"Connection Levers": Developing Teachers' Expertise with Mathematical Inquiry

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One of the challenges in research is in understanding processes and systems that enable teachers to build their expertise and commitment to reform-based pedagogies. A qualitative study documented the influence that a set of support mechanisms, or *connection levers*, had in assisting upper primary teachers over the course of a year in developing confidence in teaching mathematics through inquiry.

The research literature in mathematics education has been fairly clear that students benefit from posing and investigating meaningful, open-ended problems (e.g., Diezmann, Watters, & English, 2001; Boaler, 1997). Inquiry is one means to learning that incorporates these ideals. Although inquiry has been embraced in other content areas (e.g., National Research Council, 2000), it continues to be under-utilised in mathematics. One reason for this is likely difficulties that teachers have changing conventional practice (Stigler & Hiebert, 1999; Cuban, 1990). Calls for reform in teaching have been with us for decades (Dewey, 1938/1997; Tyler, 1949/1969; Schwab, 1978; Ball, 2002), but little is known about the *processes* by which teachers alter their practice.

This paper reports on outcomes from the first year of a research project designed to understand better the processes and experiences of teachers learning to develop expertise in teaching mathematics with inquiry. In the first year of the project statistical inquiry (Wild & Pfannkuch, 1999) was used to segue into mathematical inquiry because of its natural connections to context and interpretive epistemology, and its potential as a tool for understanding problems in multiple disciplines. The goal of this paper is to understand how a number of support mechanisms, called *connection levers*, enabled the teachers in the study to develop their expertise, confidence, and commitment to teaching mathematics through inquiry.

Literature

In inquiry, students often engage in epistemological processes of coming-to-know using ill-structured problems, where the initial definition of the problem is ambiguous or has many open constraints (Reitman, 1965). Several obstacles arise in teaching and learning with inquiry because it requires skills unfamiliar in conventional mathematics classrooms. In solving ill-structured problems, the solution phase (where nearly all teaching is focused in schools) requires a relatively small proportion of the cognitive effort compared to the process of structuring and seeing the problem through to completion. The skills required for conducting inquiry have been shown to pose multiple difficulties for learners (Diezmann et al., 2001). In statistical inquiry, for example, there are challenges in designing a measurable question (Confrey & Makar, 2002), collecting and organizing data, and relating findings back to the original question (Hancock, Kaput, & Goldsmith, 1992).

Previous research by the author suggests that *initial* experiences with inquiry pose unique challenges because learners start with a very narrow perspective of the inquiry

process. Her research found that these first experiences can result in frustration and poor outcomes and that learners need to undergo *multiple iterations* of inquiry with a number of support mechanisms – time, feedback, support, reflection, and validation – before they can begin to understand the nature of the inquiry process (Makar, 2004; Makar & Confrey, 2007). For example, inquiry often raises more questions than it answers and learners typically believe they have failed in their inquiry if their initial question (often overly simplistic and broad) is left unanswered, even if through the inquiry they have gained a much deeper understanding of the question under investigation.

Inquiry is equally challenging for teachers. It requires the ability to embrace uncertainty, foster student decision-making by balancing support and student independence, recognize opportunities for learning in unexpected outcomes, maintain flexible thinking, hold a deep understanding of disciplinary content, and tolerate periods of noise and disorganization (National Research Council, 2000). These often go against learning trajectories traditionally held in mathematics of neat and orderly classrooms with well-defined learning goals. Because mathematics is not envisioned as a field requiring inquiry, it is unusual for teachers to teach mathematics with this approach. If they do, the difficulties encountered in an initial experience likely dissuade them from continuing. Like learners, Makar (2004) speculated that teachers would need similar elements – time, feedback, support, reflection, validation, and multiple experiences – to develop expertise in teaching mathematics with inquiry in a program of effective professional development.

