Seeking Evidence of Thinking and Mathematical Understandings in Students' Writing

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This paper reports the use of three questions to guide students' discussions and reflective writing in a year 5/6 mathematics class. Journal entries and work samples were examined for evidence of students making sense of their thoughts and processes used during the completion of Space-based tasks. Reflective writings were inspected for evidence of the three functions of metacognition and Bloom's Taxonomy was used to note changes in students' levels of understanding of the content. Preliminary findings suggest that the approach and questions used in this study warrant further investigation.

Directives in curriculum that require teachers to assess and report students' thinking are complex. One approach is explained in this paper, which commences with background information about the change in emphases in recent curriculum. This is followed with an overview of the literature that informed the approach used and provided the basis for the data analysis in the investigation. Then preliminary findings are discussed.

Trends in Curriculum for Developing Thinking and Understandings in Mathematics

New directions in curriculum across Australia share a focus on preparing students for further education, work, and life (Department of Education and Children's Services, 2001; Department of Education Tasmania, 2007; Department of Education Training and the Arts, 2004; Victorian Curriculum Assessment Authority (VCAA), 2006). In 2005, the Victorian government introduced the *Victorian Essential Learning Standards* (VELS) (VCAA, 2004), a framework for planning whole school curriculum from Preparatory – Year 10. The *Learning Standards* are developed within three interrelated strands: *Physical, personal and social learning; Discipline-based learning; and, Interdisciplinary learning*. These three strands seek "to equip students with capacities to manage themselves and their relations with others, to understand the world, and to act effectively in that world" (p. 3). Each strand has a number of domains. In each domain, the essential knowledge, skills, and behaviours are identified in subcategories called dimensions. Specific standards are written for each dimension according to three broad stages of learning: P-4, Years 5-8, and Years 9-10. These standards define essential and developmentally appropriate expectations for teaching and learning programs (VCAA, 2004). The *Learning Standards* may be addressed in programs either "through explicit teaching focused on a particular strand [or] … by creating units of work which address a number of standards at the same time" (p. 3).

Since the implementation of VELS teachers have been grappling with the complex task "for ensuring that all three strands, and their domains are addressed by all schools in their teaching programs and in their assessment and reporting practices" (VCAA, 2004, p. 3). The complexity of the task is not necessarily in the planning or implementation stages but in the mandate to assess and report each of the domains. For example, Table 1 lists a possible set of domains and dimensions from the three strands included in a mathematics-based unit of work.

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J. Watson & K. Beswick (Eds),

Table 1
Strands, Domains, and Dimensions in a Mathematics-based Unit of Work

Strand	Domain	Dimension
Physical, Personal and Social Learning	Personal Learning	The individual learner
		Managing personal learning
Discipline-based Learning	English	Writing
	Mathematics	Space
Interdisciplinary Learning	ICT	ICT for visualising thinking
	Thinking	Reflection, evaluation and metacognition

A mathematics-based unit comprising these domains and dimensions may produce worthwhile experiences for students learning not only in the content but also possibly in those generic skills and strategies applicable in various contexts. Yet, one might ask: Which tools and strategies will teachers use to measure and report students' progress in the domain of thinking?

This paper reports one approach for assessing and reporting student progress given the expectations of teachers in Victorian schools using three questions addressing the three strands in VELS. The key question addressed in this study is:

- Does the use of three specific questions at the commencement of reflective writing sessions provide evidence of the development in children's thinking and mathematical understandings?

Gaining Insights into Students' Thinking and Understandings of Mathematics

A scan of proceedings at MERGA conferences suggests that teacher educators not only share a desire to help students articulate their ideas during mathematics lessons but also have various ways of encouraging the exchange of thoughts either orally and/or in writing (Beswick & Muir, 2004; Brown & Renshaw, 2004; English & Doerr, 2004; Falle, 2005). Although not necessarily building on the same theme, insights from each of these studies shaped and informed the study discussed in this paper.

In a study by Beswick and Muir (2004) comprising 20 year 6 students from five primary schools researchers examined participants' abilities to communicate their problem solving strategies and mathematical thinking. Using semi-structured interviews, each problem was read to the student by the interviewer. Students were asked to solve the problem and record the process used in writing. Concrete materials were available for students' use. On completion of the task, students were asked to explain verbally what they had done. Beswick and Muir reported that, regardless of students' abilities, students expressed their mathematical thinking more effectively in verbal than in written forms.

