Using Paper-Folding in the Primary Years to Promote Student Engagement in Mathematical Learning

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Traditionally, craft activities have been incorporated into early childhood mathematical learning experiences as an effective means of fostering curiosity and interest, and introducing abstract mathematical concepts through the use of concrete materials. However, in the primary schooling years creative and active mathematical learning experiences often give way to approaches such as computational drills and rote learning which are less than inspiring for students and often do not result in meaningful understanding. This paper shows that craft activities, in particular paper-folding, can also be valuable in the primary years as a means of promoting affective, behavioural and cognitive engagement in the mathematical learning.

In the primary classroom, the inclusion of paper-folding activities in mathematics can offer an appealing and creative means of addressing some of the goals for teaching and learning mathematics in the 21st century as identified by English (2002), namely: engaging students effectively in mathematical modelling, visualising, algebraic thinking and problem-solving. Paper-folding activities can provide a hands-on, active experience that contributes to the development of mathematical ideas, thinking and concepts; skills in communicating mathematically; and group interaction skills. The particular focus of this study is to investigate how paper-folding in primary mathematics can also be used to promote affective, behavioural and cognitive engagement in mathematical learning.

Theoretical Background

Essential to arguing the use of craft activities, most specifically paper-folding, as a valid approach to be incorporated into effective mathematical teaching and learning strategies, is the view that a learner can engage with mathematical concepts and construct their own knowledge through building and creating objects. The knowledge construction that takes place when learners build objects is emphasised in the constructionist learning theory. The theory of constructionism emerged in the 1980s from work undertaken by Seymour Papert. It builds on the "constructivist" theories of Jean Piaget that knowledge is not simply transmitted, but actively constructed in the mind of the learner. As an extension of this, constructionism suggests that learners are particularly inclined to generate new ideas when they are actively engaged in making some type of external artefact, which they can reflect upon and share with others.

> Construction*n*ism - the N word as opposed to the V word - shares constructivism's connotation of learning as "building knowledge structures" irrespective of the circumstances of learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity. (Papert, 1991, p.1)

Constructionism involves two intertwined types of construction: the construction of knowledge in the context of constructing personally meaningful artefacts, which Papert (1980) referred to as "objects-to-think-with" (p. 11). A second important aspect of a constructionist learning environment is audience, emphasising that knowledge construction is also embodied in the creation of an artefact which will be viewed and valued by others: "what's important is that they are actively engaged in constructing something that is meaningful to themselves [and] to others around them" (Resnick, 1996, p. 281).

Review of Literature

Student Engagement

A prevalent view of engagement to be found in the literature is that it comprises three interrelated components: behavioural, affective, and cognitive. Affective engagement refers to a student's emotional responses when participating in a learning activity and may be demonstrated through their enthusiasm, optimism, curiosity or interest (Chapman, 2003; Fredricks, Blumenfeld, & Paris, 2004). Student behavioural engagement has been found to be evident in their involvement in learning tasks, together with effort and persistence (Russell, Ainley, & Frydenberg, 2005). The third component, cognitive engagement, has been defined by Helme and Clarke (2001) as "the deliberate task-specific thinking that a student undertakes while participating in a classroom activity" (p. 136).

The influence of task factors is an aspect of student engagement that is of particular relevance in this study. Research undertaken by Marks (2000) identifies that *authentic* tasks contribute strongly to student engagement. Additionally, Helme and Clarke (2001) argue that engagement is more likely when students work on *novel* tasks and a strong association between *challenging* tasks and student engagement has also been identified (Marks, 2000). Finally, Blumenfeld and Meece (1988) emphasise the significance of *hands-on* tasks in promoting engagement in primary students.

Paper-Folding in the Primary Mathematics Classroom

The literature reveals many benefits that arise from including paper-folding activities in primary mathematics classroom. The principal benefit is that it contributes to the development of mathematical ideas and thinking, and the understanding of mathematical concepts (Cornelius & Tubis, 2006). Paper-folding lends itself ideally to spatial visualisation and geometric reasoning (Cipoletti & Wilson, 2004) but is also valuable in enhancing problem-solving skills (Youngs & Lomeli, 2000). Paper-folding also provides a range of opportunities for students to develop their language skills and proficiency in communicating mathematically (Cipoletti & Wilson, 2004). Sze (2005) identifies the value of paper-folding in promoting the correct use of geometric terminology, whilst Jones (1995) suggests that paper-folding can also be used to encourage writing in the mathematics classroom. Some further benefits that arise from the use of paper folding in the primary mathematics classroom are that it encourages group interaction and cooperation (Levenson, 1995) and promotes the development of fine motor skills and manual dexterity (Tubis & Mills, 2006). Of note is that investigations regarding the value of paper-folding in promoting *student engagement* in mathematical learning are seemingly absent in the previous research.