Research on teachers' learning has provided insight into principles of effective professional development. For example, in a large-scale study of relationships between teachers' professional development and their teaching practices, Cohen and Hill (2001) found that the only professional development approaches that appeared to influence teachers' classroom practices significantly involved a sustained focus on reform curriculum they were to teach, and collaborative analysis of student work. Ball (1996) has argued that professional development must provide teachers with opportunities to learn content in an environment that models effective teaching. And Elmore (2002) contends that professional development must be purposefully connected to student learning of core content, sustained for long periods of time, focus on the curriculum and pedagogy of teachers' classrooms, provide feedback, and develop within a collaborative environment.

Method

The study was developed using a design research framework (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), in which the researcher simultaneously studies and tries to improve the study context. The main question was: *How do teachers come to develop expertise, confidence and commitment to teaching mathematics with inquiry in a supported environment?* This paper reports on links between support and the teachers' development.

Four teachers of students in Years 4 and 5 (ages 8-11) at a government school in Queensland volunteered for the study. Teachers participated in four professional learning days during the year, once per term (approximately every 10 weeks). On these days, teachers were engaged as learners on various aspects of statistical inquiry. Time was also set aside for sharing of teaching experiences and planning their inquiry units. Sessions were recorded and portions transcribed for more detailed analysis. Teachers committed to teach an inquiry-based unit in their classrooms each term (see Table 1). They designed the units themselves, sometimes using published materials as a base. Lessons were videotaped to capture the flavour and content of the units, enculturate the researcher into the teachers'

classroom practices, provide ongoing support, and gather episodic evidence of teaching and learning issues that arose while teaching the units. Teachers were interviewed at the beginning and end of units to gather data on goals, challenges that arose, unexpected outcomes and opportunities, what they learned and would change next time, and particular aspects that supported and moved forward their emerging expertise and confidence. As part of the process of supporting the teachers to improve and sustain these practices, the researcher continually sought their input into elements that had impact on their practice, working to both improve on their learning and to investigate links between these supports and evidence of the teachers' development.

Table 1
Units Designed by the Teachers each Term

TERM	Year 4 Units (Kaye & Carla)	Year 5 Units (Naomi & Josh)
1	*Can you roll your tongue?* - Exploring hereditary traits	*Are athletes getting faster?* - Investigating winning times at the Commonwealth Games
2	*What's in your lunchbox?* - Investigating healthy lunches	*Kangaroos!* - Modelling and interpreting data from a predator-prey game on the oval (Naomi) *How fast is a blue-tongued lizard?* - Class negotiated investigation (Josh)
3	*Tibia mystery* - Estimating height from a tibia bone found at a archaeological dig	*Who is a "typical" Year 5 student?* Developing a survey and exploring "typical" (Naomi)
4	*How many commercials does a typical Year 4 student watch in a year?* (Kaye) *Comparing students' ages* (Carla)	*Investigating paper airplane designs* (Naomi) *Designing a parachute for an egg* (Josh)

An initial list of support mechanisms relevant to the context was developed using literature (Table 2). This list was used as a framework to code and mark episodes in the transcripts where the teachers discussed these support mechanisms or raised additional possibilities. Special focus was given to supports articulated by the teachers that helped them to connect their learning from one unit to apply to subsequent units and their evolving practice. Based on the episodes retrieved, the list was refined and illustrative examples were drawn from the interview data, focusing on those elements that demonstrated strong links to the development of the teachers' evolving practice (Figure 1). Due to the role these support mechanisms had in helping teachers to apply learning from one teaching experience to subsequent ones, they were called *connection levers*.

Table 2
Initial List of Support Mechanisms

• Developing content knowledge in an environment that models reform-based practices • Reform curriculum	• Collaborative environment • Sustained involvement • Time	• Feedback & Support • Validation • Multiple iterations • Reflection

