Exploring the Number Knowledge of Children to Inform the Development of a Professional Learning Plan for Teachers in the Ballarat Diocese as a Means of Building Community Capacity

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This paper explores the number learning in 2006 of over 7000 children in the Ballarat Diocese for the purpose of identifying any issues that may inform the development of a Diocesan professional learning plan. The data for each grade level were examined to find if there were any apparent learning, teaching, or curriculum issues. The study found that there was a spread of knowledge within each grade level, and that there were groups of students who may be vulnerable. In particular, it was found that notable numbers of students beginning Grade 6 were not yet able to read, write, order, and interpret four-digit numbers nor use reasoning-based strategies for calculations in addition and subtraction, and multiplication and division. These findings need to inform the professional learning plan.

In 2001, the Ballarat Diocese Catholic Education Office implemented a 5-year Diocesan Literacy and Numeracy Plan with the aim of building the capacity of communities (Howard, Perry, & Butcher, 2006) to improve learning for all students. Indeed, school systems throughout Australia and New Zealand have had a similar focus during the past decade. This emphasis on improving learning has been driven in Australia by the 1997 national literacy and numeracy goal that asserts that "every child leaving primary school should be numerate and able to read, write and spell at an appropriate level" (Department of Education Science and Training, 2001, p. 1). However, it is the sub-goal that "every child commencing school from 1998 will achieve a minimum acceptable literacy and numeracy standard within four years" (Department of Education Science and Training, 2001, p. 1) that focused the attention of school systems in Australia on literacy and numeracy learning in the early years of schooling. This prompted several large research projects (e.g., Gould, 2000; Clarke et al., 2002) that identified strategies for improving mathematics learning and teaching (Bobis et al., 2005).

A common feature of these research projects and also of the *Numeracy Development Project* in New Zealand (Higgins, Parsons, & Hyland, 2003) was the use of clinical interviews so that teachers could identify the current knowledge of each student and plan and customise learning opportunities accordingly. Data obtained and aggregated for a class or school were used to identify particular issues associated with enabling effective teaching, learning, and curriculum development (Clarke et al., 2002) and formed the basis of professional learning for teachers. A similar approach was adopted in the Ballarat Diocese.

This paper examines aggregated data describing number knowledge of over 7000 children attending school in the Ballarat Diocese of Western Victoria for the purpose of identifying issues associated with effective teaching, learning, and curriculum development. It is anticipated that the findings will have implications for the identification

of curriculum and professional learning needs. Of particular interest ultimately is how we may improve the capacity of communities to provide more effective learning opportunities for all.

Using Frameworks and Interviews to Identify Children's Number Knowledge

Clinical interviews are now widely used by teachers in Australia and New Zealand as a means of assessing children's mathematical knowledge. This is due to the experience of three large scale projects that informed assessment and curriculum policy formation in Victoria, NSW, and New Zealand: *Count Me In Too* (Gould, 2000) in NSW, the Victorian *Early Numeracy Research Project* (Clarke et al., 2002) and the *Numeracy Development Project* (Higgins, Parsons, & Hyland, 2003) in New Zealand.

A common feature of each of these projects was the use of a one-to-one assessment interview and an associated research-based framework to describe progressions in mathematics learning (Bobis et al., 2005). Teachers participating in each project indicated that the benefits of the assessment interview, though time-consuming and expensive, were considerable in terms of creating an understanding of what children know and can do, and for subsequently informing planning. Indeed, an important feature of clinical interviews is that they enable the teacher to observe children as they solve problems to determine the strategies they used and any misconceptions (Gervasoni & Sullivan, 2007). They also enable teachers to probe children's mathematical understanding through thoughtful questioning (Wright, Martland, & Stafford, 2000). The insights gained through this type of assessment inform teachers about the particular instructional needs of each student more powerfully than scores from traditional pencil and paper tests, the disadvantages of which are well established (Clements & Ellerton, 1995). Bobis et al. (2005) concluded that one-to-one assessment interviews and associated frameworks assisted to move the focus of professional development in mathematics from the notion of children carefully reproducing taught procedures to an emphasis on children's thinking. This is an important outcome at a time when it is broadly accepted that the traditional focus on taught procedures for calculating can negatively impact on children's number sense (Clarke, Clarke, & Horne, 2006) and may impede children's development of powerful mental reasoning strategies for calculating (Narode, Board, & Davenport, 1993). It is important to consider, therefore, when examining the data presented in this paper, whether students in the Ballarat Diocese use reasoning-based strategies for calculating or not. The evidence may highlight issues to consider when formulating the new Diocesan Professional Learning Plan and identify whether teachers may benefit from opportunities to explore methods that lead to children's development of number sense and reasoning-based strategies for calculating.

