Year Five Students Solving Mental and Written Problems: What Are They Thinking?

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This study investigated strategies used by Year five students to solve multi-digit whole number problems mentally and in written formats. The students participated in semi-structured interviews and the think-aloud strategy was used to determine the students' metacognition. The results indicated that students experience more difficulty answering oral questions than written questions. In particular subtraction and division questions proved more troublesome for students. The range and type of strategies used was indicative of the level of cognitive understanding of the students.

The increased interest in mental computation shown by researchers since 1990 is due in part to the perceived connections between strategies used by students and their conceptual understanding. Straker (1999) stated that "the ability to calculate mentally lies at the heart of numeracy" (p. 43). Knowledge and use of mental strategies may assist the development of conceptual understanding of multi-digit number calculations (Fuson et al., 1997). Unlike most written procedures, mental computation requires an individual to use conceptual understanding to select appropriate strategies.

According to Northcote and McIntosh (1999), adults predominantly use mental calculations in everyday life. Despite this knowledge, mental computation remains sidelined in many Australian classrooms (Morgan, 2000). Primary teachers often have little appreciation for the importance of mental computation in the curriculum. Traditional written algorithms continue to be emphasised at the expense of mental computation in the majority of classrooms.

This report describes strategies used by Year Five students to solve problems presented in oral and written formats and is part of a larger study that includes practicing primary teachers and preservice teachers. Although there has been an increased focus on mental mathematics over the past 15 years, many teachers remain unfamiliar with the range of strategies that students use to solve oral problems. Knowledge of strategies used to solve problems in different formats may provide teachers with more information about students' conceptual understanding. The strategies described in the following section are considered to be somewhat hierarchical and therefore indicative of the level of number sense development (Foxman & Beishuizen, 2002).

Literature Review

Mental Computation

Multi-digit addition and subtraction strategies. According to Verschaffel, Greer, and De Corte (2007) knowledge of multi-digit mental computation is not as comprehensive as that of single-digit calculations. Investigation of this area can provide valuable insight into students' conceptual understanding of multi-digit number calculation. Heirdsfeld and Lamb (2005) used the work of noted mathematics researchers including Beishuizen, Reys, and Thompson to describe classes of mental strategies used for addition and subtraction. An excerpt from Heirdsfeld and Lamb's table follows (coding is included for reference in the table of results):

- Counting on or back – 28+35 : 28, 29, 30, (c);
- Separation

1. Right to left - 28+35: 8+5+13, 20+30+50, 63 (sr);
2. Left to right - 28+35: 20+30=50, 8+5+13, 63 (sl);
3. Cumulative sum or difference - 28+35:20+30=50, 50+8=58, 58+5=63 (sc);

- Aggregation

4. Right to left - 28+35: 28+5+33, 33+30=63 (ar);
5. Left to Right - 28+35: 28+30=58, 58+5=63 (al);

- Wholistic

6. Comprehension - 28+35: 30+35=65, 65-2=63 (wc);
7. Levelling - 28+35: 30+33+63 (wl);
    - Mental image of pen and paper algorithm – Child reports using the method taught in class, placing numbers under each other, as on paper, and carrying out the operation, right to left (m).

Multi-digit multiplication strategies. Verschaffel et al. (2007) draw on the work of Baek (1998) to describe four multiplication strategies as follows:

Direct modelling strategy – child uses drawing or manipulatives (dm);

- Complete number strategies – child uses techniques such as doubling or repeated addition (cn);
- Partitioning number strategy – child splits the multiplicand or multiplier into simpler numbers and combines answers at the end (p);
- Compensating strategy – child adjusts the multiplicand and or the multiplier to make the calculation easier (cs).

Multi-digit division strategies. Likewise, Verschaffel et al. (2007) describe Ambrose, Baek, and Carpenters (2003) work on division strategies as follows:

Working with one group at a time – child repeatedly subtracts the smaller number from the larger number (og);

- Split dividend – child carries out division of units, tens and hundreds separately (sd);
- Building up strategies – child combines strategies in a complex manner (b).

