What Does it Mean for an Instructional Task to be *Effective*?

Lynn Hodge
The University of Tennessee
<lhodge4@utk.edu>

Jana Visnovska
Vanderbilt University
<jana.visnovska@vanderbilt.edu>

Qing Zhao
Vanderbilt University
<qing.zhao@vanderbilt.edu>

Paul Cobb
Vanderbilt University
<paul.cobb@vanderbilt.edu>

In this paper we discuss the considerations and challenges in designing instructional tasks that support both students' mathematical engagement and their developing mathematical competence. We draw on Dewey's work and take the perspective that cultivating students' content-related interests should be an instructional goal in their own right rather than solely serving the instrumental purpose of supporting students' conceptual understanding. We reflect on our learning from two classroom design experiments to offer illustrations of issues related to supporting students' interests. We offer these illustrations, not as exemplary cases, but instead, as points of reflection and discussion. In this paper, we focus specifically on instructional tasks by presenting a retrospective analysis on the role of tasks in supporting students' interests and access to important content ideas.

Introduction

Reform recommendations have called attention to the use of real world contexts in mathematics problems (National Council of Teachers of Mathematics, 1989, 2000) and culturally relevant pedagogy has emphasised drawing on students' local and broad communities as a source for engaging problem topics (Ladson-Billings, 1995). In this paper, we focus on instructional tasks and their role in supporting both students' mathematical interests and their developing mathematical competence. In doing so, we develop what it means for an instructional task to be effective from our perspective as mathematics educators. Our discussion centres on two ideas: (a) how a task holds potential for supporting students' development of mathematical interests and (b) how a task holds potential for providing students with access to important mathematical ideas. We believe that instructional tasks are deemed effective according to how well they respond to both of these points. We use the term *task* to refer to problems that are designed and presented to students in mathematics class. We use the term instructional *activity* to refer to how these tasks become realised in the course of discussions and interactions in the mathematics class. In focusing on the design of instructional tasks we emphasise intent and potential. Additionally, we must examine how instructional activities become constituted in a classroom in order to test and refine what we understand about designing effective tasks. Therefore, our focus is on the considerations and challenges in the design of effective instructional tasks while at the same time exploring tensions that might emerge as these tasks become realised in the classroom.

In order to discuss effective tasks in this way, it is important for us to delineate more specifically how we might evaluate tasks in how they provide access to interests and content ideas. To this end, we initially lay the foundation for the analysis to come. We do so by clarifying an orientation toward students' development of content-related interests

Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia
J. Watson & K. Beswick (Eds), © MERGA Inc. 2007

that draws heavily on the ideas of John Dewey (1913/1975). This orientation has implications for how we think about the specific role of tasks in supporting students' mathematical interests. Secondly, we provide background to two design experiments from which the retrospective analysis draws data. We then share insights from the analysis in order to

- clarify a two-part process of cultivating students' mathematical interests
- examine the potential of task situations in supporting students' development of mathematical interests
- explore the role of tasks in supporting the emergence of particular mathematical topics in whole-class discussions.

These three parts of the analysis relate to each other in that the first describes a way of cultivating students' mathematical interests whereas the second and third parts clarify the role of tasks in supporting this process.

An Orientation on Cultivating Students' Interests

As we have indicated, our purpose in this paper is to examine characteristics of instructional tasks that can contribute to supporting both students' interests and their access to important mathematical ideas. For this reason, we draw on the work of Dewey since his perspective encourages us to think about the resources teachers can draw on to support students' interests within the context of the classroom. In this paper, we focus primarily on tasks, but as will become apparent, classroom discourse and the role of teacher serve as resources in this process as well.

Dewey's ideas have been helpful in that he describes interests as something that individuals can cultivate rather than characteristics that are inherent aspects of people. From his perspective, students' current interests act as leverages from which students' content-related interests could be developed. In this process, current interests could afford opportunities from which content interests, such as mathematics, could be developed. Dewey used the term *cultivation* to indicate that he regarded it a teacher's responsibility to support the development of students' disciplinary interests. He argued that disciplinary interests are an inherent aspect of disciplinary literacy, and as such their development should be an instructional goal in their own right.