The Early Numeracy Interview and Framework of Growth Points

The *Early Years Interview* (Department of Education Employment and Training, 2001), developed as part of the *Early Numeracy Research Project* (ENRP) (Clarke et al., 2002), is one example of a clinical interview and a research-based framework of growth points that describe key stages in the learning of various aspects of mathematics. This interview and the associated growth points were used in the Ballarat Diocese to gather data explored in this paper, so an understanding of them is important. The principles underlying the construction of the growth points were that they would:

1. describe the development of mathematical knowledge and understanding in the first three years of school, through highlighting important ideas in early mathematics understanding in a form and language that was useful for teachers;
2. reflect the findings of relevant international and local research in mathematics (e.g., Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & von Glasersfeld, 1988; Fuson, 1992; Boulton-Lewis, 1996; Mulligan & Mitchelmore, 1996; Mulligan, 1998; Wright, Martland, & Stafford, 2000; Gould, 2000);
3. reflect, where possible, the structure of mathematics;
4. allow the mathematical knowledge of individuals and groups to be described; and
5. enable a consideration of students who may benefit from additional assistance.

The growth points formed a framework for describing children's development in Counting, Place value, Addition and Subtraction, Multiplication and Division, Length, Mass and Time, Properties of Shape, and Visualisation and Orientation. The processes for validating the growth points, the interview items and the comparative achievement of students in project and reference schools are described in full in Clarke et al. (2002).

To illustrate the nature of the growth points, the following are the points for Addition and Subtraction. These emphasise the strategies children use to solve problems.

1. Counts all to find the total of two collections.
2. Counts on from one number to find the total of two collections.
3. Given subtraction situations, chooses appropriately from strategies including count back, count down to & count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding 9, build to next ten, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

Each growth point represents substantial expansion in knowledge, or key "stepping stones" along paths to mathematical understanding (Clarke, 2001). It is not claimed that every student passes all growth points along the way, nor should the growth points be regarded as discrete. However, the order of the growth points provides a guide to the possible trajectory (Cobb & McClain, 1999) of children's learning. In a similar way to that described by Owens and Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children" (p. 4).

In summary, the framework of growth points can help teachers to understand a possible trajectory for describing children's learning, identify where any child is currently positioned, identify any children who may be vulnerable in a given domain, identify the zone of proximal development for each child in each domain so as to customise planning and instruction, and identify the diversity of mathematical knowledge in a class. Professional learning programs for teachers who use such frameworks may need to build teachers' capacities to use this information to more effectively teach each child.

The interview takes between 30-40 minutes per student and is conducted by the regular classroom teacher. The full text involves around 60 tasks, although no child is presented with all of these. Given success with a task, the interviewer continues with the next tasks in the given mathematical domain (e.g., Place Value) for as long as the child is successful.

The *Early Numeracy Interview* provided teachers participating in the ENRP with insights about children's mathematical knowledge that they reported might otherwise not have been forthcoming (Clarke, 2001). Further, the project found that teachers were able to use this information to plan instruction that would provide students with the best possible opportunities to extend their mathematical understanding. This is important to consider when developing a professional learning plan for the Ballarat Diocese.

Focus on Place Value Knowledge and Reasoning-Based Strategies

A factor in providing effective mathematics learning opportunities for children is the teacher being able to anticipate the difficulties that some children may encounter in order to assist them. Many studies have provided insight about such difficulties. Important to consider in regard to the data presented in this paper are issues associated with children's understanding of Place Value ideas and use of reasoning strategies for calculating.