Connection Levers

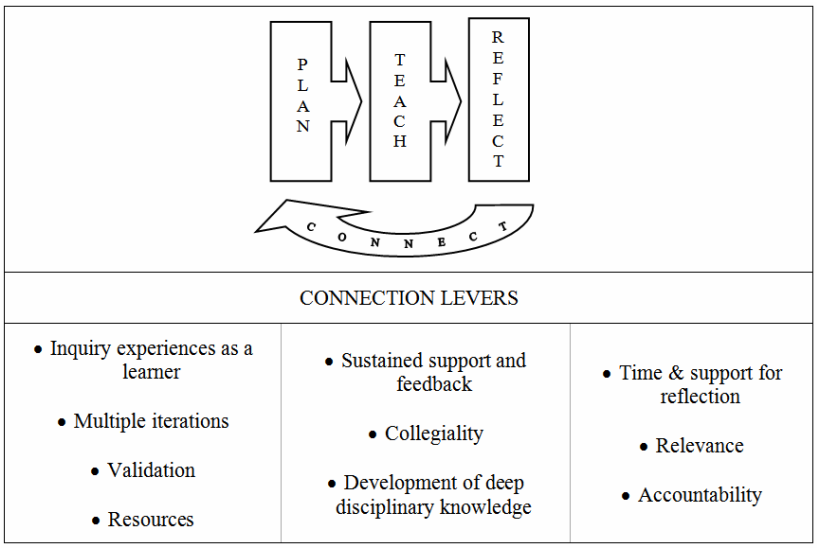

Figure 1. Connection levers to support teachers' learning to teach with innovative pedagogies.

Results

Inquiry Experiences as a Learner

One of the most compelling experiences for the teachers in learning to teach using inquiry was having the opportunity to work through inquiry problems themselves. The first learning seminar focused on the ambiguity and uncertainty associated with ill-structured problems by having the teachers work together to design an ergonomic chair (adapted from TERC, 1998). They spoke throughout the year about the impact the activity had on them.

Kaye: I thought it was helpful to actually physically throw us into the deep end and say "I want you to investigate chairs". And for a lot of us that's very different to what we've done before and for us, even as a group, it was quite a hard task for us to maintain some sort of focus and to have a direction moving forward. And I think putting us in that situation was good because I think it showed us that some of the things the kids can happen – it gave us a little bit of an insight as to where we might need to help kids move forward.

Carla: You know it made you see sort of phases [of an inquiry process] didn't it? It made you see well, perhaps you need to just brainstorm this part first.

The teachers believed in principle that inquiry was a beneficial approach for learning, but before being immersed in a problem as learners they were unsure what an inquiry-based problem felt like. Experiences with the open-endedness of the initial activity therefore raised a number of issues they had not considered. They worried about teaching students to work collaboratively, managing student diversity in dealing an inquiry, and coming up with good problems. For one pair of teachers, they reflected on their own struggles managing ambiguity and decided they should carefully structure the first unit for their students.

Kaye: We've seen how difficult it is for us, that we'll try to make the introductory process less stressful for them. ...

Carla: Yeah, it might have to be more constrained the first time.

Kaye: Or they would probably need more teacher input or adult input or someone just to sit and focus them. Like you had to come back ... [and] focus on trying to subtly pulling us back to where you want us to go without dominating our investigation but you would hope that we even now, given the same task or a different task after lunch, would be more focused.

After teaching their first unit, Carla and Kaye again mentioned their experiences with the chair problem and how it caused them to decide to scaffold the first unit.

Carla: I was going to do less guidance or less modelling at the beginning but I'm glad I haven't. Otherwise yeah I could foresee that my kids would just go 'oh well, I don't know what I'm supposed to do so, oh well, why bother doing it', those kinds of questions.

Kaye: It's not only children. Let's have a look at four of us up a few, two or three weeks ago when we were given 'Do an investigation on a chair'. How much time did we spend, really without any direction? We were going off in all different planets. But we, as adults, we found it difficult to do, so children will find it difficult to do. I guess even as adults we like structure and we like a scaffold. I guess that's why Carla and I went for a scaffold and we're pleased that we did.

The reaction to the chair activity was quite different for the other pair of teachers. They wanted to give students more control and designed their unit to incorporate this.

Naomi: I think to a large extent this is how it does work in the world. ... It's not as if the boss is standing there saying, 'well this is what the end product has to look like and these are the steps you're going to take', which is what we do in the classroom.