Mental Computation versus Written Calculation

Verschaffel et al. (2007) explain that "standard algorithms have evolved over centuries for efficient, accurate calculation and for the most part are far removed from their conceptual underpinnings" (p. 574). Mental calculation strategies, however, are typically closely related to the conceptual nature of the calculation. While most researchers advocate for students to be exposed to both mental and written calculation strategies, an inordinate amount of classroom time focuses on the practice of traditional written algorithms (Verschaffel et al., 2007).

An algorithmic approach to mathematical computations promotes passive responses from students (Thompson, 1999). Mental computation in contrast requires more active participation. Students, when using mental computation, are more likely to thoughtfully engage with the problem and apply conceptual knowledge to assist in the selection of appropriate strategies. According to Callingham (2005) further research is needed into the relationship between algorithms and conceptual understanding in the area of mental computation.

Metacognition

Whilst an in-depth discussion of metacognition is beyond the scope of this paper, some comments are pertinent. According to Panaoura and Philippou (2007), "metacognition is a multidimensional construct with two main dimensions: knowledge about cognition and regulation of cognition" (p. 149). Assessing students' competence in appropriately selecting and using strategies to solve mathematics problems is an important component of metacognition.

Wilson and Clarke (2004) describe metacognition as having three parts: (a) awareness of thinking, (b) evaluation of that thinking, and (c) regulation of that thinking. In order to enhance students' metacognition in mathematics, teachers must have knowledge of the process of strategy choice and application. This understanding will potentially result in more effective selection of problems and instruction by teachers.

Purpose

The study was designed to provide teachers with information about the mental and written strategies that students use to solve multi-digit addition, subtraction, multiplication and division problems. The research questions answered in this paper are:

What strategies do Year Five students use when solving mental and written multi-digit whole number problems?

- What metacogonitve awareness of the strategies used do the students exhibit?

Methods

Study

This study took place in an Australian primary school where the researcher had previously conducted professional development and co-taught activities with some classroom teachers. Following a verbal description of the study by the researcher and teacher (for the students), an information letter was sent to parents/guardians and students volunteered to participate in the research. Students were not selected on the basis of ability.

Eleven Year Five students (four boys and seven girls) participated in semi-structured interviews that were conducted in a small private room by the researcher. Fontana and Frey (2005) describe interviews as "one of the most common and powerful ways in which we try to understand our fellow humans" (pp. 697-698). A semi-structured format was used to provide a basis for comparison between the participants while allowing for some flexibility to cater for individual differences. To this end, all students were asked identical base questions but subsequent interaction varied slightly according to the students' responses. Wilson and Clarke (2004) describe a multi-method interview (MMI) as an appropriate method for assessing metacognition. MMI involves an interview, self-reporting, a think-aloud strategy, observation, and video recording. The method was adapted for this study, as the emphasis needed to be on the strategies used to solve the problems rather than on metacognition.

Students were individually asked a series of four oral questions and four written questions (see appendix). The written questions were presented using a horizontal format (for example, 600 – 35) and students were able to rewrite the questions in any way they wanted to. Each interview was videotaped and transcribed for purposes of analysis.

During the interview, students were asked to solve one question at a time and subsequently asked to describe how they solved the problem. In particular they were asked to describe what they were thinking when they solved the problem. The "Think-Aloud Strategy" provides an insight into cognitive processes and is suitable for problems that require more than an automatic response and therefore necessitate some thinking (Van Someren, Barnard, & Sandberg, 1994). It is suggested that the Think-Aloud Strategy be used for a small set of problems, as the technique can be time consuming.

Analysis

Transcriptions of students' answers were analysed and coded using the framework of strategies defined in the literature review. The interviews were scrutinised for the whole group and for individuals. Patterns of strategies used were examined according to accuracy of answers and the metacognition of the students.

A comparison of students' success with oral and written questions provided another perspective for the analysis. Table 1 details the number of correct answers for all problems and indicates the strategies used for the oral questions. In order to maintain anonymity, pseudonyms have been used for student names.

