written as $8x$). The tendency can be attributed to various causes, such as: students wanting to "close" or "finish" an algebraic expression (Booth, 1984, 1988; Tirosh et al., 1998; Hall, n.d.); students making false generalisations from an arithmetic context (e.g., $30 + 4$ becomes 34, or, $3 + 1/4$ becomes $31/4$ (Matz, 1982)); or students interpreting brackets in an expression as indicating that the expression inside the brackets is to "be done first" (e.g., when $2(x + 5)$ becomes $10x$) (Linchevski & Herscovics, 1994). The tendency for students to conjoin terms inappropriately appears when they first encounter algebra. If this remains unremarked, and uncorrected, and possibly masked as students deal with more complex algebraic expressions, further development of their algebraic understanding must be inhibited.

The question addressed in this paper is whether students' ability, as measured by their success on a test of algebraic techniques is associated with their tendency to conjoin terms. The discussion draws on data from items in a test given to students as part of a study of their thinking as they carried out simple algebraic techniques. Only the data from students' responses to particular items in the test are discussed in this paper. The items under consideration are those in the test that required students to simplify expressions by collecting like terms or first expanding brackets and collecting like terms, as well as "semi-literal" items that required students to rewrite an algebraic statement[1]. The data discussed

[1] The term "semi-literal" is used to describe items that ask for an algebraic form of a statement, that still uses some numbers. These items are those used, or similar to those used, by Küchemann (in Hart, 1981).

in this paper are a small part of the data collected during the main study, which is described in the methodology.

Methodology

Data Collection

The main study involved participants from three private secondary schools in a regional town (n = 222). The participants were students from Years 8 and 9 when the study began. These students were in the second and third years of secondary school, and so had been studying algebra for two or three years. The study aimed to find associations between language structures used by students to describe their thinking as they carried out various types of algebraic processes and their mathematical ability. The study consisted of two phases. The first phase was a test consisting of forty items based on the beginning algebra techniques outlined in the *Mathematics 7 – 10 Syllabus* (Stage 4, Board of Studies NSW, 2002) and associated textbooks used by the participating schools. Also included, to provide a well-documented basis for comparison, were items from Küchemann's study (1981), or adaptations of those items. The tests were administered in Term 4 of the school year by the class teachers and collected and marked by the researcher. The results were Rasch modelled using QUEST software (Adams & Khoo, 1994).

The second phase of the study consisted of interviews with students from each of the schools. Because of organisational constraints, this phase occurred in the first term of the year following the test. Students were selected for interview on the basis of their test performance so that a range of abilities would be represented at the interviews. The students who were finally interviewed were those for whom the relevant permission and consent had been obtained, and who were available at times suitable to the school, the teachers, and the researcher. These students were representative of the range of abilities as described by the Rasch model.

The interviews were structured using the test items grouped according to syllabus topic areas (Stage 4, Board of Studies NSW, 2002). Students were interviewed individually using a prepared protocol of questions supplemented by further probes or prompts or requests for clarification by the interviewer, depending on the response given to the initial question. The students were presented with each group of items, one group at a time, and asked the initial stimulus question, "What goes on in your head when you see questions like these?" Responses were audio-taped, and transcribed for later analysis.

Results from the interviews were used to complement the test responses. A particular aspect of those responses, namely the conjoining of terms and the language used by students during the interviews, is discussed in this paper.

Data Analysis

Test Items

The test items were marked and the results analysed using Rasch modelling, and later, an analysis of errors. The test responses were coded as either correct (1) or incorrect (0). Test items were marked by the researcher. Only algebraically "complete" answers were marked as correct. Responses where intermediate steps only were written were also counted as "incorrect", as were those instances where students left a blank (baulk).

The Rasch model uses dichotomous data (e.g., correct/incorrect) from a set of items that test a single construct (unidimensional). Item difficulty and participant ability scores are based on a probabilistic scale of successful response to each item by each participant.

