Numeracy in Action: Students Connecting Mathematical Knowledge to a Range of Contexts

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This qualitative multiple case study involved eight Year 6 and 7 students and six classes and investigated their capacity to recognise, apply, and question the use of mathematical ideas embedded in a range of contexts. It also considered the extent to which students' capacity to connect mathematical knowledge to other contexts could motivate them to learn mathematics. In particular, it investigated the effect of the *Mathematical Search* strategy in achieving these ends. It found that student thinking about mathematics and their attitudes towards it could be enhanced by targeting mathematical connections through the use of the *Mathematical Search*.

In recent times, much has been written about numeracy. One common aspect in most definitions of numeracy is the disposition and ability to apply widely one's mathematical knowledge. In her discussions of the notion of statistical literacy, a related concept, Watson (1995; 2004) was concerned not only with the quantity of statistical information that continually bombarded the general population, but also that many people ignored it, misunderstood it, or did not bother to check if its associated claims were valid. In a similar vein, Peter-Koop (2004) found that, when working with worded problems, primary school students often failed to identify the key mathematical ideas involved and tended to randomly apply numbers contained in the text of such problems to arbitrarily chosen mathematical operations. Both of these ideas are encapsulated in Perso's (2006) statement that "since numeracy involves both the mathematics you know and the disposition to use it, teaching must focus on both of these" (p. 25).

The inference for teachers is clear – it is not only necessary to teach the mathematical content but also important to provide students with strategies for recognising and applying mathematics in a range of contexts. Therefore, a main research issue addressed by this study is the investigation of the effectiveness of teaching and learning strategies in helping students to connect their mathematical knowledge to various contexts and situations.

The overall study (Hurst, 2006), on which this paper is based, investigated three ideas:

- The effectiveness of strategies like the *Mathematical Search* in enhancing student ability to recognise mathematics in context.
- The extent to which such strategies enhance student motivation towards mathematics.
- The value that teachers see in using such strategies to enhance student thinking and motivation.

This paper focuses on the first of the above issues, as embodied in the following research question: To what extent does the *Mathematical Search* enhance student capacity to recognise mathematical ideas embedded in a written context, and to display contextual and strategic thinking about mathematical ideas embedded in written contexts?

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Theoretical Framework

This study drew upon a wide range of research-based writing in developing the research questions and methodology and there were several key points that emerged. First, at the very heart of the numeracy debate, are the notions of situated cognition and transferability of learning. Boaler (1993) noted that traditional approaches to developing student numeracy were based on the assumption that "mathematics can be learned in school, embedded within any particular learning structures, and then lifted out of school to be applied to any situation in the real world" (p. 12). However, as Kemp and Hogan (2000) pointed out, "evidence suggests that students do not automatically use their mathematical knowledge in other areas" (p. 13). Indeed, if learning were freely transferred from the mathematics classroom to any of a number of outside situations, it is unlikely that the numeracy debate would have begun, or at least, reached the proportions it has.

Second, the idea of teaching "numeracy across the curriculum" emphasises that numeracy is more than mathematical knowledge and that students learn best when "the richness of a context helps them to make sense of mathematical ideas" (Willis, 1998, p. 8). This is closely allied to the previous point as students who tackle mathematics in restricted contexts will be likely to develop limited cognitive structures (Coles & Copeland, 2002). The importance of embedding mathematical learning in a range of contexts was underlined by Morony, Hogan, and Thornton (2004):

> Education must be about enabling people to understand and interact with the world. The skills, habits of mind and dispositions developed through effective attention to numeracy across the curriculum are clearly key components of understanding and interacting with the world. (p. 2)

The above ideas about numeracy are encapsulated in the *Numeracy Framework* developed by Willis and Hogan (Hogan, 2000; Morony et al., 2004; Willis, 1998). The framework incorporates three perspectives on numeracy, a blend of which was required for students to display intelligent mathematical action in context. The three types of knowledge are:

- Mathematical knowledge – the knowledge needed for intelligent mathematical action
- Contextual knowledge – the ability to link mathematics to experiences
- Strategic knowledge – the ability to ask questions about the application of particular mathematical knowledge

A Conceptual Framework – The Model for Teaching Numeracy in Context

The ideas related to numeracy outlined above, particularly the *Numeracy Framework* (Hogan, 2000; Willis, 1998), informed and were incorporated in the *Model for Teaching Numeracy in Context* (Figure 1) that became the conceptual framework for the study. This model was based on the notion that the different modes of thinking in the *Numeracy Framework*, that is mathematical, contextual and strategic thinking, could be developed by using the *Mathematical Search* and associated teaching and learning strategies. The *Mathematical Search* was devised by the researcher and was used on four occasions by the researcher during the course of the study. It was developed with the intent of ascertaining whether or not a specific strategy of that type could enhance the capacity of students to recognise and use mathematical ideas embedded in a variety of contexts. In the study, only written contexts were used. Students had not used the *Mathematical Search* prior to their involvement in the study.