Importantly, Dewey's view on interests also highlights the *nature of students' interests*. His focus was on students' interests in particular content ideas that could be cultivated over time in a class, and subsequently a series of courses. His view is in contrast to the more typical emphasis on engaging students to participate in particular activities in the classroom without necessarily noting what students are becoming interested in as they engage in such activities. This orientation on cultivating mathematical interests reflects a developmental perspective that emphasises the deeply cultural nature of students' interests. In this way, Dewey anticipated Vygotsky's argument that interests cannot be adequately accounted for by either biological desires or skill acquisition but are culturally developed (compared with Hedegaard, 1998; Vygotsky, 1987).

From this orientation, cultivating students' mathematical interests becomes a challenge for both instructional design and teaching. As instructional tasks are the most visible means of organizing students' mathematical activity, we examine their potential as a resource in cultivating students' mathematical interests. In doing so, we attempt to discern characteristics of tasks that support students' long-term interests in learning mathematics.

The kinds of tasks we identified as effective are quite different from activities and problems that connect with what can be identified as students' current interests but are weak in providing access to significant mathematical ideas.

The Design Experiments

The classroom design experiments on which we draw focused on supporting students' increasingly sophisticated forms of statistical reasoning. A member of the research team served as the teacher in both experiments, which were conducted in an urban middle school in the United States. Twenty-nine seventh-grade students participated in the first experiment that was conducted over a 12-week period and involved 34 classroom sessions of approximately 40 minutes in length. This experiment was conducted in the students' regular mathematics classroom and focused on the analysis of univariate data. The following school year, a smaller contingent of students from the same class (now eighth graders) participated in a 14-week experiment involving 41 classroom sessions of 40 minutes that focused on the analysis of bivariate data.

Analyses that we have reported elsewhere indicate that the teacher was generally successful in supporting students' development of increasingly sophisticated forms of statistical reasoning (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003). The relatively impressive nature of the students' learning encompasses both the sophistication of the data-based arguments that they developed and the depth of their understanding of issues related to the process of generating data such as the representativeness of samples and the control of extraneous variables (P. Cobb & Tzou, 2000). Additional analyses (P. Cobb, Gresalfi, & Hodge, 2007; P. Cobb, Hodge, Visnovska, & Zhao, 2007) reveal that students during the course of the design experiments came to view analyzing data as an activity that was worthy of their engagement. The findings of these prior analyses indicate that the design experiments provide a rich context from which to examine the role of instructional tasks in supporting students' mathematical engagement and their developing competence.

Instructional Tasks in the Design Experiment Class

A basic design principle that guided the development of instructional tasks during both experiments was that they should support students' analyses in involving the investigative spirit of exploratory data analysis from the outset (cf. G. W. Cobb & Moore, 1997). As a consequence, we attempted to develop instructional tasks in which the students analyzed data sets that they viewed as realistic for purposes that they considered legitimate. Most of the instructional tasks involved comparing two data sets in order to make a decision or judgment (e.g., determining whether installing airbags in cars does have an impact on automobile safety). To support the students' engagement further in what might be termed genuine data analysis, they were required from midway through the first experiment to write a report of their analyses for a specific audience that would act on the basis of their reports (e.g., a police chief who wanted to know whether a speed trap had been effective in reducing traffic speed).

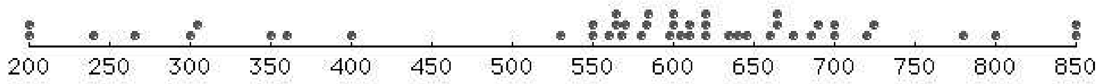
In most of the instructional tasks, the students did not collect data themselves. Instead, the teacher introduced each task by engaging the students in an introductory discussion that was often times lengthy. In the course of these discussions, the class talked through the process by which data might be generated. Specifically, the teacher and students together delineated the particular phenomenon under investigation, clarified its significance, identified relevant aspects of the phenomenon that should be measured, and considered how they might be measured. The teacher then introduced the data as having been generated by this process and the students conducted their analyses individually or in small groups. The final phase of an instructional activity consisted of a whole-class discussions of the students' analyses. The resulting organization of an instructional activity often spanned two or more class sessions.