Rank order of item difficulty and participant ability are then mapped on the same equal interval scale in *logits* (scale units) (Bond & Fox, 2001). The software used to model the data (QUEST, Adams & Khoo, 1994) enables the reliability of the data, and the extent to which each item fits the construct, to be calculated. These statistics are summarised in Figure 1. Reliability of the *item difficulty estimates* was calculated at 0.99, and of *student ability estimates* at 0.93.

Summary of Item Difficulty Estimates and Fit Statistics					
Item Difficulty Estimates		Item Fit Statistics			
Mean	0.00	**Infit Mean Square**		**Outfit Mean Square**	
SD	1.75	Mean	1.00	Mean	
SD (adjusted)	1.74	SD			
Reliability of estimate	0 .99	**Infit *t***		**Outfit *t***	
		Mean	-0.05	Mean	-0.05
		SD			
0 items with zero scores		0 items with perfect scores			
Summary of Case Ability Estimates and Fit Statistics					
Case Ability Estimates		*Case Fit Statistics*			
Mean	-0.64	**Infit Mean Square**		**Outfit Mean Square**	
SD	1.89	Mean	0.99	Mean	0.99
SD (adjusted)	1.83	SD	0.22	SD	0.77
Reliability of estimate	0.93	**Infit *t***		**Outfit *t***	
		Mean	0.02	Mean	0.17
		SD	0.94	SD	0.73
0 cases with zero scores		0 cases with perfect scores			

Figure 1: Summary statistics for item difficulty and case ability estimates (QUEST, Adams & Khoo, 1994).

The scale of item difficulty and student ability ranged from –5 logits to +5 logits with the mean set at 0. A student with an ability estimate that is the same as the difficulty level of a particular item has a 50% chance of correctly answering that item. Students with an ability estimate greater than the difficulty level of an item have a better than 50% chance of answering that item, in proportion to the linear scale difference.

The software also produces a map of student ability (case estimates) and item difficulty (item estimates). The map, in Figure 2, is a modified version of that produced by the QUEST software. It illustrates a developmental hierarchy of student understanding (ability estimates, designated by an "x" to the left of the vertical line) and concept difficulty (item difficulty estimates, represented by item numbers to the right of the vertical line) within the construct being tested. The construct in this instance is that of algebra.

Distinct clusters of item difficulty and student ability are apparent. There are three main clusters of items (numbers corresponding to items in the test to the right of the centre line in Figure 2). Cluster 1, consisting of 7 items, has a mean difficulty estimate of -2.7 logits; Cluster 2, containing 21 items, has a mean difficulty estimate of -0.32 logits, and Cluster 3, containing 12 items, has a mean difficulty estimate of 2.09 logits. The differences in the means of difficulty estimates for each cluster are significant at the $p<0.05$ level. There are also three distinct clusters of student ability (shaded "x" clusters to the left of the centre line in Figure 2). These clusters are labelled Ability Groups. The mean for Ability Group 1 is -2.34 logits; for Ability Group 2, -0.15 logits; and, for Ability Group 3, 2 logits. These means are significantly different at the $p < 0.05$ level, and closely align with those of the item difficulty means for each of the clusters of items (no significant difference). These data are summarised in Figure 3.