Although the teachers had different responses to their experience with the ambiguity of an open-ended task in designing their first unit, it was clear that it was an important experience for them to think back to during the year.

Multiple Iterations

Regardless of whether they were structured or open-ended, all of the teachers ran into difficulties in their first units.

Naomi: The first one, we were more uncomfortable with it. ... We wanted something that was absolutely, you know, out of this world and we didn't, we didn't plan properly where it was going and whether or not we had the tools to get it to go in the right direction. ... *that* was a *steep* learning curve!

Kaye: It's like all things that we introduce to kids to start, we think the results you get on the first thing you do are probably not going to follow what we want, but probably the more that we do the better they get.

In the second unit, both pairs of teachers designed units that were more balanced between structure and open-endedness. Over the course of the year, they experimented with different phases of the inquiry cycle, sometimes focusing on data collection and other times on interpreting findings or communicating results. At the end of the year, Naomi reflected on how through multiple iterations, both she and her students came away with a robust sense of what statistical inquiry could do.

Naomi: The first unit we looked straight at data collection really, and the interpretation of that data. ... [The second unit] was, yeah, just collecting data and having a look at the data. Then the third unit we extended it a little bit further and we looked at devising our own [survey] forms with which to collect data. And then, interpreting the data to the extent of saying, well, you know, "What was a typical Year 5 student?" But the last one is by far my

> favourite one because it went right from collecting the data all the way through using that data. And then creating something from that data then using, um, taking more measurements and using that data to see what could be improved and keeping a cycle going. So the children could actually then look at the data and say, "Ok, well, this is what we can realize from it and this is what we need to do next time". It was so much more of a practical use in how we would really use that sort of data in the outside world.

Naomi's statement was indicative of observations by the other teachers as well. In nearly every case, the final unit was the most complex and well-designed. This suggests the iterations were central to the teachers' abilities to build their expertise.

Validation

Having the support of the researcher and the other teachers in the study helped them to build confidence and persistence. Particularly in the beginning of the project, the teachers had concerns about whether they were "doing it right". When things did not go well, they often blamed themselves for not anticipating issues in advance.

> Naomi: I said it was the worst day because it was all the stuff I should have anticipated and allowed for so I was blaming myself. You have lessons where something goes wrong and it's outside your control—that's one of those things. But this was well within my control and I didn't account for it.

When I asked the teachers what helped them persist through the units, Josh commented that the validation that their experiences were normal was important to his ability to persist when things did not go as anticipated or unexpected school events disrupted the plans.

> Josh: Well, to start with, … you're always there saying, "look, this is a normal classroom".

The students also validated the teachers' efforts through their enthusiasm and learning. Naomi recounted a particularly challenging day for her, but when she reflected on the kind of lifelong skills the students had gained from the unit, she felt validated.

> Naomi: There was one day I could have thrown my hands up and said 'I'm not doing this' but I could see that the children were enjoying it. ... [And] the way they're now approaching things and saying "yes, but, what if - ? Could it be that - ?" And that's just wonderful.

Resources

Several times during the study, the researcher asked the teachers what they would suggest to someone attempting to teach mathematical inquiry.

> Kaye: I do believe where teachers feel a bit threatened or are doing something new, they work better if they've got a structure to work from. They're more inclined to have a go at it. Like I don't know if we would have gone down the path that we have or had the ideas to go down the path that we have without the resource that we've used.

Naomi and Carla both talked about how they used the resources for inspiration and guidance to generate ideas.

> Naomi: The other thing that really helped is that *TeachStat* book [Gideon, 1996] because just flipping through there was a really good place to start to get ideas. Because right from the start, it was well, "Ok, this is a great principle, great in theory. How do I do it? ... What do I *do*? How do I come up with ideas?" So that *TeachStat* book was actually full of some really good ideas. And one of them gave me the idea for "The Typical Year 5 Student" [her third unit]. ... [Otherwise] the ideas are hard to generate sometimes.

Carla: I'm sure that if Kaye and I didn't have that resource we'd be racking our brains trying to think of a good one that's going to try and interest as many people as possible.