Data Sources and Method of Analysis

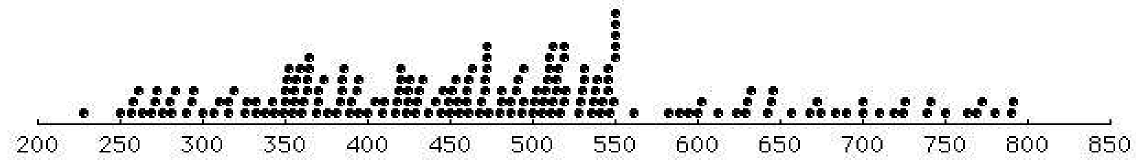
Our analysis of instructional activities draws from data that include video-recordings made with two cameras of classroom sessions, copies of all student work, and two independent sets of field notes of all the classroom sessions. Our central question had to do with discerning which instructional tasks were constituted as worthy of students' engagement and those that were not. Three members of the research team used video-recordings of one productive and one unproductive introductory discussion from the second design experiment as test cases initially in which to develop, test, and refine these criteria. They focused on these introductory discussions because it was during these discussions that the teacher and students negotiated the intent of the activities by talking through the significance of the problem at hand and the relevance of analyzing the situation from a mathematical point of view. This procedure was repeated by reexamining two further productive introductory discussions. As a result, the following criteria were established to determine whether an instructional task was constituted as worthy of students' engagement: (a) at least half of the students contributed to the data generation discussion, (b) the number of turns taken by students in the discussion was equal to or greater than the number of turns taken by the teacher, and (c) the majority of student contributions concerned ways to address the question under investigation by generating and analyzing data (e.g., relevant aspects of the phenomenon that should be measured, how these aspects might be measured, and how data might be generated). These criteria are generally consistent with Engle and Conant's (2002) contention that evidence of engagement can best be seen by considering questions such as: "How are students participating? What proportion of students is participating? And how are students' contributions responsive to those of other students?" (p. 402). Three members of the research team subsequently used these developed criteria to analyze the video-recordings of introductory discussions of all 14 tasks presented in the first experiment independently in order to determine which of these tasks were constituted as worthy of students' engagement. All researchers agreed that eight of the tasks were constituted as worthy of students' engagement whereas six were not. A comparative analysis was conducted to gain insight into the characteristics of the instructional tasks that contributed to the differences documented in students' engagement. We discuss findings from this analysis at a later point in this paper.

A Two-Part Process: Cultivating Pragmatic Interests and Mathematical Interests

Our learning in the design experiments sheds some light on processes that are involved in supporting students' mathematical interests (P. Cobb et al., 2007). One aspect of our learning concerns a two-part process of supporting students' development of disciplinary interests. This two-part process involved first cultivating students' *pragmatic* interests or interests in the problem situation presented in the instructional task. These pragmatic interests we describe relate to an interest in pursuing the specific problem at hand. To illustrate what we mean, one of the instructional activities in which students engaged in the latter part of the seventh-grade design experiment involved analyzing data on the T-cell counts of AIDS patients who had enrolled in a standard treatment program and an experimental treatment program. The datasets presented to students are shown in Figure `1.



Experimental Treatment



Traditional Treatment

Figure 1. AIDS Data.

A pragmatic interest that we encouraged students to develop related to investigating which treatment was more effective rather than solely an interest in the broad topic of AIDS. It seemed from our observations that the issue of AIDS was relevant to few if any of the students' personal daily lives. In other words, they did not know anyone, including family and friends, who had been diagnosed as having AIDS. However, they appeared to have developed a genuine interest in the issue as they engaged in an introductory whole-class discussion that clarified the instructional task and took place prior to the students conducting their own analyses. The teacher typically initiated these introductory discussions by posing a general problem or issue. In the ensuing conversation, the teacher and students clarified why this problem or issue would be significant to them or to a particular audience.

During the AIDS introductory discussion, the teacher and students talked about the general topic of AIDS, the importance of finding an effective treatment, and how data might be collected to help the class decide which of the two AIDS treatments had better results. The initial focus on the students' knowledge of AIDS led to a conversation about both the relevance of finding an effective treatment for AIDS and measures that could indicate to what extent an applied treatment is effective. We conjecture that many students became interested in the instructional activity as they came to see the relevance of developing effective treatments for AIDS within the context of wider society. In this way, students' pragmatic interests were cultivated as they engaged in a discussion that clarified the overall relevance of the task investigation and how data might be used to address this issue. This first phase of cultivating students' pragmatic interests in issues of social relevance was crucial in students coming to see a reason for analyzing the data sets with which they were presented. As we later discuss, our analysis of effectiveness of instructional tasks indicated that the tasks, which did not afford leverage for cultivation of students' pragmatic interests in the problem at hand, were not instructionally effective. As will become apparent, although critical, cultivation of students' pragmatic interests was only one part of cultivating students' interests in mathematics.