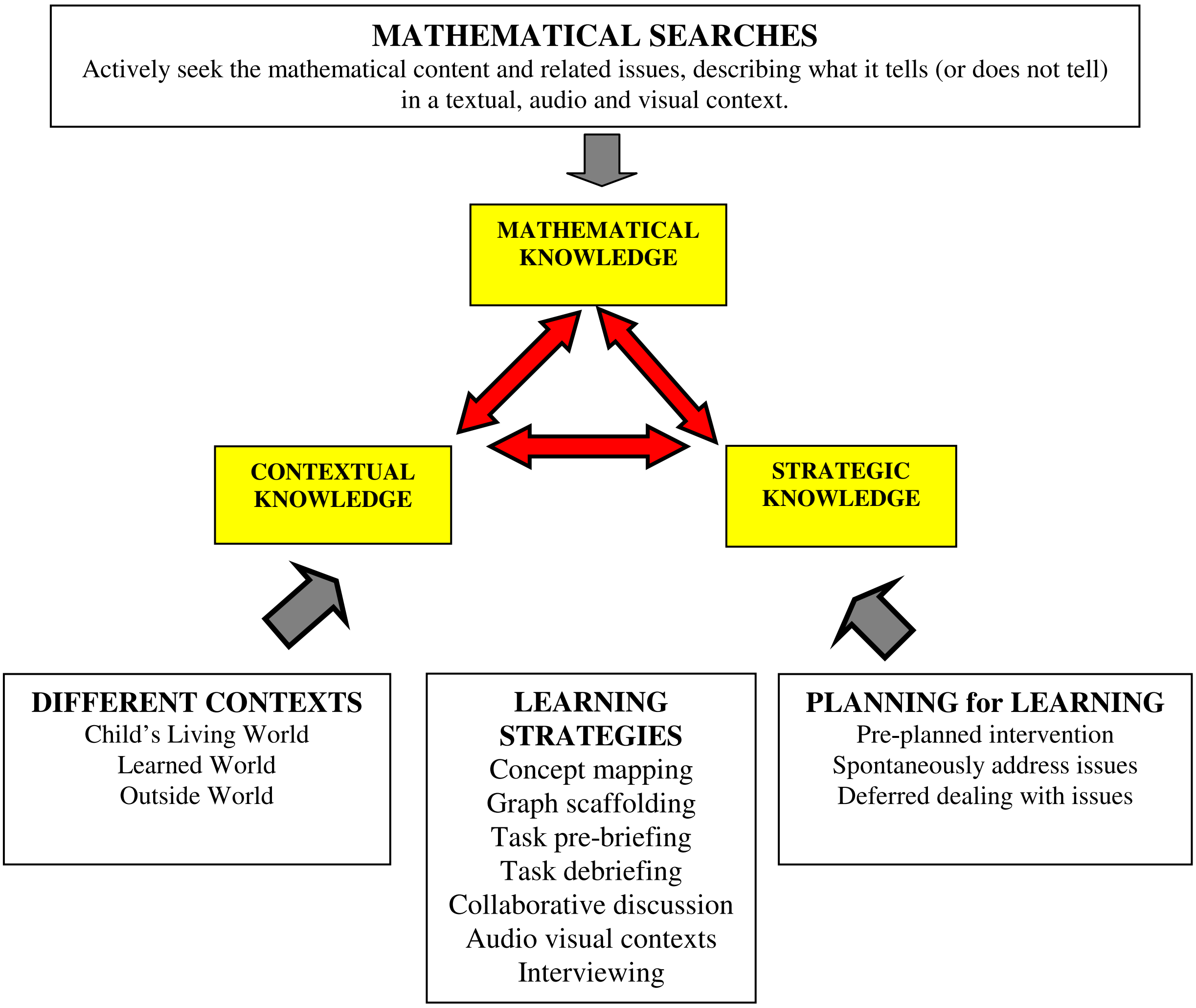

Figure 1. Conceptual framework (Model for Teaching Numeracy in Context).