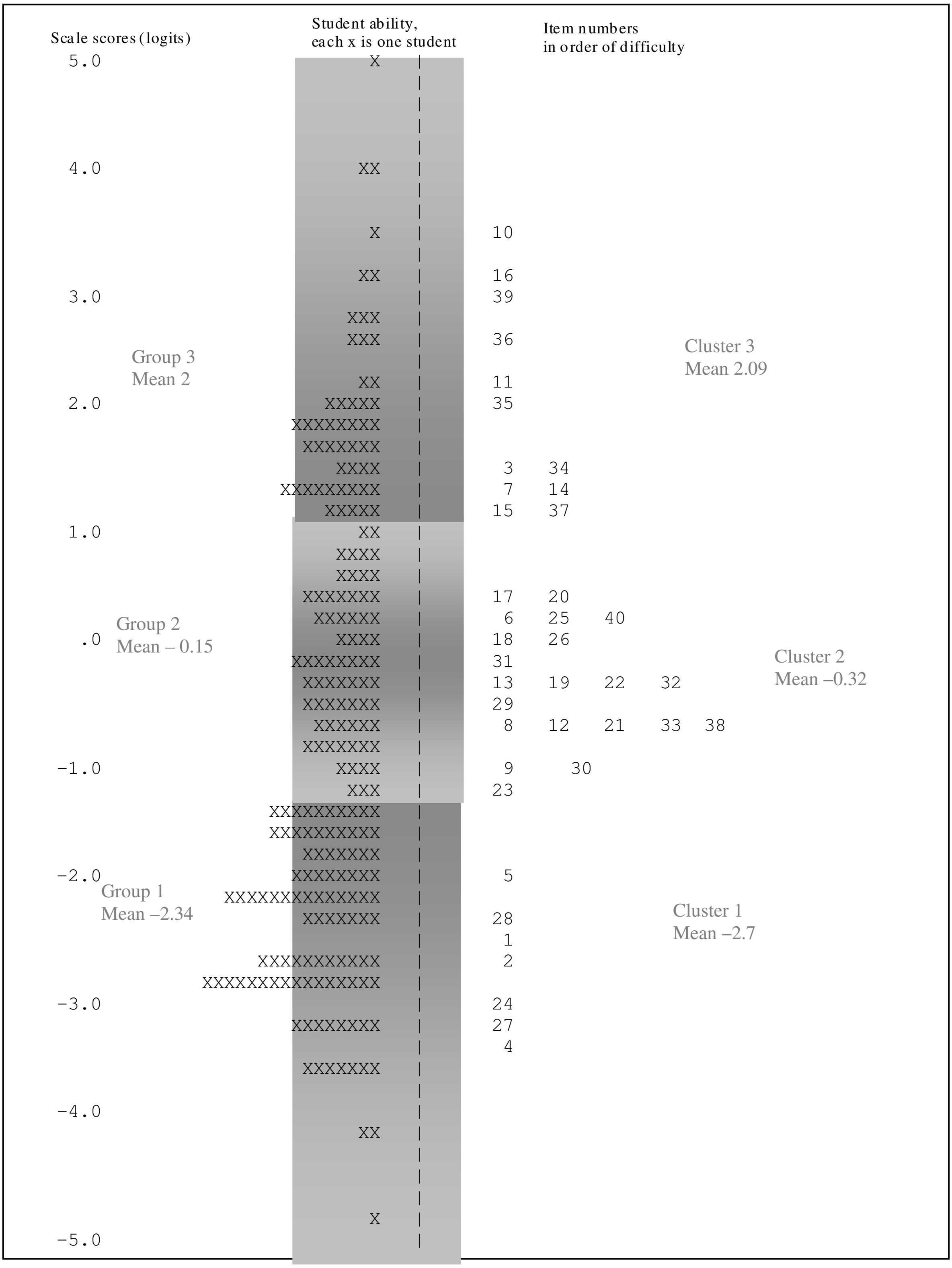

Figure 2: Map of Rasch modelling of algebra test items, showing clusters of items and clusters of student ability estimates (modified from QUEST print out).

Ability Group	**1**	**2**	**3**	**t-test: group ability means**	
Ability Range (Logits)	-1.28 to – 4.89	0.93 to -1.1	4.93 to 1.1	**Group 1-2**	21.75
Number in Group	102	69	52	**Group 2-3**	24.28
Mean ability	-2.34	-0.1472	1.995		
Item Cluster	1	2	3 & 4	**t-test: item difficulty means**	
Difficulty Range (Logits)	-3.27 to –1.98	-1.17 to 0.46	1.18 to 3.56	Cluster 1 - 2	20.49
Mean Difficulty	-2.7	-0.302	2.09	Cluster 2 – 3&4	18.76
t-test: group ability means/item difficulty means	1.8	1.72	0.664		

Figure 3: Summary of student ability group means, item difficulty cluster means and t-test significance at $p<0.05$.