Beswick and Muir (2004) concluded that learners would benefit from instruction that encouraged visualisation of their thinking and "efficient and meaningful ways of recording their thinking in writing" (p. 101) and this is one of the goals of the study discussed in this paper.

Another source shaped the approach and the design of the tasks used. Brown and Renshaw (2004) argued that "success in school mathematics is often measured in terms of a student's capacity to reproduce others' inventions and justifications" (p. 135) and advocated the need to link students' experiences and processes with the more formal content knowledge in the domain of mathematics. They proposed an alternative format to teachers for initiating class discussions and for developing deeper understandings of mathematics by incorporating both everyday and scientific notions of mathematics into their discussions.

Two terms, *replacement* and *interweaving*, were recommended as ways for students "to make sense of the mathematics being presented to them and about linking students' inventions to the conventions of mathematics rather than about teacher and/or textbook evaluations of student answers" (Brown & Renshaw, 2004, p. 142). *Replacement* referred to using "an everyday understanding with a more sophisticated conventionalised understanding" (p. 135). *Interweaving* seemed to refer to an acceptance of and interchange between informal and scientific concepts and/or language.

Also contributing to the teaching approach, English and Doerr (2004) claimed that recent research necessitates teachers to be "more attentive and responsive to their students' mathematical reasoning" (p. 222). Teachers who display a hermeneutic disposition in their teaching tend to use tasks that provide opportunities for students to explore mathematical ideas, carefully listen to students' ways of thinking, and adopt various roles in their interactions with students. Such teachers observe, listen, and ask students questions for further clarification.

Similarly, Falle (2005) reported that students' explanations reveal not only the degree of their mathematical thinking but also the linguistic features used by students in their responses that may serve as indicators of their level of understanding. Falle noted that less successful students resort to "parroting" mathematical rules even though they may not be able to use them. In contrast, students who are more mathematically capable tend to experiment with logic and have greater control over the language needed to express themselves. This provided further justification for the attention to developing students' expressive skills in mathematics.

Monitoring Metacognition

An overview of processes for monitoring students' thinking processes is also relevant to the discussion given the focus on developing thinking skills in several curriculum policies. Wilson and Clarke (2004) referred to metacognition as the "awareness individuals have of their own thinking; the evaluation of that thinking; and the regulation of that thinking" (p. 26). Given this definition Wilson and Clarke noted three functions of metacognition: awareness, evaluation, and regulation. "Metacognitive awareness relates to individuals' awareness of where they are in learning process or in the process of solving the problem, of their content-specific knowledge, and of their knowledge about the personal learning contexts or problem solving strategies" (p. 27). "Metacognitive evaluation refers to judgments made regarding one's thinking processes, capacities and limitations as these are employed in a particular situation or as self-attributes" (p. 27). "Metacognitive regulation occurs when individuals make use of the metacognitive skills to direct their knowledge and thinking" (p. 27).

Wilson and Clarke (2004) assumed that promoting metacognition was a valuable exercise in mathematical learning contexts and that some strategies encouraged metacognitive acts. To address the known difficulties with monitoring metacognition, they refined a multi-method clinical interview that involved self-reporting and a think-aloud technique, observation, and audio and video recording. The clinical interview involved a card-sorting procedure enabling the participant to reconstruct his/her "thought processes during a problem solving episode just completed" (p. 29).

Wilson and Clarke's study (2004) comprised 90 one-on-one interviews with year six students from six different classes across Victoria using three different types of tasks: numerical, spatial, and logical. A series of metacognitive action statement cards varied according to the task but were categorised according to the three functions of metacognition identified in their earlier definition: awareness, evaluation, and regulation. For example, statements from the awareness category included: I thought about what I already know; I had tried to remember if I had ever done a problem like this before; I thought "I know this sort of problem". Sample statements from the evaluation category included: I thought about how I was going; I checked my work; I thought "is this right?" In the regulation category some statements included: I thought about what I would do next; I made a plan to work out; I changed the way I was working.

Wilson and Clarke (2004) reported that it seemed reasonable to expect a particular pattern in these three functions: awareness first, followed by an evaluation and finally a regulatory act. However, students used various sequences, many of which were non-linear. Generally, sequences commenced with awareness. Regulatory and evaluative statements were often arranged in different combinations. Students concluded tasks with an evaluation statement regardless of whether the task was completed successfully.