As part of their attempts to cultivate students' mathematical (or, specifically statistical) interests, the research team supported students' participation in the emergence of practices consistent with those in which data analysts might genuinely engage. The students' participation in these practices involved identifying relevant patterns in the data, presenting data-based arguments, writing a report to a decision maker summarising their analyses, and judging the adequacy of arguments presented by others. During the whole-class discussion that focused on the students' analyses of the AIDS data, it became apparent that all the students in the class had concluded that the new treatment was more effective than the traditional, standard treatment. However, a lengthy, whole-class discussion ensued that focused on different ways of structuring and organizing the data. It appeared in this discussion, at least on the surface, that students were becoming interested in developing data-based arguments and judging the adequacy of these arguments in the context of this class session in spite of their consensus on which treatment was more effective. We refer to these developing interests, related to practices of *doing* mathematics, as mathematical interests. The following excerpt illustrates the nature of the whole-class data analysis discussion students were afforded. This excerpt focuses on one group's analysis (Figure 2), in which the students proposed an inscription to show the global differences in the way the two sets of data were distributed.

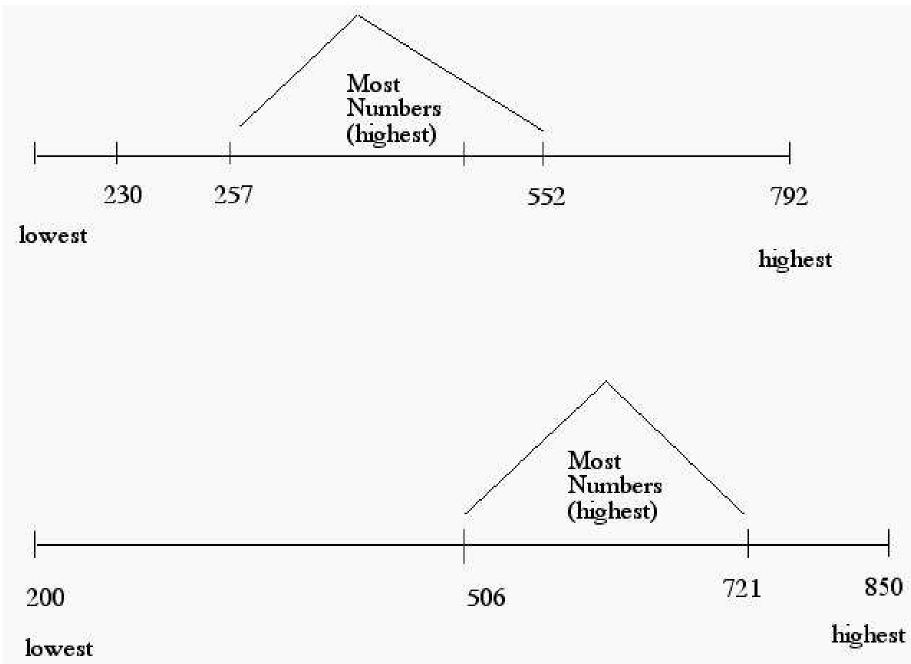


Figure 2. One student group report.

Janet:	I think it's an adequate way of showing the information because you can see where the ranges were and where the majority of the numbers were.
Dan:	What do you mean by majority of the numbers?
Teacher:	Dan doesn't know what you mean by the majority of the numbers.
Janet:	Where the most of the numbers were.
Teacher:	Sue, can you help?
Sue:	What she's talking about, I think what she's saying, like when you say where the majority of the numbers were, where the point is, like you see where it goes up.
Teacher:	I do see where it goes up (indicates the "hill" on Figure 2)
Sue:	Yeah, right in there, that's where the majority of it is.
Teacher:	Okay, Dan.
Dan:	The highest range of the numbers?
Sue:	Yes.
Teacher:	The highest range?
Students:	No.
Teacher:	Valerie.
Valerie:	Out of however many people were tested, that's where most of those people fitted in, in between that range.
Teacher:	You mean this range here (points to lower and upper bounds of one of the "hills")?
Valerie:	Yes.

In this excerpt, students clarified Janet's use of the term "majority" in relation to the datasets. In doing so, majority as related to the notion of relative proportions became an explicit topic of conversation in the classroom. This opportunity to clarify statistical ideas was prompted by both the task situation and the design of the specific data sets to make comparisons of unequal data sets necessary. Furthermore, this excerpt is illustrative of the discussions that constituted the second part of a two part process that sought to cultivate students' interests in learning mathematical ideas. As we reiterate later, the tasks that would not allow for a meaningful mathematical discussions to develop based on students' mathematical contributions make it difficult for teachers to cultivate students' mathematical interests effectively.