Research Focus and Significance

This study describes a constructionist approach to teaching and learning whereby paper-folding is used to create and explore mathematical models and investigates whether using this approach in primary mathematics can promote affective, behavioural and cognitive engagement in the mathematical learning.

The significance of this study is, firstly, that it extends the range of benefits that can arise from including paper-folding activities in the primary mathematics classroom in that student engagement in the mathematical learning inherent in such activities has not been addressed in previous studies. Additionally, paper-folding has been incorporated into mathematical learning activities in this study in original and inventive ways not readily evident in previous studies. In some cases (Coad, 2006; Turner, Junk, & Empson, 2007), whilst the paper-folding activities incorporated mathematical explorations, they do not involve the creation of a model. Whilst other examples (Cipoletti & Wilson, 2004; Cornelius & Tubis, 2006) did involve the creation of a model to be used for various mathematical investigations, once the explorations had been completed the paper model afforded no on-going mathematical purpose. This study takes the use of paper-folding one step further in that the students create various models that are used for mathematical explorations during the course of the learning activities, including ultimately the pieces of the Soma Cube puzzle, which then continues to offer a mathematical challenge as the students explore solutions to the puzzle. Finally, the significance of this study also lies in its contribution to research about constructionist approaches to teaching and learning. Kafai and Resnick (1996) contend that constructionist researchers are continually seeking new educational activities and tools to expand their endeavours. Previous constructionist research has focused overwhelmingly on the digital learning environment (Papert, 1980; Stager, 2001). This study contributes a fresh perspective on new constructionist educational activities through a non-digital construction medium: paper.

Research Method and Procedures

A classroom teaching experiment approach to research was adopted for this study. This research method involves a teaching agent (either the researcher or the collaborating teacher) conducting a sequence of teaching episodes over a period of time (Cobb, 2000; Kelly & Lesh, 2000). What transpires during each episode is recorded and this data is used in preparing subsequent teaching episodes, as well as in conducting a retrospective analysis of the teaching experiment (Steffe & Thompson, 2000). In this study, the research site was a Year 5 classroom. This site was purposefully selected because the classroom teacher and the researcher had a well-established communicative and collaborative relationship. The class consisted of 26 students, 14 boys and 12 girls, aged between 9 and 11 years. The mathematical ability of the students in the class varied significantly. Some students would be considered to be of high intellectual potential ("gifted") whilst others had diagnosed learning disabilities. During the classroom teaching experiment the researcher acted as the teacher, to order to draw upon personal expertise in the paper-folding techniques that were required. The teaching sequence, which was devised and reviewed in collaboration with the classroom teacher, incorporated three modules and comprised one or two teaching episodes each week over eight school weeks.

In the first module, the students learned how to construct paper cubes using unit origami techniques, then small groups of students paired with small groups from another Year 5 class to teach them how to construct the paper cube. The mathematical focus in this module was to review of the features of geometric figures such as faces, vertices and edges, together with line and angle properties. This occurred incidentally whilst the construction techniques for the cube assembly were being demonstrated. Additionally, the appropriate use of mathematical language was emphasised, particularly in designing the set of written instructions for the peer teaching episode. The second module involved the students extending their paper cube to create paper-folded rectangular prisms in order develop further their paper-folding skills. The mathematical learning in this module took the form of an investigation that focussed on patterns and algebraic reasoning. In this investigation the students exploring the "growing" pattern associated with the number of squares to be found on the faces of the rectangular prisms they had created by joining one, two or three cubes, that is the sequence $\{6,10,14,\ldots\}$. The students were asked to predict, extend and generalise the pattern by considering the total number of squares on the faces of larger rectangular prisms, without constructing such prisms from paper. The mathematical learning in the third module focussed on spatial visualisation and geometric reasoning. The module commenced with the students constructing one piece of the Soma Cube puzzle, a three cube L-shaped model. This model was used to examine the property of irregular figures: the possibility of joining two points on the surface of the figure with a line that lies outside of the figure. Using small wooden blocks, the students investigated the six different irregular figures comprising four cubes, which are the remaining pieces of the Soma Cube puzzle. This module culminated in the students working in groups to paper-fold the seven pieces of the Soma Cube puzzle. Rather than distributing written and diagrammatic instructions for the construction, each group was just provided with a colour photograph of each of the seven pieces. The successful completion of the Soma Cube pieces required that the students utilise their skills in spatial visualisation to construct the three-dimensional object associated with a two dimensional photograph. To culminate the teaching sequence, the whole class collaborated to plan and organise a Paper Folding Expo for parents and students from other classes, where they exhibited the models they had created and demonstrated the skills and understandings they had acquired.