Responses from the Test Scripts

The test responses were also analysed for the types of incorrect responses and the frequency of occurrence of those errors. Blank responses (baulks) were counted separately from other, written, incorrect responses. These data are described only for those responses pertinent to the discussion in this paper. Errors resulting from misreading or misapplication of signs were not considered. Nor were errors resulting from an inability to distribute the multiplier correctly over the brackets and then collect like terms considered. Responses by students are described firstly with respect to the interview sets, and then with respect to the student ability groups.

Responses with respect to the interview sets. The items from the forty-item test that are here discussed were included in interview Sets 1, 3 and 8. These sets are listed in Figure 4, where the particular items are identified, together with their Rasch difficulty estimates.

Set 1: Simplify			**Set 3: Simplify**		
Item No	*Item*	*Difficulty*	*Item No*	*Item*	*Difficulty*
1	3m + 8 + 2m - 5	- 2.53	7	(a – b) + b	1.38
2	5p – p +1	- 2.6	11	8p – 2(p + 5)	2.28
5	2ab + 3b + ab	-1.98	18	2(x + 4) + 3(x – 1)	0.13
6	5a – 2b + 3a + 3b	0.33	19	2(x + 5) - 8	- 0.27
Set 8: Read aloud and tell me how the following could be rewritten?					
Item No	*Item*		*Difficulty*		
20	Multiply x + 5 by 4		0.46		
21	Add 4 on to n +5		-0.58		
22	Add 3 on to 4n		-0.34		
25	Take n away from 3n + 1		0.2		
26	If p + q = 5, then p + q + r =?		0.07		

Figure 4: Items where students conjoined terms arranged in the sets used in interviews, with Rasch difficulty estimates.

For items in Set 1, the number of baulks was very low – from one only for Item 2 [$5p - p + 1$], to nine for Item 6 [$5a – 2b + 3a + 3b$]. For Set 3 the number of baulks was greater, on average, 36 per item. In both sets 1 and 3, the number of Year 8 students who gave no response, was almost the same as the number of Year 9 students who also baulked. For Set 8 baulk numbers varied from 46 on Item 26 [If $p + q = 5$, then $p + q + r$ =?] to more than 20 for Items 21 [Add 4 on to $x + 5$], 22 [Add 3 on to $4n$], and 25 [Take n away from $3n + 1$]. Baulk numbers were higher for items requiring some multiplicative reasoning that also involved the use of brackets, such as Item 20 [Multiply $x + 5$ by 4], or for those requiring logical, but arithmetic, deduction, such as Item 26. In this set, more Year 8 students gave no response than Year 9 students. (e.g., there were 40 baulks for Item 20, 30 of which were Year 8 students, 10 Year 9.)

Some of the most common errors in Set 1 were those in which students conjoined terms inappropriately. For Item 1, 17 responses (out of the 50 errors) were given as $8m$; in the case of Item 2, 38 of the 49 errors involved responses such a 6, $5p$ or $6p$. Item 5 elicited a greater variety of errors than other items in the set; there were 65 incorrect responses, and 33 different responses. The most common error however, involved the conjoining of terms, although there were many different representations. The conjoining of terms was not a common erroneous response to Item 6, and only students in Ability Group 1 gave such responses.

In Set 3, the most common errors were not those that involved the conjoining of terms in Items 7 and 11. However, the conjoining of terms as responses to Items 18 and 19 was common. Item 18 elicited a considerable variety of errors (60 different versions out of 103 incorrect responses), many of which involved conjoined terms either within the brackets, or as a final answer. Item 19 elicited 86 errors, with 17 of those being the response $15x$. Other individual answers also involved the conjoining of terms.

In Set 8, the conjoining of terms was a common error, particularly for students in Ability Groups 1 and 2.