Task Situations and Their Potential for Cultivating Students' Pragmatic Interests

Students' development of pragmatic interests was critical in providing a reason to engage in discussions about specific mathematical ideas. We conjectured that "effective" task situations drew from topics that were located within students' zones of proximal development. These situations and topics were located within a space of topics that students were likely to find engaging when supported through discourse and interactions within the classroom. During the design experiments, we found issues that were of a personal or societal relevance to be the most effective in engaging students. This finding is understandable given adolescents' growing interest in their place in society and their sense of power in affecting change on society and their immediate community (Hedegaard, 1998).

During the design experiments, we made a number of modifications to the instructional tasks in light of the instructional agenda, students' mathematical learning, as well as what we learned about ways to cultivate students' interests. In a retrospective analysis on instructional tasks, we found four distinguishing characteristics of effective instructional activities. As an illustration, we draw on the AIDS task that was deemed as a success in engaging students in both pragmatic and mathematical issues. We discuss the four characteristics of task situations that were engaging to students:

- Students have developed some familiarity with or awareness of the phenomenon either in school or out-of-school (e.g., the topic of AIDS, batteries, etc.)
- Students have developed a prior awareness of the specific question to be investigated and initial familiarity with the processes involved (e.g., finding an improved treatment for AIDS patients, AIDS involves your immune system, the physical effects of AIDS on the body).
- Students came to view the specific question to be addressed as significant during the course of a discussion that introduced the instructional activity (e.g., finding a more effective treatment for AIDS would be important to patients and to medical staff).
- Students came to view addressing the question from a mathematical perspective as reasonable during the course of a discussion that introduced the instructional task (e.g., the analyses of AIDS patients' T-cell counts to assess the effectiveness of the two treatments).

It is important to note that we documented examples of ineffective instructional activities in which different ones of the four listed key characteristics were violated. In this sense, we propose that each of the characteristics was *necessary* for cultivating students' interests in the statistics design experiment classroom.

Many would argue that statistics lends itself to real world task situations whereas this is not the case with all mathematical topics or ideas. At this point, we would not make the claim that all effective instructional tasks require a real world scenario; however, we would make a two-fold argument that (a) an introductory discussion that clarifies the intent of the task and its significance (to society or to the students' mathematical learning) is critical in providing all students opportunities to understand the task and to become engaged in it and (b) a real world situation may be useful in engaging students, but the task situation must also be scrutinised in terms of the mathematical ideas that it affords.

Interests, Learning, and the Space of Possible Mathematical Topics

In retrospect, we found it helpful to consider task situations and questions posed in these tasks specifically in light of the space of possible mathematical issues that might emerge in whole-class discussions. This would involve considerations of how students might interpret and reason about the task and what conversations might come about from clarifications and comparisons of these ways of reasoning. It is not surprising that instructional tasks that do not adequately support teachers' efforts in building on students' reasoning towards instructional goals are also generally not effective in supporting students' mathematical learning. Similarly, in order to cultivate students' mathematical interests, it is critical to provide students with access to mathematical ideas that would enable them to solve problems that they come to see as pragmatically important.

In the case of the AIDS activity, the research team purposefully constructed data sets with a significantly different number of data points when we developed the activity so that the contrast between absolute and relative frequency might become explicit. This in turn required a task scenario in which the inequality in the size of the data sets would seem reasonable to the students and which they would view as significant and engaging. The data sets for this activity were therefore designed so that 46 people enrolled in the experimental treatment and 186 people enrolled in the traditional treatment. Additionally, the total number of data points in the larger data set was not a multiple of the total number of points in the smaller data set. These design decisions were made in order to support students' examination of the data in proportional terms. During the whole-class discussion of the students' analyses, a number of significant mathematical issues emerged during the conversation. These include the meaning of the term majority, the distinction between absolute and relative frequency, the usefulness of percents in specifying relative frequencies and the interpretation of graphs in which data sets were partitioned into four groups that contained the same number of data points.