Data Sources and Analysis

Two data sources were utilised to provide evidence of the students' affective, behavioural and cognitive engagement in the mathematical learning inherent in the activities. Firstly, qualitative student self-report data in the form of written reflections were collected at the conclusion of each module in the teaching sequence. In the written reflections, the students were provided with guiding questions designed to elicit their affective, cognitive and behavioural responses to the learning activities. The indicators of student engagement used to analyse the written reflections have been drawn from overlapping elements found in the work of others. Several previous studies have identified indicators of affective engagement in learning activities, such as: enthusiasm, optimism, curiosity and interest (Klem & Connell, 2004), together with enjoyment, happiness and excitement (Marks, 2000). Concentration and persistence have been characterised as indicators of behavioural engagement (Skinner & Belmont, 1993), and student claims to have learned something is one indicator of cognitive engagement utilised in the work of Helme and Clarke (2001). The students' written reflections were initially coded using these previously identified indicators for affective, behavioural or cognitive engagement. Further analysis of the written reflections highlighted additional indicators of student engagement, seemingly absent in previous research. Helme and Clarke (2001), however, recommend that sole reliance on student self-reporting ought to be avoided when seeking evidence of student engagement. Some researchers have also used student work samples to determine engagement in learning tasks (Chapman, 2003). Student work samples, completed in the second and third modules, contribute particularly to the evidence of cognitive engagement in this study. These work samples were analysed to illuminate the students' willingness to explain procedures and reasoning which has been identified as another indicator of cognitive engagement (Helme & Clarke, 2001).

Results and Discussion

Affective Engagement

The most apparent evidence of affective engagement associated with the paper-folding activities was the sense of *enjoyment* that the students expressed: "It is fun and you don't feel as if you are learning maths at all"; and "I didn't even notice it was maths I was having so much fun". For one student paper-folding also enhanced an *interest* in mathematics: "I enjoy it because it is something I wouldn't usually learn because it teaches me many maths skills I didn't learn in school maths". Pride and contentment as indicators of affective engagement are seemingly absent in the findings of previous studies. Yet in this study a number of students expressed their *contentment* upon completing their first cube: "…it was a good moment"; and *pride* in their achievement: "I felt really good and proud of myself".

In reflecting the key principles of the constructionist approach to learning, the creation of artefacts and the opportunities to share them with an audience elicited the most significant expressions of affective engagement. Upon successfully completing their first paper cube many students clearly indicated their *happiness* with the artefact they had created: "I felt happy because it was hard"; and "I felt glad and happy because I thought I was never going to make it". Whilst constructing the first paper cube was a significant accomplishment for many students, completing the Soma Cube pieces and solving the Soma puzzle was the ultimate goal and the *happiness* and *excitement* expressed in the student reflections once this had been achieved was palpable: "I felt really excited and really happy that we accomplished that goal"; "I WAS SO EXCITED!!!!!"; and "Soooo happy! I was laughing so much I cried! The best feeling!". The sense of excitement and achievement in the classroom as one-by-one student groups solved the Soma Cube puzzle was almost euphoric.

The data also exposes the significance of audience in promoting affective engagement. The student responses to the peer teaching experience range from expressions of *contentment*: "I felt good to share my knowledge with others"; *happiness*: "I felt happy because I had taught other people things that could make them happy"; *pride*: "I got a bit frustrated, but in the end I felt proud and glad"; and simply *enjoyment*: "It was fun teaching them". The Paper Folding Expo elicited an equally rich range of indicators of affective engagement in the student reflections. Expressions of *pride* in their achievements were common: "I felt proud that other people looked at what our class had achieved"; and "I felt proud and special because only we knew". Others expressed their *enjoyment* of the occasion: "I really had fun showing my mum and dad things we've made in our paper-folding class"; their *happiness*: "I felt happy that I could show my skills to my family"; and their *contentment*: "I felt good because everyone was congratulating us on our good job".

Behavioural Engagement

The data revealed that, by their own recognition, the paper-folding activities provided the students with a challenge: "It's fun but hard to teach and learn". However, *persistence* was evident as a key indicator of behavioural engagement with the challenges in the learning activities: "It can be difficult the first time but after a while it's easy"; "I felt glad and happy because I thought I was never going to make it"; and "You don't always get it the first time you try". *Concentration* was also a significant factor in the students' willingness to persist in the face of the challenges that arose, and photographs taken during the learning activities provided clear visual evidence of student concentration levels.