Analysing Levels of Understandings

Although not specifically from the mathematics education field of research, some reference to the levels of understanding using Bloom's Taxonomy (Anderson, 1999) is helpful with the data analysis in this investigation. Bloom's Taxonomy was first published in 1956 with six categories knowledge, comprehension, application, analysis, synthesis, and evaluation. These were considered to be increasingly complex behaviours that cumulated in a hierarchical structure (Anderson, 1999).

Over the past 50 years there have been variations of the original model (Houghton, 2003). Changes in the new taxonomy include the recognition of the role of social learning and "cultural-specificity of knowledge" (Anderson, 1999, p. 7). Another is the qualification of the premise "that the categories form a cumulative hierarchy in all cases … depends on a series of factors" (Anderson, 1999, p. 8). For example, Anderson (1999) reported that an individual may use several cognitive processes such as recall, understand, analyse, synthesise, and evaluate in selecting an appropriate strategy to solve a problem. However, there are other cases in which one may apply a given or known strategy in a routine manner. The difference is that metacognition is evident in the former but not necessarily in the latter.

Houghton (2003) compared models of the taxonomy. The version that listed verbs for each category was helpful for inspecting and assessing student work samples in this investigation.

In summary, over recent years various authors cited in this paper, have suggested ways in which teachers may link students' experiences with mathematical learning through their interactions and discussions with students. Some offered a way to help identify the functions of an individual's metacognitive processes in completing a task. Others suggested that teachers provide guidance to help students visualise and record their thoughts in writing, or ask questions so that students may clarify their ideas. Insights from such authors provide the basis for the analysis. Yet, perhaps more is needed for gathering and analysing children's written records of their thinking and mathematical understandings. One approach is to use three specific questions as prompts for children's reflective writing within mathematics lessons and examine these for evidence of cognitive processes used and mathematical understandings gained.

Investigation of the Effectiveness of Three Questions

The study examined the changes in year 5 and 6 students' perceptions of themselves as learners, their knowledge and skills in an aspect of Space, and their ability to use specific tools and strategies over a 2-week period. The specific research question addressed in this paper is:

- Does the use of three specific questions at the commencement of reflective writing sessions give evidence of the development in children's thinking and mathematical understandings?

Participants

Twenty-three students in the year 5/6 class attended a small inner city school where 94% of the student population came from Culturally and Linguistically Diverse (CALD) backgrounds and 67% of the families received financial assistance. One student had recently arrived in Australia with limited English skills and several had learning disabilities.

The classroom teacher had 2 years teaching experience and chose to work along side the researcher. The researcher had taught for 14 years in primary schools.

Overview of the Planning, Lesson Format, Tasks and Approaches used

In the week prior to the study commencing the classroom teacher collected students' prior knowledge of the content and discussed these and the content to be taught with the researcher. The researcher planned and delivered four lessons. The classroom teacher was always present in the room and interacted with students as they worked on the activities.

Each lesson was between 60 – 80 minutes in duration and followed a similar format. The researcher introduced the focus of lesson to the whole class and invited the students to accept a challenge posed in tasks. Students investigated the open-ended tasks for ten minutes, were asked to share their ideas and strategies, and then were directed to resume working on the tasks being mindful of shared insights. Lessons concluded with the researcher summing up key points and students reflected on their experiences of the lesson and wrote personal reactions in their workbooks.

During the fourth lesson, students were invited to consider what knowledge, skills, and feelings they had that were somewhat different to those which they had prior to these lessons. Students wrote for approximately 40 minutes in response to three specific questions:

- What have you learnt which is somewhat different to what you already knew about **mathematics**? Give examples.
- What have you learnt which is somewhat different to what you already knew about the **program, tools and games** used?
- What have you learnt which is somewhat different to what you already knew about **yourself or the way you learn**?

Although 40 minutes is not realistic in many classrooms these students predominantly from non-English speaking families needed the time to reflect and write.