Responses to test items with respect to ability groups. The patterns arising from the error analysis are reflected in the patterns of student responses when considered by the ability groupings of the Rasch model (see Figure 3 and Figure 2). These data are summarised in Figure 5. All errors that are considered the result of terms being inappropriately conjoined are included in the raw numbers. The Rasch difficulty estimates, in logits, are those calculated using QUEST Software (Adams & Khoo, 1994).

Item	Rasch difficulty	Group 1		Group 2		Group 3	
		Numbers	%	Numbers	%	Numbers	%
1	-2.53	*24*	24	*1*	1	*0*	0
2	-2.6	*29*	28	*2*	3	*0*	0
5	-1.98	*24*	24	*4*	6	*3*	6
6	0.33	*19*	19	*0*	0	*0*	0
7	1.38	*14*	14	*5*	7	*0*	0
11	2.28	*31*	30	*14*	20	*2*	4
18	0.13	*34*	33	*4*	6	*0*	0
19	-0.27	*34*	33	*5*	7	*0*	0
20	0.46	*42*	41	*20*	29	*3*	6
21	-0.58	*47*	46	*15*	22	*0*	0
22	-0.34	*69*	68	*20*	29	*1*	2
25	0.2	*57*	56	*20*	29	*4*	8
26	0.07	*23*	23	*5*	7	*1*	2
Students in each group		n = 102		n = 69		n = 52	

Figure 5: Numbers of students who incorrectly conjoined terms in response to items, by ability group and item number [The items are arranged in groups as presented in interviews (see Figure 4). The “groups” are ability groups (see Figure 3).]

Given items in Set 1, students in the Ability Group 1 (mean ability: -2.34 logits) tended to conjoin terms in this set of some of the least difficult items (mean difficulty estimate: -1.7 logits), which required terms to be added and subtracted. No student in the Ability Group 3 (mean ability estimate: 2 logits) did so; with the exception of Item 5 [$2ab+3b+ab$] Ability Group 2 did so (mean ability estimate: -0.15 logits). Item 5 also elicited the greatest number of errors and the greatest variety of incorrect responses that indicated misconceptions and confusions other than that of the appropriateness of conjoining terms.

When required to expand brackets, as in Set 2 (mean difficulty estimate: 0.88 logits), students in Ability Group 2 also tended to conjoin terms, particularly with Item 11 [$8p–2(p+5)$], but not to the extent evident for those in Group 1. Few students in Ability Group 3 did so. The greatest number of conjoining errors occurred with Item 11, although these were of such a varied nature that no particular response could be counted as occurring with great frequency.

The third set of items discussed here were those in Set 8 (mean difficulty estimate: -0.38 logits) of the interview. These items were adapted, or used unchanged, from those in the study by Küchemann (1981). It was in response to these items that the greatest number of conjoining errors occurred in each of the three groups. The absolute numbers remained small in the case of students in Ability Group 3, but greatly increased in the other two ability groups.

Analysis of Interviews: Items in Sets 1, 3, and 8 (Figure 4)

Examination of the transcripts of students in each of the ability groups revealed differences in the verbal responses to the main interview question when the students were directed to the groups of items in Sets 1, 3, and 8 by the instruction to describe their thinking as they dealt with the items in the sets. These responses are described set by set.

Set 1 (Items 1, 2, 5, and 6). Students in each group typically replied: “It’s like terms”, “You put the same/like terms together”; “You add like terms”, etc. Students in Ability Group 1 (mean ability estimate: –2.34 logits) used informal strategies or language such as “Circle the like terms”, “ I use the ones with letters first”, “You put the letters/numbers together”. Only rarely did a student in this group use terms such as “add or “subtract” to describe what they did with the terms. None verbally offered the finished answer to any item. Students in Ability Group 2 (mean ability estimate: -0.14 logits) and those towards the lower end of Ability Group 3 (mean ability estimate: 2) tended to use a mix of both formal language and informal language. Students in Ability Group 2 tended to describe just the sequence of steps involved, although some gave the completed response. Students at the top end of Ability Group 3 (ability estimate >2 logits) tended to use language of a high modality only, describing the steps in the simplification using mathematical terms for the operations, and completing the item.