Cognitive Engagement

Indicators of cognitive engagement, namely a willingness to explain procedures and reasoning and most evidently student claims they have learned something, provide evidence that the students connected with the mathematics inherent in the paper-folding activities. Many students identified mathematical concepts they had learned: "I learnt about irregular shapes, normal shapes, angles and 3D shapes"; and "I learnt number patterns". The responses also indicated that the students had learned something about the nature of mathematics: "That all maths has patterns"; or "Maths can be stuff other than numbers and sums". Some students identified that they had learned new mathematical skills: “I learned how to make a cube and explain how to do it”; and “I learned how to draw shapes on isometric paper”, whilst others recognised more generic skills they had learned through the paper-folding activities, such as: “How to write instructions”. One student clearly articulated his cognitive engagement by simply declaring “I put my mind to it”. Analysis of the students’ written work samples provided evidence regarding their willingness to explain procedures and reasoning. Most students were prepared to explain the procedures they used in the mathematical investigations, for example how they determined all of the irregular figures that comprised four cubes. Some described a trial and error approach: “Get some blocks and shuffle them around to see what irregular shape I can get”. Others were more systematic: “I would make a regular shape and then move 1 block, then another block and then another block to make irregular shapes. Then I recorded it”.

The written work samples, though, were less conclusive with respect to the students’ willingness to explain their reasoning. For example, the students were asked to predict the total number of squares on the faces of a rectangular prism that comprised four cubes. In extending the growing pattern {6,10,14,…} almost all of the students correctly offered eighteen squares as their response. However, only a few were able to explain their reasoning: “Each time you add on a cube the square faces go up by 4 and the last one was 14 and 14 + 4 = 18”; or “There will be 4 square faces on each side which is 16 and then 2 bottom and top makes 18”. One the other hand, whilst many other students were able to correctly extend the numerical pattern they offered only vague explanations of their reasoning: “I followed the pattern and added 4”; or “The number goes up by 4”. Possible reasons for the unwillingness of the students to clearly explain their reasoning in writing can be found in the work of others. In her study, Warren (2005) encountered similar difficulties in encouraging primary students to rationalise and generalise patterns in writing, instead the students in that study found it easier to verbalise generalisations. Others have also pointed out that encouraging students to write in mathematics can be difficult, and that success in doing so appears to be determined by the students’ ability to express their thinking in the written word (Anderson & Little, 2004).

Sustained Engagement

This study does not make the claim that paper-folding is a guaranteed means of ensuring that mathematics will be more engaging and meaningful for all primary students. Whilst the data in this study have revealed the value of paper-folding in promoting affective, behavioural and cognitive engagement in mathematical learning, not all of the students will wish to approach mathematical learning through this means on future occasions, and nor should they. However, some students in this study continued to explore and extend the classroom learning activities in out-of-class contexts, exposing an additional mode of engagement: sustained engagement. In describing sustained engagement, Eisenberg and Nishioka (1997) use “musical instruments” as an analogy. They explain that whilst most students have the opportunity to try a musical instrument during their schooling, the instrument will become a passion for some. Paper-folding could be similarly considered as a mathematical “instrument” capable of fostering a long-term passion for creative mathematical practices.

Over the course of this study the sustained engagement of some students with the paper-folding activities became increasingly apparent. It was first evident when a group of students asked that I hold optional lunchtime sessions so they could learn to construct more complex models. Included in this group was Tom, a student with a diagnosed learning disability and a form of autism. The collaborating teacher was amazed that Tom not only attended an optional lunchtime session but successfully completed a reasonably complex model with little assistance. Tom had never been known to remain in the classroom for a minute longer than he had to! Several weeks later Tom proudly brought to school a “super-sized” cube that he had designed and decorated at home. Other members of the lunch time group also continued exploring and extending their skills at home with students excitedly bringing into class increasingly complex models, some exquisitely small, and others large and decorative. Matt was a particular student whose sustained engagement with the paper-folding classroom activities transformed into a growing passion for this “mathematical instrument”. Matt’s skills and fascination blossomed over the teaching sequence and in recognising his passion Matt’s parents purchased several of the resource books I had been using in order for him to continue on his paper-folding journey. At the conclusion of the study, Matt’s parents wrote: “It’s been great seeing him come alive with the paper folding”.

Conclusion

The purpose of this study was to investigate the value of paper-folding in the primary mathematics as a means of promoting affective, behavioural and cognitive engagement in mathematical learning. Drawing on overlapping elements in the work of others, a range of indicators were utilised to discern evidence of student engagement in the data sources: written student reflections and student work samples. Some indicators of affective engagement, as identified in previous research, were similarly evident in this study, specifically: enjoyment, interest, happiness and excitement. Further, additional indicators of affective engagement, seemingly absent in the findings of previous studies have also been revealed, namely: contentment and pride. The data have also exposed persistence and concentration as the key indicators of behavioural engagement with the learning activities and the students' willingness to describe their mathematical learning and explain the procedures that they used in the mathematical investigations provide strong evidence of their cognitive engagement with the mathematics inherent in the paper-folding activities. Finally, the study has revealed an additional mode of engagement in some students, sustained engagement, evident when they continued to explore and extend classroom learning activities in out-of-class contexts.

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