Because there were only three desktop computers in the classroom students worked in pairs rotating through planned tasks. These involved either the manipulation of concrete materials and discussions at students' tables or completing a computer-based task. The series of lessons were designed to link students' everyday experiences with mathematical content. Three additional aims were:

1. to draw on students' interests in arcade-type computer games and in programs such as *MS Powerpoint* (Microsoft Corporation, 1995),
2. to develop students' use and understandings of mathematical language when transforming two-dimensional shapes such as flip/reflection, slide/translation, turn/rotation, resize/enlarge/reduce/dilation,
3. to provide opportunities for students to discuss, reflect and write their thoughts at the conclusion of each lesson.

Table 2 summarises the tasks completed.

Table 2
Computer-based and Table Tasks Completed over Four Lessons

Type of task	Brief description of activity
Computer-based	Individuals play two games of *Tetris* (2M Games, 2004).
Table	Using multi-link cubes make 12 shapes which could be used in a game like *Tetris*. In pairs, one person plays the game and fills as many whole lines as possible gaining 10 points each time. The other person provides the pieces one by one. (No flipping allowed). Is the game better or worse if you are allowed to flip the pieces?
Table	Create a picture using 7 tangram pieces. Trace around the outline. Make a small scale drawing of your solution. Recreate another person's picture. Check the answer sheet.
Table	Groups of three complete a barrier game using tangram pieces/picture. A tells B how to make his/her picture by giving verbal instructions only. C acts as observer and records the language used.
Table	Create mosaic picture/pattern using pattern blocks. Then using grid paper, create a tiled floor. After a few attempts create a piece of art work using Escher's style.
Table	Make a picture flick note pad to show an image moving.
Computer-based	In pairs, create a series of four/five slides which show shapes moving (flipping, sliding, rotating, resizing).
Table	Draw a simple picture onto grid paper. Enlarge and reduce the picture according to a scale.

Data Collection and Analyses Techniques, Tools, and Approaches

Prior to the series of lessons commencing the classroom teacher asked students to write what they knew about the topic and in which situations one might use the content or related terms. During the four lessons students' computer-based work files were saved on the class server and samples of their book work were collected. Researcher took anecdotal notes of significant events and discussions with students. Researcher and classroom teacher each kept journals with their reflections of each lesson and later shared their thoughts via email communication.

After the lessons, dated work samples were examined in two ways. First, for evidence of levels of understandings about concepts in transforming 2D shapes using Bloom's Taxonomy from written responses to questions in pre-lesson and from the fourth lesson. Analyses of data were tabulated to provide an overview of the levels of understandings about concepts in transforming shapes for each student. Second, the work samples were inspected for evidence of the three functions of metacognition (Wilson & Clarke, 2004). The *Monitoring Metacognition Interview* (MMI) multi-method interview technique (Wilson & Clarke, 2004, p. 29) was not used in this study.

Results and Discussion

A small group of participants attending a professional development session were provided with the adapted version of Bloom's Taxonomy used in this investigation and asked to look for evidence of understandings in students' work samples. Their responses were similar to those independently categorised by both the researcher and teacher. Table 3 presents frequencies of level of understanding of concepts and vocabulary related to transforming 2D shapes using Bloom's Taxonomy in students' responses recorded pre-lessons and in fourth lesson. This was the first way work samples were examined.

Table 3
Students' Levels of Understanding of Topic using Adapted Version of Bloom's Taxonomy in Written Responses

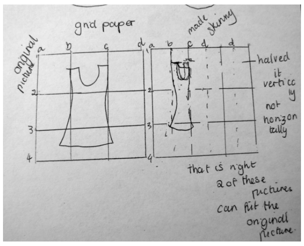
		Remembering	*Understanding*	*Applying*	*Analysing*	*Creating*	*Evaluating*
Pre-lesson	No evidence			1			
Student	Limited example	3	4	9			
(n = 14)	Multiple examples	11	10	4			
Fourth Lesson	No evidence				2	4	11
Student	Limited example	6	7	6	6	6	7
(n = 20)	Multiple examples	14	13	14	12	10	2

The figure 14 in the bottom left hand cell indicates that 14 of the 20 students either listed or described two or more examples related to the topic in their reflective writing from the fourth lesson. There was evidence of students' increased levels of understandings about concepts in transforming 2D shapes using the adapted version of Bloom's Taxonomy from written responses to questions in pre-lesson 3 Nov (n = 14) and from journal entries dated 17 Nov (n = 20). For example, although a group of 10 or 11 students began the series of lessons with a reasonable knowledge of the terms and were able to describe or define the terms, only four gave examples of when the terms were used in both mathematical and everyday settings. In contrast, by the fourth lesson there was evidence that 14 students saw applications for these terms. There was also evidence that students (n = 12) were synthesising their understandings that went beyond the tasks or saw connections between them.