In a *Mathematical Search,* students were given a body of text to read. These were based on themes and topics that were being taught in classes, such as Indigenous Australians, Gold Rushes, and Environmental Pests. Their task was to describe the mathematical ideas in the text and what the mathematics told about the main ideas in the text, and to use the mathematical ideas to explain some of the patterns, trends and any apparent inconsistencies in the text. The purpose of the *Mathematical Search* was to encourage students actively to seek mathematical concepts and facts embedded in any of a variety of contextual situations. In this study, students were also asked to pose questions about the text using the mathematical ideas described. The *Mathematical Search* was supported by other teaching and learning strategies such as concept mapping, graph scaffolding, debriefing discussions following a *Mathematical Search*, and one-to-one interviewing.

Design and Methodology

In order to generate the rich data required, the study made use of qualitative methods, specifically, a multiple case study approach. This involved a group of eight female Western Australian primary school students, aged 11 or 12 years, in six Years 6 and 7 classes.

Frankel and Wallen (2003) and Yin (2003) noted that evidence from multiple case studies was generally more convincing compared to that from a single case study and could lead to useful and valid generalisations.

Over a period of 6 months, evidence gathered from the multiple case study was supported by evidence from a *General Sample* of students, this consisting of the remaining students in the six classes from which the case study students were drawn. In order to ensure the validity of the data, and increase the possibility of making reasonable generalisations from the results of the study, data triangulation was achieved using multiple sources of evidence, as shown in Table 1.

Table 1
Data Collection Instruments Used During the Study

Instrument	Purpose
Pre-Project Student Survey	Identify students who display a positive attitude towards mathematics, for possible selection in the case study group.
Pre-Project Benchmark Task in Mathematics	Provide a benchmark for later comparison in order to have a basis for assessing changes in student thinking.
Western Australian Literacy and Numeracy Assessment (WALNA) data for each student	Enable selection of students to be confirmed on the basis of a combination of researcher-generated criteria (Student Survey, Pre-Project Interview) and standardised testing.
Pre-Project Teacher Interview	Provide an understanding of the level of experience, commitment to numeracy teaching, teaching style and general philosophy of project teachers. Act as a reference point for later comparisons after implementation of project tasks.
Pre-Project Student Interview	Provide an understanding of current student thinking about mathematics in context, the importance of mathematics and how success in mathematics is judged. Act as a reference point for later comparisons after implementation of project tasks.
Project tasks *Mathematical Searches* (four) and other tasks	Provide students with opportunities to identify, discuss meanings of, and apply mathematical knowledge in a variety of contexts. Generate work samples to serve as indicators of student thinking and progress.
Researcher's Reflective Journal and Anecdotal Notes	Record details of observations made during classroom visits to administer project tasks. These visits occurred at least monthly over a six month period.
Teacher Progress Interviews	Provide anecdotal information about case study students from the perspective of the class teacher.
Post-Project Benchmark Task in Mathematics	Provide a benchmark for comparison with Pre-Project Benchmark Task in order to have a basis for assessing changes in thinking.
Post-Project Teacher Interview	Act as a reference point for comparisons with earlier interview after implementation of project tasks. Ascertain extent of changes to teacher thinking about the value of the project tasks.
Post-Project Student Interview	Act as a reference point for comparisons with earlier interview after implementation of project tasks. Ascertain extent of changes to student thinking about the value of the project tasks, mathematical learning in context, importance of aspects of mathematics, and how mathematical ability is recognised.

The interviews with students were in part "task-based" in that students were given samples of articles, maps, and advertisements, about which they were asked questions to probe the development of their thinking. The benchmark tasks were based on tabular information and students were to identify key ideas that the information showed and also give possible explanations for the variations in that information. Benchmark Task 1 contained a table of information based on school fund raising and Benchmark Task 2 was about a school traffic counting activity. One associated strategy used in the study was task debriefing. This followed each *Mathematical Search* and consisted of whole class discussions in which the possible responses and thinking were modelled by the researcher.

During the initial phase of data analysis, interview transcripts, work samples, and field notes were analysed and some thirteen empirical assertions were developed from the data. An empirical assertion could be described as a contention, statement, declaration or claim that something in particular is likely to occur, based on the contender's observations and experiences (Erickson, 1986). Two of the empirical assertions generated from Research Question 1 that are discussed in this paper are contained in Table 2.