Set 3 (Items 7, 11, 18 and 19). When presented with expressions containing brackets to be expanded, students, regardless of ability level, responded, “You do them first”. Of the 32 students interviewed, three only directly stated that brackets indicated some form of grouping. All three students had ability estimates greater than 0.75 logits. Most students also described the process of expanding brackets as “getting rid of the brackets”, an informally phrased instruction which implied that the brackets were “unnecessary”, or “you times the outside by the inside”. Most students described the steps in multiplying out the brackets, but did not verbally describe the end result. Only one student (ability 3.8 logits) described what would be done in general, and gave examples, with justifications of the procedural steps. Two out of the seven students interviewed from Ability Group 1 explicitly conjoined terms as they explained their thinking, as did one student in Ability Group 2. Another student in this group seemed unsure of the difference between $5x$ and $x + 5$.

Set 8 (Items 20, 21, 21, 25 and 26). When students were asked to express orally how expressions such as those in Set 8 could be rewritten, many simply repeated the expression, reading it from left to right. This did lead to a "correct" version, although little or no mathematical change occurred, particularly with items such as "Add 4 to $n + 5$", where many students responded with " Four plus *n* plus five". Only students in Ability Groups 2 and 3 completed the items verbally. Some supplied the answer only without describing the steps in their thinking. Students in Ability Group 1 tended to read aloud the items only, from left to right, and make no mathematical changes. Those in Group 2 tended to make some changes and also were uncertain as how to express, for example, the answer to Item 26 as "five plus *r*" or "5*r*".

Discussion of Results

In many cases, when explaining how they dealt with examples such as those in Set 1 and Set 3, the students spoke about "putting together like terms". However, students in Ability Group 1 tended to "put terms together" by conjoining all the terms. Having identified and isolated "like terms" in Items 1, 2, 5, and 6 circling them, or by rearranging the expression, or simply acting sequentially on each, students in Ability Group 1 "put them together" in a different way to those students in Groups 2 and 3. Students in these two Ability Groups did not tend to conjoin terms in these items. Students in Ability Group 2 tended to do so when dealing with items in Set 3 [those with brackets, Items 18, 19, and 7 and Item 11] and particularly those in Set 8 [Items 21, 22, 25, 26, and Item 20].

Item 11 also prompted some students in Group 2 and Group 3 to conjoin terms. This may be because they failed to take account of the fact that the item indicated a difference between $8p$ and $2(p + 5)$ rather than a multiplicative relationship between the terms, and so multiplied throughout – a case of a stimulus causing an automatic response: when there are brackets in an expression the procedure is to "multiply what is inside by what is outside". This procedural thinking also caused students to have problems with Item 7 [$(a – b) + b$]. Some students simply multiplied $(a – b)$ by b, because the b was outside the brackets. This procedure resulted in the errors such as $ab – b^2$, or ab^2.

The conjoining of terms by students in Ability Group 2 became much more frequent when they were required to answer Items 20, 21, 22, 25, and 26, the "semi-literal" items. These items required students to translate from words to mathematical symbols on their test scripts, showing an awareness of appropriate mathematical syntax and possible ambiguity in the written statement. Students in Group 3 did not tend to make this type of error. In the case of students in Groups 1 and 2, there was a marked increase in the numbers of conjoined-term errors as they responded to these items, compared with that for items in Sets 1 and 3 (Figure 5).

One possible explanation for this is that items in Sets 1 and 3 were typical textbook examples and students could respond to them by carrying out a well-rehearsed procedure, where they had been trained not to "put together" all the terms. Faced with an unfamiliar context, students with little understanding of the mathematical relationships conveyed by arithmetic operators in an algebraic context provided a closed response. The tendency to conjoin terms may help to explain why the group of "semi-literal" items had a higher average degree of difficulty than the group of addition and subtraction items, but which was lower than that for the items with brackets (Set 3) and why the successful response rate for students in Ability Group 2 dropped.