An excerpt from student N1's fourth lesson written response provides a sample of the evidence identified for the *creating* category.

> I also learnt that by stretching a picture, the picture would look very different because your (sic) only changing the width, but if you change both height and width the picture will look the same but bigger.

It seems that this student is developing a generalisation about ratio and proportion.An excerpt from another student N2's fourth lesson written response provides a sample of the evidence identified for the *evaluating* category.



> I learn't (sic) how to draw a particular picture on grid paper and then making it skinny. Since my original picture was drawn on a 2cm scale I wanted to make it skinny. First I halved the 2cm which would be 1cm but I didn't halve it horizontally only vertically and drew my picture (sic). That is right 2 of these pictures can fit the original picture.

There are two comments added to the diagram in which student N2 justifies her thoughts: "halved it vertically not horizontally" and "That is right two of these pictures can fit the original picture".

Insights from Wilson and Clarke's (2004) three functions of metacognition and action card statements provided the basis for the second form for data analysis. The culturally diverse group of students, who refrained from participating in class discussions, were willing to write in journals at the end of the fourth lesson. Written responses from ten students indicated that they noted changes in their own thinking, skill level and/or attitude towards aspects related to these activities.

Many students wrote about increased awareness of the applications of the mathematics being studied in everyday activities. For example, student A wrote:

> I never knew that I was using mathematics when on (sic) powerpoint but I [now know] that I was estimating sizes when [I was] changing [resizing] pictures [to use in slides] for [creating] animations. When I play tetris, I play it for fun but I was using flip, slide and rotate to fit shapes into gaps.

Although this student had some difficulties with clear expression, the entry provides evidence of the awareness the student gained as a result of these lessons. Without the opportunity for writing such insights would be more difficult to capture.

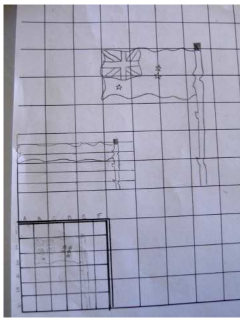
The following excerpts are all from student D's fourth-lesson written response:

> I learnt that the game *Tetris* involves maths because when we use the shapes to make lines/rows, we are using tessellation.

Similarly, the student seems to be reflecting on the activity and drawing on the metacognitive function, awareness.

> I also learnt that when allowing the person to *flip* in the game, it is sometimes easier [to get higher scores].

This sentence could be within the evaluation category.



> Drawing on grid paper also involves maths because we use scales when either enlarging or reducing the size of images/pictures. When we make our drawings flatter, we divide the grid that goes horizontally, smaller.
>
> I've learnt that when doing an animation on powerpoint (on the computer), you only move each picture a bit on each slide to make it moving when the entire slide was played. It was one of the best things I learnt because I have never done it before.

Again, in both of these sentences there is evidence of some metacognitive awareness and regulation occurring. The student is aware of the new knowledge and indicates that he will use the knowledge to plan and complete similar tasks in the future.

Conclusion

To an extent the goal of the investigation was successful. The approach and the three specific questions provided students with opportunities to discuss and write responses gathering evidence of students' progress in the three interwoven strands central to the Victorian Essential Learning Standards (VCAA, 2004). Even students with limited skills in English were able to communicate their thought processes and some deepened their mathematical understandings about aspects of Space over four lessons.

There were also limitations to using the approach. Reflective writing is a text-type and a generic skill that needs to be explicitly taught. As with other text-types teachers need to model the language features used in such forms of writing (Derewianka, 1990). For this group of students reflective writing was a new text-type and skill. Part of the mathematics session was spent explaining the questions and expectations of the writing which was non-mathematics specific learning.

The tools and techniques used for data analyses seemed helpful in identifying changes in students' written responses. Having said that, it might be useful to expand the list of verbs in the table of the version of Bloom's Taxonomy used.

Given these preliminary findings, it would be useful to replicate this investigation or conduct further research using these three questions with students and teachers P-10 classrooms to check whether similar trends emerge.

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