Table 2
Empirical Assertions Generated from Research Question 1

1. Students will display an improved capacity to recognise mathematical ideas in a written context, and to use contextual and strategic thinking when considering mathematical ideas embedded within a written context, having used the *Mathematical Search* strategy on several occasions.
2. Students will display a greater capacity to recognise mathematical ideas embedded in a written context, and to use contextual and strategic thinking when considering mathematical ideas embedded in a particular written context, when they are personally interested in that context.

It was important not to set the boundaries of the research too wide and therefore some potential variables were eliminated from the sample. For example, it was not intended to make wide-ranging comparisons involving gender, different year levels, different types of schools (such as Government, Catholic, or Independent), or other issues such as school policy, socio-economic characteristics of school intake areas, and student ethnicity. Consequently the sample for the study was restricted to Years 6 and 7 female students.

Findings and Discussion

The discussion that follows is based only on Research Question 1 and the two empirical assertions listed in Table 2.

Empirical Assertion 1

> Students will display an improved capacity to recognise mathematical ideas in a written context, and to use contextual and strategic thinking when considering mathematical ideas embedded within a written context, having used the *Mathematical Search* strategy on several occasions.

In attempting to warrant or reject Assertion 1, evidence from the multiple case studies is presented here. To begin with, the responses to Benchmark Tasks 1 and 2 are considered. It is apparent from a comparison of responses by the eight case study students to Benchmark Tasks 1 and 2 that gains were made in terms of the various modes of thinking, that is, mathematical, contextual, and strategic thinking. Mathematical thinking is characterised by the recognition, reiteration, and/or application of specific mathematical information to perform a mathematical operation. For example, a student working with an advertisement showing a price reduction and "new" price for a sale item might use the information to calculate the "normal" price of the item. Contextual thinking may involve the interpretation of data or the posing of questions that require such interpretation. For example, a student working with a similar advertisement to the above might consider a claim made in the advertisement that the product "whitens in fourteen days" and pose the question such as "Does the container last for fourteen days?" Strategic thinking may involve the synthesis of data to produce a new idea or the evaluation of data for consistency and the identification of anomalies. For example, a student working with an advertisement claiming that "Everything is reduced by 15%" might test the claim by comparing original and discount prices to see if the claim was accurate.

The basis on which "gains" are considered to have been made is whether or not a student has displayed modes of thinking that were not displayed earlier in the project. For example, a student displaying mathematical thinking on Benchmark Task 1 is deemed to have made "substantial gains" if, on Benchmark Task 2, he/she displayed contextual thinking, as well as mathematical thinking. A student is considered to have made "reasonable gains" if, for example, emerging contextual thinking on Benchmark Task 1 had developed into established contextual thinking on Benchmark Task 2. Similar criteria described "very substantial gains", "no gains", or "loss".

Five of the eight students made "substantial" or "reasonable" gains and three made "no gain". For each of the eight students, the quality and frequency of responses for Benchmark Task 2 were higher than for Benchmark Task 1. In Benchmark Task 1, students may have displayed emerging contextual thinking without applying mathematical ideas or they may have displayed genuine contextual thinking but only gave one example. For Benchmark Task 2, all students provided multiple responses incorporating mathematical ideas relevant to the context of the task. Responses by the student Tania were typical of those of the other seven students and are shown here in Table 3. It can be seen that Tania gave more responses and more detailed responses to the second task compared to the first. In addition, during the second task, she displayed strategic thinking that was not evident in her responses to the first task.

Table 3
Comparison of Responses by Tania for Benchmark Tasks 1 and 2

Benchmark 1	Benchmark 2
Contextual Thinking – Learner User	Contextual Thinking – Learner User
I think Year Seven raised the most because there are more children in that year.	Brett took 28 minutes and Craig took 12 minutes. Is this because Brett was on a road with a traffic jam or the speed limit was low, or made up some of the answers? Maybe they were at different times of the day or more populated cities.
	Strategic Thinking – Critical User (emerging)
	There must have been at least four emergencies because it shows four emergency vehicles on the chart. But that might not be true because it says at the top that they're all from different schools so they might not be in the same city or did it on a different day. Each time the sedan cars were the most seen. Maybe because they were the cheapest or the most useful?