One important finding is that children who have not constructed grouping and place value concepts often have difficulty working with multi-digit numbers (Baroody, 2004). This is an important idea to explore when examining the data presented in this paper. Also, being able to interpret numerals to order them from smallest to largest is another Place Value challenge for some children. Griffin, Case, and Siegler (1994) observed that this involves integrating the ability to (1) generate number tags for collections, and (2) make numerical judgments of quantity based on the construction of a mental number line (Griffin & Case, 1997; Griffin et al., 1994). This becomes more complex as children encounter two-digit numbers.

Other studies have found that successful problem solving with two-digit numbers depends on children's ability to construct a concept of ten that is both a collection of ones and a single unit of ten that can be counted, decomposed, traded, and exchanged for units of different value (e.g., Cobb & Wheatley, 1988; Fuson et al., 1997; Ross, 1989; Steffe et al., 1988; Young-Loveridge, 2000). Cobb and Wheatley (1988) found that some children develop a concept of ten that is a single unit that cannot be decomposed, and proposed that this type of concept is constructed when children learn by rote to recognise the number of tens and ones in a numeral, but do not recognise that the face value of a numeral represents the cardinal value of a group.

The counting and reasoning strategies children use to solve addition and subtraction problems have also been the focus of many studies (e.g., Clarke et al., 2002; Fuson, 1992; Griffin et al., 1994; Steffe et al., 1988). Counting strategies identified include count-all (including perceptual counting and counting by representing), count-on (from largest and smallest addend), count-back-all, count-down-to, and count-down-from. Reasoning strategies include doubles, near doubles, adding ten, adding nine, commutativity, combinations for ten, part-whole strategies, and retrieving answers from memory (e.g., Clarke, 2001; Fuson, 1992; Griffin et al., 1994; Steffe et al., 1988). Once children have developed a range of strategies, it becomes important to choose wisely among these strategies to fit the characteristics of a strategy to the demands of a task (Griffin et al., 1994). However, not all children choose wisely or have each strategy available.

In order to think multiplicatively, children need to shift from viewing groups as being composed of single items, to viewing the group itself as a countable unit (Clarke et al., 2002; Mulligan, 1998). This is difficult for some. Sullivan, Clarke, Cheeseman, and Mulligan (2001) found that constructing knowledge for abstracting multiplication and division problem solutions provides a significant barrier for many children, and Clarke et

al. (2006) found that 16% of children at the end of Grade 4 did not use reasoning strategies in multiplication. These difficulties provide a lens for examining the data presented later.

Improving Mathematics Learning in the Ballarat Diocese

In 2001, the CEO Ballarat implemented the Ballarat Diocese Numeracy Strategy (2001-2005) to improve mathematics learning for primary school students within the Diocese. The strategy was informed by the findings of the ENRP (Clarke et al., 2002) and in a similar way to the ENRP, adopted the Hill and Crévola *Key Design Elements* (Hill & Crévola, 1999) as a means of building the capacity of school communities to provide more effective learning opportunities for all students. These were beliefs and understandings, leadership and coordination, standards and targets, monitoring and assessment, classroom teaching programs, professional learning teams, school and class organisation, intervention and special assistance, and home, school, and community partnerships.

From 2002, schools began to use the *Early Numeracy Interview* to assess all students' number knowledge. All schools were using this interview for all children by 2006. Teachers were encouraged to analyse the data to determine any school-based issues and to identify and assist those students who were at risk of poor learning outcomes. To facilitate this, teachers were invited to train as specialist intervention teachers, so that they could introduce the *Extending Mathematical Understanding* (EMU) intervention program (Gervasoni, 2004) in Grade 1, and provide specialist advice for teachers and parents..

From 2004 onwards, all schools developed a numeracy action plan that addressed each of the nine Key Design Elements. Schools were also funded to enable the appointment of a Numeracy Co-ordinator to guide the implementation and evaluation of the school plan. From 2002, the Diocese provided a professional learning program for all teachers (P-6) and Numeracy Co-ordinators. This included a mix of regionally-based whole-day programs, school cluster workshops, and school-based professional learning team meetings.