Sustained Support and Feedback

Ongoing feedback and technical support were also important for the teachers. In an interview at the end of the year, Kaye recalled a suggestion to consider stacked plots instead of a single graph to allow students to compare, not just describe, their data.

Kaye And actually the support, the throwing in of things that we could do, I appreciated it. A couple of times when you came in, [and suggested] 'this is how you can do this'. ... For me, somebody that, I often learn a lot better and work a lot better when there is input. ... A classic example was stacked line plots, which was something that, you know, I hadn't even registered that stacked line plots made it so easy for the students to interpret the data. And from there that's something that they have been able to do a lot easier, doing it that way rather than putting it [a single graph] on their presentation. Yet in *all* the books I read through, it hadn't *mentioned* stacked line plots! So without your input there, I wouldn't have been able to fly the way I did.

Collegiality

The teachers also expressed how important it was for them to interact together and how this contributed to their ability to develop.

Josh: I think one of the most beneficial things about today, has just been listening to each other.

Naomi: We all had problems, it was ok because we could learn from each other's problems.

Kaye: I think has been one of the major aspects of [Carla] and I just actually working, and bouncing off [ideas] – "oh well this is what we can do, let's try it with this" or "let's use this resource", so that has been professionally very good for us.

Both the professional sharing of teaching in teams and the opportunity to share their experiences with others trying the same innovation was important to their development. It not only helped them continue the momentum, but also enabled them to learn each other.

Development of Deep Disciplinary Knowledge

Another connection lever that the teachers said helped them to sustain and develop expertise in teaching inquiry-based mathematics was their new understanding of statistics. This new learning changed the way that they focused their students' learning.

Carla: Now at the end of the year, I know what it might mean to understand a statistical investigation or working with data, where at the beginning of the year [I only considered] "can they draw that graph?" ... [But now we know] what to look for to say this child understands what working statistically means. [*To the others*] Wouldn't you say?

Naomi: Oh, definitely. I'll be honest, I used to look at chance and data and say, yeah, "if they can draw a graph – good, if they can work out the probability of tossing a head when tossing a coin – that's done. Chance and data's out of the way".

Carla: But now you can say, "Wow, this person can interpret that data and make this assumption".

Time and Support for Reflection

The time to think, to plan, to talk, to try things, and to generate ideas away from school was talked about by the teachers. This was time for them to reflect on what they had learned in a supportive environment with others sharing in the same experience.

> Naomi: Once you're out at the university, or anywhere else that's away from school, you stop thinking about what's going on at school. ... We could just shut out school completely, and just sit and talk and focus completely on maths. And that was really valuable.

Through supported reflection, the teachers drew on their experiences of each unit in planning subsequent units and to stand back, abstract from their experience, and consider how they would apply it to improving their practice.

> KM: What about for you professionally? What do you think that you gained?
>
> Naomi: Well a couple of things. First of all, I'd never actually thought to use an inquiry approach in mathematics before. We use it in science commonly but not in mathematics. So to see that there was a way that we could incorporate that into the classroom was wonderful. It was, uh, a learning curve for me though because I've realised now there's a lot more planning that I have to do in inquiry maths than I would in a normal maths unit. Simply because I have to try and anticipate now where the unit could go to make sure the children have those underlying skills.

Relevance

The project immersed the teachers in thinking about teaching with inquiry. The way that the professional learning opportunities were directly linked to the teachers' classroom practice and were sustained throughout the year became important support mechanisms for the teachers. Taken together, the inquiry experiences they had during professional development, the opportunity to participate in a community of learning about what they were doing in their classroom, and knowing that others were thinking through the unit with them as they were teaching it, all contributed to their ability to build their expertise. The opportunity to integrate their learning with their teaching was relevant to their classroom work and day-to-day practice. They were excited when they saw that the work they were doing was at the forefront of teaching mathematics and that the inquiry approach they were teaching was being promoted as well by state and local initiatives.