The development represented in Table 3 was typical of the case study students. Where there was not a “reasonable” or “substantial” gain in modes of thinking, there was at least an increase in the quantity and variety of responses. Similar gains in modes of thinking were noted when case study student responses for the Pre-Project and Post-Project Interviews, and responses to the first and final *Mathematical Search*es, were compared. All case study students displayed both mathematical and contextual thinking during the first interview and all three modes of thinking, mathematical, contextual, and strategic, during the second interview. For six students, this represented a “substantial” gain, for one a “very substantial” gain, and for one, a “reasonable” gain. A summary is contained in Table 4 where “M” represents mathematical thinking, “C” represents contextual thinking, “S” represents strategic thinking, and (em) represents emerging thinking.

Table 4
Comparative Gains for Student Responses to Mathematical Searches 1 and 4, and Interviews 1 and 2, for the Case Study Students

Student	Mathematical Search #1 to #4	Gain	Interview #1 to Interview #2	Gain
Mary	M(em) to M/C	Very Substantial	M/C to M/C/S	Substantial
Sara	M(em) to M	Reasonable	M/C to M/C/S	Substantial
Jenny	M to M/C/S (em)	Very Substantial	M/C to M/C/S	Substantial
Tania	M to M/C	Substantial	M/C to M/C/S	Substantial
Kerryn	M/C to M/C	No gain	M/C(em) to M/C/S	Very Substantial
Louise	M to M/C	Substantial	M/C to M/C/S	Substantial
Lexie	M/C(em) to M/C	Reasonable	M/C to M/C/S(em)	Reasonable
Sonia	M/C to M/C/S (em)	Reasonable	M/C to M/C/S	Substantial

Responses from the Post-Project Interviews support Assertion 1 in that the eight case study students unanimously thought that the *Mathematical Search* helped them to develop their thinking about mathematics. The following responses were made in reply to the interview question “Do you think that doing these tasks [*Mathematical Search*es] helped you to understand mathematics better and if so, how did they help you with your thinking about mathematics?”

> I think they’ve helped my mind expand and look at things in a different way that I haven’t seen them before, to make it easier and different to learn, and I think it’s helped a lot. Instead of just looking at a picture or something once, I look at it closely and see if I can find any maths in it. (Jenny, student, Post-Project Interview, November 18, 2005)

> Well, ever since the first task, it really made me think, just looking around at things. It really, really did make me think about everywhere maths is and I talked about it a lot to my parents and they realised a lot too. I know some things I probably wouldn’t have noticed as well about maths and I realised that there was heaps of maths everywhere. (Kerryn, student, Post-Project Interview, November 20, 2005)

> Yeah, ‘cause it helped me understand maths because I didn’t know there was maths in writing. I thought there was just maths in numbers, but there’s maths in writing as well. (Lexie, student, Post-Project Interview, November 25, 2005)

The level of gain in student thinking as well as sentiments expressed by students during Post-Project Interviews provide sufficient evidence to establish a warrant for Empirical

Assertion 1. That is, capacity to recognise and use embedded mathematical ideas and to display contextual and strategic thinking is enhanced by using the *Mathematical Search* on several occasions.

Empirical Assertion 2

> Students will display a greater capacity to recognise mathematical ideas embedded in a written context, and to use contextual and strategic thinking when considering mathematical ideas embedded in a particular written context, when they are personally interested in that context.

In attempting to warrant or reject Assertion 2, examples of evidence from teacher and student interviews, and the Researcher's Reflective Journal are presented here. The interview responses from teachers support the assertion that context is an important consideration. The following comment from Karen (teacher) was made in response to a question about the level of reading involved in the *Mathematical Search* tasks. Around the time that her class completed the first *Mathematical Search* task, the context of which was about Indigenous Australians, an indigenous student of a similar age, and known to her students, had died. This gives the following comment considerable weight in terms of the importance of context.

> The reading with the first one [*Mathematical Search* task 1] . . . the level was fine, but I'm not sure if they found the content engaging until this child's death, because then it became more interesting to them because it was their real world. (Karen, teacher, Post-Project Interview, November 25, 2005)

Another teacher, Georgie, made the following comment in response to an interview question about the value of the *Finding the Maths* task. This task was the third *Mathematical Search* where students chose the context and samples to analyse. Typical things chosen by students were "junk mail" catalogues, advertising material, and newspaper articles.