The Diocese is now evaluating the effectiveness of the Strategy and considering key issues to focus on to inform a new professional learning program for teachers to build community capacity further to provide effective mathematics learning for all.

Analysing Children's Number Knowledge in the Ballarat Diocese

The data presented in this paper were collected in 2006 from over 7000 children from all 52 Catholic Primary Schools within the Ballarat Diocese. This enabled a rich picture of these children's number knowledge to be formed. The practice in this region is for teachers to assess each student in the first week of school using the *Early Years Interview* for the purpose of gaining insight about each child's current mathematical knowledge. The interview was developed during the ENRP (Clarke et al., 2002). Its development and the associated framework of growth points are reported in detail elsewhere (e.g., Bobis et al., 2005; and Clarke, 2001). However, it is important to note that the growth points describe major learning along a hypothesised learning trajectory (e.g., Cobb & McClain, 1999) and formed the basis for the development of interview assessment items.

Children's responses to assessment items were analysed by the teacher to determine the growth points children reached. To increase the validity and reliability of the data, each teacher followed a detailed interview script, recorded children's answers and strategies on a detailed record sheet, and used clearly defined rules for assigning growth points. Children's growth points were entered into an excel spreadsheet and each school's data were

aggregated to form the data set reported on here. The region's Numeracy Advisors and each school's Numeracy Co-ordinator managed this process.

Issues Arising from Examining Children's Number Knowledge

The purpose of the examination of data collected in 2006 within the Ballarat Diocese is to identify any important issues related to learning, teaching and curriculum that need to be addressed to improve learning opportunities for children and that might inform the Diocesan Mathematics Professional Learning Plan. This paper will focus on issues related to the Place Value, Addition and Subtraction, and Multiplication and Division domains.

The percentage of children in each grade reaching each Place Value growth point (GP) is shown in Figure 1. Of particular interest is children's knowledge of multi-digit numbers.

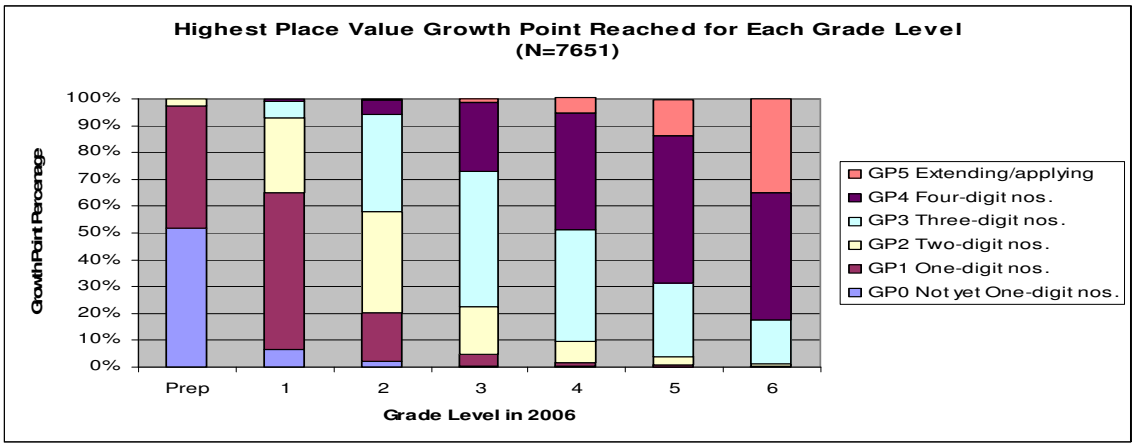

Figure 1. Percentage of children in each grade reaching each growth point at the beginning of 2006

An issue highlighted in Figure 1 is the spread of growth points at each level. This finding has been noted elsewhere (e.g., Gervasoni & Sullivan, in press; Bobis et al., 2005) but highlights the complexity of the teaching process and the importance of teachers identifying each child's current knowledge and knowing ways to customise learning opportunities that meet each child's needs. This has important curriculum and instruction implications for any plan to strategically improve learning outcomes for students.