> Kaye: We've had to really look deeply at what an investigation really is and investigations really do form a major part of the new maths syllabus. ... One of the new [mathematics] outcomes ... was about children creating and interpreting and analysing data, which is all what we've been doing the whole year. So I guess this whole thing we've been doing has been excellent for us getting a handle on the sorts of things that we can do.
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> Josh: There was a classroom magazine that a friend of mine had the other day and there was a big [article on] inquiry. ... I looked at it and I thought, "Oh! That's what we did!"

Accountability

A big issue for these teachers was juggling the demands on their time. With good teachers, there is intent to try new things, but sometimes the best intentions get buried. Naomi spoke about the fact that she would not have gone beyond the first iteration had I not been there expecting a unit to watch each term.

Naomi: [The accountability] kept me going. Otherwise, ... you go to the conference, you sit there and you write it all down. You say 'this looks wonderful', and you go back and you drop it on your desk. And about six months later when you sort out the pile of things that's built up on your desk. You go, "oh, that looks interesting, I'll put it in a file and I'll try and read that later". And that's kind of it. Whereas this was good. The first one [unit], yep, we did it. We did what we were supposed to do. It was good, I can see some value in it and I can honestly say, that I probably would have then said, "ok, well, I'll try that next year". Maybe! And then probably forgotten. Whereas because there was an expectation to do one every term, by the time you got to the last one, you felt comfortable with it, the unit was great, the kids took it to places that I just, and showed understandings that I didn't think they would be capable of. ... So, I'm completely sold, but it would have taken more than one to do that. ... The accountability, and the fact that you had to rehearse it, effectively, over and over, kind of solidified the skills.

Discussion and Implications

Over the past two decades, there has been a paradigm shift in the teaching and learning of mathematics. In this shift, the ideal for mathematical instruction transforms from an emphasis on skills, facts, and procedures towards greater stress on developing children's mathematical conceptions and proficiency at applying mathematical tools to new situations: in particular, open-ended, complex and everyday problems. In order for teachers to make these shifts in designing innovative learning experiences for their students, they must develop capability with this approach and be able to envision and embrace it. This project examined the process of learning to teach mathematical inquiry in a supported environment. The preliminary results presented here suggest that these connection levers enabled the teachers to reflect on their iterative experiences in teaching mathematical inquiry towards building their emerging expertise. The teachers described how these connection levers supported their ability to persist beyond the challenges encountered during the initial teaching experiences, and continue to sustain them, building their confidence and commitment in the process.

The teachers in this study developed a great deal of expertise in the course of a year, more than was predicted. It must be cautioned, however, that this is partially due to the fact that the teachers in the study already possessed beliefs about learning that were consistent with an inquiry-based environment. Quite possibly progress would be slow unless teachers first commit to an inquiry-based epistemology. Similar work in research on middle schooling suggests that unless teachers' philosophy is consistent with the reform, any apparent change in practice is not sustainable (Pendergast et al., 2005).

Although these findings are tentative and preliminary, many of the connection levers named by the teachers were consistent with research on good professional development (Elmore, 2002; Ball, 1996; Cohen & Hill, 2001). There was no magic in these levers; none are beyond the reach of schools or districts with creative leadership. The challenges the teachers faced and the supports they named were in the context of work in authentic classrooms with diverse student needs. The use of a design experiment further supported the applicability of the research and layers of iterative learning by the researcher, teachers, and students. On one hand, the excerpts from the teachers and the support mechanisms they list point to the complexity of moving teachers from a stage of orientation about teaching mathematical inquiry towards a commitment to teaching with this approach. On the other hand these supports are consistent with moves in education to support more collaborative engagement of teachers throughout their careers in the learning profession.

Postscript

An additional support that has been discussed and will be trialled this year is having the researcher model particular teaching approaches with the teachers' students in their classrooms. This kind of interaction, if equally effective, would further support recommendations for expanding partnerships between schools and universities (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). The teachers are already being utilised by their schools to begin training their colleagues in this approach. In addition, they are presenting their work at teachers' conferences both locally and nationally.

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