> When they actually found the context, they became active learners and they were putting their skills into practice. I thought that was the most valuable task, but they had to have experienced texts presented to them to begin with but then when they did that [pause] in fact if we gave them that task now, having done two more practices at presenting them with texts, I think the results would be even better. (Georgie, teacher, Post-Project Interview, November 25, 2005)

The following excerpt from the Researcher's Reflective Journal, compiled immediately after a Post-Project Interview with Nick (teacher), provides another example of the importance of considering the context in which mathematics may be embedded.

> The idea of context has arisen again. Today's interview with Nick was very enlightening from several viewpoints; one being that Nick considered that the choice of context for written texts was very important when devising text samples to use with the *Mathematical Search* tasks – he felt that student interest was quite dependent on the information contained in the text. (Researcher's Reflective Journal, December 3, 2005)

Responses from students also supported the assertion that context was an important consideration when considering whether or not students might be able to recognise and apply mathematical ideas contained in that context. The following exchange from a Post-Project Interview provides an example of this view.

> Interviewer: Was there any one of the tasks that was more useful for you than others or more enjoyable for you to do?
> Louise: I really enjoyed the Finding the Maths where you could go out and think where you could find it yourself in the real world, so that's like, real world things you can do.

Interviewer: So because it was real world thing, you thought it was particularly good?
Louise: Yeah, that way you think of things outside the class, things like catalogues and things.
Interviewer: So, if it's something that you're interested in do you tend to think more about it maybe?
Louise: Yep.
(Post-Project Interview, November 30, 2005)

On the basis of the above evidence presented, a warrant for Assertion 2 was established. Teachers and students both indicated that familiarity with, or interest in a particular context enhanced student capacity to recognise and use embedded mathematical ideas. It seems as though the concerns about student numeracy that were illuminated in the review of research literature may have been partly addressed by using the *Mathematical Search.* For instance, the inability of people to recognise embedded mathematical ideas, and to understand and apply them (Peter-Koop, 2004; Watson, 1995, 2004), and the lack of disposition by people to use such mathematical ideas (Perso, 2006) inferred that teachers need to use specific strategies designed to address those problems. On the basis of empirical evidence presented in this study, it appears that the *Mathematical Search* may be such a strategy that could be used successfully.

It is also important to note that the Conceptual Framework for the study, the *Model for Teaching Numeracy in Context*, incorporates a number of other teaching and learning strategies.When used in tandem with the *Mathematical Search*, these strategies, such as task debriefing, concept mapping, graph scaffolding, and interviewing can be effective in enhancing the capacity of students to recognise and apply embedded mathematical ideas. Task debriefing was conducted by the researcher following each *Mathematical Search* task and involved modelling of how to recognise and apply the embedded mathematical ideas. As well, the task debriefing sessions incorporated concept mapping in which typical examples of embedded mathematical ideas were developed around the central theme of the particular *Mathematical Search* context.

Conclusions and Implications

This study has shown, through the warranting of Empirical Assertions 1 and 2, that student thinking and capacity to connect mathematical learning to a range of contexts can be enhanced by using particular dedicated strategies. In other words, the *Mathematical Search* strategy can enhance student performance, subject to some qualifications. These qualifications included regular use of the strategy, application of associated strategies such as task debriefing, and choice of context in which mathematical ideas are embedded. Other aspects such as teacher style and philosophy, and student reading ability had an impact on student performance. Hence, this study has begun to address the important research issue of investigating the effectiveness of teaching and learning strategies in helping students connect their mathematical knowledge to various contexts and situations. The following implications can be made for both teaching and research.

Implications for Teaching Practice

The *Mathematical Search*

- has been shown to be an effective link between classroom mathematics and other learning areas and contexts in which mathematics might be embedded;
- is an effective tool in helping students recognise and connect their own mathematical knowledge;

- helps students develop mathematical, contextual and strategic thinking when working with a variety of contexts;
- could be successfully applied to audio visual and pictorial contexts, as well as written texts; and
- is effective when used in tandem with a range of other strategies, shown in Figure 1 as "Learning Strategies".

Implications for Further Research

Further research could replicate the study or focus on the use of the *Mathematical Search* and associated strategies where other variables could be considered such as

- both male and female students,
- different age groups,
- socio-economic status of students,
- students with varying reading ability,
- use of the *Mathematical Search* in audio-visual contexts, and
- use of the *Mathematical Search* over extended periods of time, perhaps beginning at a younger age.

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