Another interesting point is that almost half the children beginning Prep, the first year of school in Victoria, can already read, write, order, and interpret one-digit numbers. These children already need opportunities to explore two and three digit numbers, an issue that needs to be addressed in curriculum development and planning. The remaining students require the more traditional Prep experiences that firstly emphasise exploring and constructing knowledge about one-digit numbers. However, right from the beginning of schooling, the data highlight differences in children's knowledge to which the community needs to respond to optimise learning. It is also important to acknowledge that some teachers may not have been able to identify the extent of some children's knowledge because this is sometimes culturally specific, and may not be obvious to the teacher (Gervasoni, 2003). This issue may be another focus for professional development.

Figure 1 also shows that nearly half the Grade 2s and three-quarters of the Grade 3s were already able to interpret three-digit numbers and needed opportunities to explore and construct understandings about four-digit numbers and greater. School communities need to consider how this can be best achieved.

Another feature of the data is the number of students in Grades 4 to 6 within the Diocese who have not yet reached GP 4 and GP 5 (52%, 32%, and 18% respectively). Further examination of these students assessment responses shows that many were able to read and write four-digit numbers, but were not able to either order four-digit numbers and/or answer the questions, "What is 10 more than 2791?" and "What is 100 less than 3027?" As highlighted by Baroody (2004), these tasks require children to appreciate the *quantity* associated with number names and numerals and either to use their mental number line (Griffin & Case, 1997) to find 10 more or 100 less, or to use a reasoning-based strategy that draws upon their number sense. Difficulty with this type of task typifies the children who experience difficulty in Place Value. Certainly, a curriculum emphasis on understanding these numbers as quantities and numbers with positions on the number line is important. A Diocesan professional learning plan may need to address this issue.

A further implication of this finding is that some children in Grades 4 to 6 may be required to solve problems requiring calculations with four-digit numbers and greater (a prominent feature of the curriculum at this level), without an understanding of these numbers as quantities and their position on the number line. It seems fair to assume that many of these children may be reliant on learning procedures for performing calculations without constructing the conceptual underpinnings, and perhaps before they have developed reasoning based strategies for calculating. To explore this conjecture, we first examined the highest growth point reached by students in the Addition and Subtraction Strategies domain (see Figure 2).

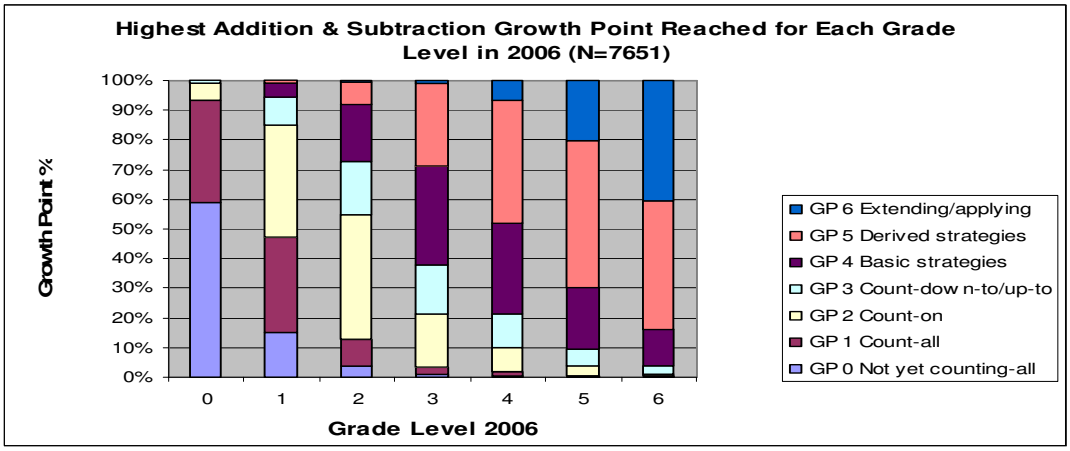

Figure 2. Percentage of children in each grade reaching each growth point at the beginning of 2006 in addition and subtraction (N=7651).