Results

Students were able to provide descriptions for a vast majority of questions, although there were five instances when students could not describe the strategies used. In two of these cases students were unable to answer the question, probably due to lack of conceptual understanding. On the three other occasions, students successfully answered questions but could not describe how they had solved the problems.

Some patterns are discernable from the type of questions answered correctly. The students were far more capable of using mental strategies to solve addition questions than subtraction questions. Likewise oral multiplication questions were answered more successfully than division questions. It is interesting to note that these features were not evident for the written questions.

While an examination of the strategies used by students does not reveal any clear pattern, there are many interesting observations to be made. Seven of the nine students who described a strategy for the oral subtraction question chose to use a separation strategy. In contrast, only four students selected the separation strategy for the oral addition question. The students appeared to experience more difficulty with the subtraction problem and resorted to the lower level separation strategies. All but one of the students correctly answered the addition question and a wider range of strategies was used by the group. In fact four students selected the higher level aggregation or wholistic strategies.

The partitioning strategy was favoured by the majority of students for the oral multiplication question. Of the three students who gave an incorrect answer for the multiplication question, one used the lower level complete number strategy and one attempted to mentally use the written algorithm. The oral division question proved to be the most troublesome for students. Reasons for incorrect answers for the mental division question can be categorised as follows: (a) inaccurate multiplication tables knowledge; (b) unsuccessful attempt to mentally use the written algorithm; and (c) lack of conceptual understanding of division.

Table 1

Summary of Students' Results

Name	Oral -	Oral +	Oral x	Oral /	Written -	Written +	Written x	Written /	Total
Alan	N (sr)	Y (wl)	N (cn)	N (n)	N	Y	N	N	2
Lisa	N (c)	Y (ar)	Y (p)	N (dm)	N	Y	Y	N	4
Serina	Y (sr)	Y (al)	N (p)	N (dm)	N	Y	N	Y	4
Teresa	N (sl)	N (sl)	N (m)	Y (b)	Y	Y	Y	Y	5
David	Y (sl)	Y (sl)	Y (p)	N (n)	Y	Y	Y	N	6
Robert	N (sl)	Y (sl)	Y (p)	N (t)	Y	Y	Y	Y	6
Alice	N (n)	Y (n)	Y (p)	Y (t)	Y	Y	N	Y	6
Georgia	Y (n)	Y (sl)	Y (p)	N (t)	Y	Y	Y	Y	7
Leanne	Y (m)	Y (m)	Y (p)	Y (t)	Y	Y	Y	Y	8
Helen	Y (sl)	Y (m)	Y (m)	Y (t)	Y	Y	Y	Y	8
Edward	Y (sr)	Y (wl)	Y (p)	Y (t)	Y	Y	Y	Y	8
Total	6	10	8	5	8	11	8	8	64

Y = correct answer N = incorrect answer (code) = uses the strategy code defined in the literature review (n) = no explanation given, (t) =multiplication tables, codes such as (m) were used across a range of questions

Students answered more of the written questions correctly than the oral questions. Only 11 of the 22 oral subtraction and division questions were answered correctly. In contrast, 19 of the 22 written subtraction and division questions were answered correctly. Students appeared equally capable of answering oral and written addition and multiplication questions.

Discussion

Success in using written algorithms is not necessarily indicative of students' conceptual knowledge. This is illustrated by the different success rates for oral and written subtraction and division questions. From descriptions given of strategies used for the oral questions, some conceptual misunderstandings were evident. These same students however, correctly answered written forms of the questions and described in detail how to use an algorithm.

Lisa's Response to Questions One and Five

Researcher: A chocolate frog costs about 45c. How much change would you get from $5?

Lisa: $4.95… I just took, I went forty-five, then I went all the way to a hundred and then I just counted and took $1 off.

Researcher: Can you work out the answer to this question? (shows written form of 600-35)

Lisa: (Lisa rewrote the problem in a traditional written vertical ormat) 565 … With the zero you can't do it so you have to borrow a number, except the next number is zero, so I went to the six and then I went to the five. Then I brought the one across and then I cross that out and that became nine and then I took one and added it on to zero. Then I took nine and take three which is six and five take zero, which is five.