These data suggest that students in the middle and lower ability groups, according to the model of the test responses, have a limited procedural understanding of the algebra presented to them. They have learnt a particular procedure that can be applied to particular examples that have a surface similarity. Tall (1994) suggested that the role of the visual structure of an expression is important in learning algebra, but cautions that the image cannot provide the entire concept. Where students, "search their memories for something previously learnt", as one student explained in the interview, they are often seeking an image that matches the appearance of the expression in front of them. The image need not encapsulate mathematical meaning, but acts as a visual cue to prompt a series of mathematical manipulative steps whereby the student changes the form of an expression. No meaning need be attached to the steps, or to the expression itself. Responses to Item 11, Item 7 and Item 19, other than those where terms were conjoined, indicate many students see expressions such as these with brackets and react in one way regardless of the structure and the meaning of the expression. This is also evident when students described their procedures in visual terms such as "circling" the like terms, or when they explained their thinking by simply pointing to parts of the expression when being interviewed.

Questions such as those in Set 8 (Kuchemann, 1981) probe the conceptual understanding of the various forms of algebraic expressions without the visual clues provided by more usual examples encountered by students. Such questions are rare in texts and often only appear in the introductory (Year 7, NSW) phases of algebra teaching.

Conclusions: Implications for Teaching

Rasch modelling of algebra test responses resulted in three clusters of student ability estimates. One of the characteristics of the students in these groups is the diminishing tendency for students to conjoin terms as their ability to deal with conceptually more difficult items develops. In other words, in order for students to be able to deal successfully with more complex algebra they need to learn when it is appropriate to conjoin terms (as in algebraic multiplication) and when not. If the tendency to conjoin terms results from students' understanding arithmetic as much of the literature suggests, then teachers need to become aware of this persistent difficulty and use appropriate teaching strategies, such as those suggested by MacGregor and Stacey (1996) and Tirosh et al. (1998). In particular, students need to encounter arithmetic expressions in different equivalent and unclosed ("unfinished") forms (Linchevski & Herscovics, 1994).

The data discussed in this paper suggest that students of lower "ability" tend to conjoin terms more often than other students. However, a great number of reasonably successful students have a limited procedural understanding of algebraic techniques. Provided that they have only to deal with standard or familiar examples, they can do so. When challenged by examples requiring an understanding of ways in which mathematical meaning and mathematical structure are connected, they expose their reliance on visual cues (or oversimplified schemata) that prompt the exercise of a particular procedure. In order to provide students with a more comprehensive schema, students need to encounter a variety of forms of expression and to experience being able to write them in several ways without the meaning being altered. Perhaps the use of the instruction "to simplify" is too limiting. Asking students to rewrite expressions in many ways and discussing the mathematical usefulness of their responses may help students to attend to the structure and meaning of expressions and so develop their conceptual understanding.

The data from interviews also suggest that the use of informal language in the classroom may serve to obscure the mathematical ideas. Statements such as “Get rid of brackets”, “Do the brackets first” or “Put the like terms together” may not always be correctly interpreted by students, and may contribute to their tendency to conjoin terms because these statements do not convey an exact mathematical message.

The students in Ability Group 3 did not tend to display any marked tendency to conjoin terms in any of the sets of items presented to them. This implies that they have a conceptual understanding of these types of algebraic expression. However, their descriptions of their thinking, although high in modality when they described procedures, lacked depth of explanation or justification. Thus, it could be inferred that their understanding remains largely tacit and, hence, can be articulated only with difficulty. It might also have been that the situation of having to explain their thinking was unfamiliar to the students. This would suggest that class discussion of the various ways in which expressions can be written is a necessary part of developing deeper mathematical understanding. Just as students need to develop a rich vocabulary in their everyday language, so too they also need to experience, and use, a variety of mathematical language and symbols in order to explore and express mathematical meanings. Without this, their algebraic development must be inhibited.

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