The data show that 51% of children beginning Grades 4 and 30% of children beginning Grade 5 were not yet using derived strategies (GP 5). This is consistent with the findings of a longitudinal study of 323 children who participated in the ENRP (Clarke et al., 2006). Their study found that when children reached Grade 4 and 5, respectively 53% and 37% had not reached GP 5. However, note that in the longitudinal study, data refer to assessment at the end of Grades 3 and 4, so comparisons are indicative only. Figure 2 also highlights that 16% of Grade 6s were not yet using derived strategies. This suggests that these children may rely on rote procedures for performing calculations.

To explore this issue further, we determined the number of Grade 6 students who had not yet reached GP 4 in Place Value, nor used reasoning-based strategies in Addition and Subtraction (GP 5) and Multiplication and Division (GP 4).

Figure 3 shows the number of children who had not reached these growth points and the combinations of domains for which this was the case (N=1195, n=371). It is important to note that 69% of children beginning Grade 6 had meet these minimum targets. Conversely, 31% were vulnerable in at least one of these domains, and these children are the focus of Figure 3. In summary, Figure 3 shows that of the 31% of Grade 6s who were vulnerable in at least one of these domains, 18% were vulnerable in all three domains, and nearly half (45%) were vulnerable in at least 2 domains.

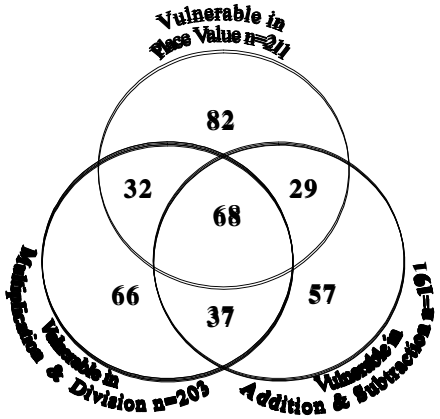

Figure 3. The number and combinations of domains for which Grade 6 children had not yet reached targets in Place value, addition and subtraction, and multiplication and division, (N=1195, *n*=371).

In relation to the question about whether children who had not yet reached GP 4 in Place Value used reasoning-based strategies in Addition and Subtraction and Multiplication and Division contexts, Figure 3 shows that of the 211 Grade 6 children who had not yet reached GP 4 in Place Value, 61% had also not yet reached the growth points associated with using derived strategies in Addition and Subtraction and reasoning strategies in Multiplication contexts. A focus for increasing the capacity of communities to provide effective learning opportunities for these students will include professional learning opportunities that enable Grades 4 to 6 teachers to identify and develop instructional approaches to identify and assist these students. This may also include intervention-style programs aimed at accelerating children's number learning in these aspects.

Conclusion

Examination of the current number knowledge of over 7000 children in the Ballarat Diocese highlights some important issues to consider for developing a professional learning plan to improve mathematics learning outcomes for students. Key issues are the need for communities to provide more effective learning opportunities to assist children interpret four-digit numbers, and reasoning based strategies in Addition and Subtraction and Multiplication and Division. However, it is acknowledged that in formulating a professional learning plan for teachers throughout the Ballarat Diocese, it will be important to explore the views of those living and working in the various communities, and to identify the characteristics of communities that already make a difference.

Discussions with School Numeracy Co-ordinators within the Diocese suggest that although considerable change has occurred in the curriculum and teaching approaches of those involved in the early years of schooling (P-2), and for many teachers working in the

later years, some Grades 3 to 6 teachers continue to adopt a more traditional approach to number learning that is based on the rote learning of calculation procedures and number facts. Another point raised was the need for ongoing monitoring and assessment of children's knowledge. Numeracy Co-ordinators suggested that whereas all teachers use the one-to-one assessment interview and framework of growth points at the beginning of the year to inform their curriculum planning, some teachers do not continue to use the framework to monitor children's knowledge and differentiate curriculum and instruction throughout the year. This is another possible focus for the Diocesan professional learning plan.

Overall, it seems that building the capacity of communities to provide more effective learning environments for Grades 3 to 6 children will be an important factor in addressing the learning, teaching, and curriculum issues highlighted by the examination of children's number knowledge, and will be an essential focus for a new professional learning plan.

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