When mathematics becomes a tool for students to solve significant problems they can be supported to see mathematics as relevant and interesting in its own right. We concur with Clarke (2005) that mathematics as it becomes realised in the classroom can be *relevant* in different ways when situated within multiple contexts. In his description of Chinese classrooms, mathematics can be seen to be situated within the broader cultural context in which it is respected and valued as both a pragmatic and intellectual tool (Svan & Clarke, 2007). Additionally, Clarke describes classrooms in South Africa in which mathematical learning is in the service of informing a broader agenda, that of addressing social issues such as substance abuse or AIDS (Sethole, Adler, & Vithal, 2002). In our reflection, we have emphasised the importance of the task situation, the mathematical ideas, and the relationships between the two. When constructing effective mathematical tasks, the multiple ways in which mathematics can become relevant to students should be considered. For our part, we have focused on what can be done *in the classroom* to support students' development of mathematical interests in situations when the students do not necessarily see mathematics as relevant to their lives from the outset.

Discussion

In closing, we refer to two points that we have emphasised in this paper. First, we have argued that when designing instructional tasks, it is important to consider how the task holds potential for cultivating both students' pragmatic and mathematical interests. We have described both of these aspects as closely related and as phases of a process that serves to cultivate students' mathematical interests. We acknowledge that considering both of these aspects at the same time when designing a task is challenging. Similarly, as an instructional activity becomes constituted, addressing both of these aspects in teaching is challenging as well. Tensions can and often do arise between addressing pragmatic interests and content-related interests (Azevedo, 2002). This emphasises the need for analyses that investigate how instructional tasks can serve as resources for teachers as they navigate such tensions and how classroom practices mediate this process.

Second, critics of the use of real world contexts argue that not all students have experiences that support their understanding of such contexts. Some would say that some students are advantaged over others (Lubienski, 2002). Introductory discussions and the ideas of *pragmatic interests as accomplishments* emphasise topics that are located within a zone of proximal development and substantive discussions that support students' access to understanding the task context and its significance. In this way, the *meaningfulness* of a task is seen to be supported and developed through discussions, interactions, and other resources within the social context of the mathematics classroom.

References

Azevedo, F. (2002). *Personal excursions: Investigating the dynamics of student engagement*. Unpublished manuscript, University of California Berkeley.

Clarke, D. J. (2005). *Essential complementarities: Arguing for an integrative approach to research in mathematics classrooms*. Paper presented at the 28th annual conference of the Mathematics Education Research Group of Australasia. Retrieved January 2007, from http://www.merga.net.au/publications/counter.php?pub=pub_conf&id=33.

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly*, *104*, 801-823.

Cobb, P. (1999). Individual and collective mathematical learning: The case of statistical data analysis. *Mathematical Thinking and Learning*, *1*, 5-44.

Cobb, P., Gresalfi, M. S., & Hodge, L. (2007). *An interpretive scheme for analyzing the identities that students develop in mathematics classrooms.* Manuscript submitted forpublicaion.

Cobb, P., Hodge, L., Visnovska, J., & Zhao, Q. (2007, April). *Cultivating students' mathematical dispositions*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Cobb, P., McClain, K., & Gravemeijer, K. (2003). Learning about statistical covariation. *Cognition and Instruction*, *21*, 1-78.

Cobb, P., & Tzou, C. (2000, April). *Learning about data creation*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Dewey, J. (1913/1975). *Interest and effort in education*. Carbondale, IL: Southern Illinois University Press.

Engle, R. A., & Conant, F. C. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners' classroom. *Cognition and Instruction*, *20*(4), 399-483.

Hedegaard, M. (1998). Situating learning and cognition: Theoretical learning and cognition. *Mind, Culture, and Activity*, *5*, 114-126.

Ladson-Billings, G. (1995). Making mathematics meaningful in multicultural contexts. In W. G. Secada, E. Fennema & L. B. Adajion (Eds.), *New directions for equity in mathematics education* (pp. 329-348). New York: Cambridge University Press.

Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning*, *4*, 103-127.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Sethole, G., Adler, J., & Vithal, R. (2002, September). *When AIDS goes to a mathematics class: Is it a hindrance or a source for revising the mathematics classroom culture*. Paper presented at the ICMI Study - Mathematics education in different cultural traditions: A comparative study of East Asia and the West, Hong Kong.

Svan, K., & Clarke, D. J. (2007). *Expanding the relevance paradox*. Manuscript in Preparation

Vygotsky, L. S. (1987). The development of thinking and speech. In R. W. Rieber & A. S. Carton (Eds.), *The collected works of Vygotsky, L.S. (vol.1): Problems of general psychology* (pp. 191-220). New York: Plenum.