There appeared to be some connections between students' success rates with questions and the ability to clearly describe the strategies used to answer oral questions. Panaoura and Philippou (2007) concluded that more capable students exhibit better metacognitive skills**.** The following example illustrates the metacognition of a student who correctly answered all eight questions.

Edward's Response to Question Two

Researcher: There is a school trip for two classes and they need to know how many students will be on the bus. Mr T's class has 19 students and Mrs D's class has 27 students. How many students altogether?

Edward: 47 … I took three from nineteen to get thirty, so it is easier to work out. It's actually forty-six.

This response also supports the theory that more capable students tend to select higher level strategies (Foxman & Beishuizen, 2002). In this example, Edward used a wholistic levelling strategy whereby he subtracted three from nineteen and added three to twenty seven (19-3 = 16, 27 + 3 = 30, 16 + 30 = 46). Conversely, students producing few correct answers tended to select lower-level strategies to answer questions. Alan only successfully answered two questions and his response to question three exemplifies this notion.

Alan's Response to Question Three

Researcher: A local team scored 14 goals. How many points did they score?

Alan: 100 … I did it a kind of long way. I kept adding six until I got to fourteen.

It is interesting to note that some of the more capable students chose to use mental strategies for selected written questions. For example, Georgia (who gave seven correct answers) used a mental strategy to answer question five.

Georgia's Response to Question Five

Researcher: Can you work out the answer to this question? (shows written form of 600-35)

Georgia: 565 … I know that hundred take thirty is seventy and take five is sixty-five and then I just add five hundred.

An unexpected observation was the fact that some capable students solved the oral questions by visualising a written sum in their heads. This is illustrated by Leanne's description of the strategy she used to answer questions one and two.

Leanne's Response to Questions Two

Researcher: There is a school trip for two classes and they need to know how many students will be on the bus. Mr T's class has 19 students and Mrs D's class has 27 students. How many students altogether?

Leanne: I think that would be forty-six students.

Researcher: You worked that out in your head very quickly, how did you do that?

Leanne: Well in my head I pictured a sum and I added the two together and I got an answer.

Some struggling students used written strategies to solve oral problems, because they did not appear to have a repertoire of mental strategies. In contrast to Foxman and Beishuizen's (2002) findings, the type of strategies used do not always equate to a student's mathematical ability. There are numerous factors, including background experiences and teachers that impact the choice of strategy to solve oral questions.

Conclusion

This was a small-scale study and it is important therefore the results are not generalised. It is true, however, that the results do support the findings of previous research described in the paper. Students' conceptual understanding largely corresponds to the strategy choice described to solve multi-digit whole number problems mentally. Furthermore, cognitive ability is evident in the students' metacognitive fluency.

The lack of skills and understanding of multi-digit subtraction and division displayed by some Year five students is concerning. Poor conceptual knowledge has been masked by their ability to perform written calculations. This deficiency has implications for the students' future success with mathematics.

The next step in this study will engage practicing primary teachers with the data. Teachers will view the DVD of the students being interviewed and discuss the strategies used and the implications this has for teaching. The role of mental computation in developing conceptual understanding will be explored by the teachers. Following this, the teachers will develop pedagogy to effectively incorporate mental computation into the classroom.

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Appendix

Verbal Questions

1. A chocolate frog costs about 45c. How much change would you get from $5?
2. There is a school trip for two classes and they need to know how many students will be on the bus.
   Mr T's class has 19 students and Mrs D's class has 27 students.
   How many students altogether?
3. How many points in a football goal? (answer given if the student did not know this)
   A local team scored 14 goals. How many points did they score?
4. You have 56 lollies to share with 6 of your friends. How many will you each receive?

Written Questions

5. 600 – 35 =
6. 6. 79 + 33 =
7. 17 x 6 =
8. 84 / 7 =

Following each question the students were prompted to describe how they solved the problem. Some of the prompts used were:

- How did you work that out?
- Can you tell me what you thought in your head when you worked out the answer?
- How did you know what to do?
- How could you check your answer?