Advancing Research Into Affective Factors in Mathematics Learning: Clarifying Key Factors, Terminology and Measurement

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The literature on affective factors in the learning of mathematics is difficult to interpret because of differences and inconsistencies in terminology and measurement. To advance research in this field of affect, I compare and clarify terminology, and reconcile scales for measurement by examining the factors and research instruments targeted by four research teams. The findings reveal two distinct broad primary areas of interest, namely self-concepts about mathematics, and intrinsic motivations for learning mathematics. The instruments used to measure a range of underlying factors within these two areas are analysed and reconciled, terminology is clarified, and further recommendations are made.

It is both timely and imperative to explore the potential of self-beliefs and attitudes to inform educational planning and practice. It is known that self-concepts and related attitudes are important influences in the learning choices students make, and play a role in learning behaviour and performance. For example, in a study of the factors that influenced more than 500 first-year Australian students, Cretchley, Fuller, and McDonald (2000) found that self-beliefs about mathematics ability were a major influence behind their choice to study mathematics at university. Given the need to attract students to mathematics, the legacies of low mathematics self-esteem and low interest in mathematics are serious.

Despite the need, there have been few attempts to clarify the role of mathematics self-concepts and attitudes in learning. A number of researchers have explored students' attitudes via case studies and journal entries, but few have accepted the challenge of quantifying affective factors, exploring relationships with learning approaches and progress, and monitoring changes. Reviewing literature in the field of affect in mathematics education, Leder and Grootenboer (2005) found '*few studies in which the difficult task was attempted of exploring the relationship between affect and a range of other important factors including cognition, learning and achievement.'*. This neglect is easy to understand, given the difficulties researchers face in this new field: theories not yet well-developed, terminology used differently and ambiguously, and varying research instruments, some untested, make the literature difficult to interpret, and leave researchers open to criticism.

Research findings also vary. Correlations between affective factors and performance vary widely. Leder and Grootenboer described *'tantalizing'* findings and *'provocative glimpses'* of the interaction between affect, teaching, and learning. and causal directions found in relationships between affective and cognitive learning factors are inconclusive, and little has changed since Marsh (2002) argued that early research supported a model of '*reciprocal effects'*, with prior academic self-concept having a positive effect on subsequent achievement beyond that which can be explained in terms of prior academic achievement; and vice versa, subsequent academic self-concept is affected by priori achievement beyond what can be explained in terms of prior academic self-concept.

Nevertheless, some reports find significant relationships between self-concepts and achievement. Cretchley and Galbraith (2003) found Pearson correlation coefficients of up to 0.60 between mathematics confidence and performance on a range of different types of assessment tasks. It is clear that there is much still to be learned about self-concepts and other affective factors related to learning. To facilitate understanding of students' self-beliefs and learning attitudes, to investigate the ways in which these shape learning behaviour, and to ensure that the research literature is accessible to educators, there is an urgent need to advance the field. This research report aims to:

- identify affective factors that are important for research into mathematics learning;
- examine instruments for their measurement;
- clarify the terminology.

Self-Concept Terminology and Measurement

Researchers and educators frequently refer to students' self-beliefs about their ability to do or learn mathematics as *mathematics confidence*. Some use the terms confidence and self-efficacy synonymously . Clearly we need to distinguish.

Probably because *confidence* generally is widely understood to refer to self-beliefs and judgments about one's capabilities, definitions of mathematics confidence are hard to find. *Self-efficacy* is a term coined relatively recently by the architect of social cognitive theory, Albert Bandura (1977, 2005) for *'people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances*'.

Bandura's theories propose that there are hierarchies of increasingly context specific self-efficacies, and that individuals have multiple self-efficacies. But he warns strongly that measures of self-efficacy must be closely task-specific. This warning surely provides the strongest means of distinguishing between confidence and self-efficacy, for an examination of the literature reveals that measures of mathematics confidence generally tap broader self-concepts. However, the term confidence is often used within specific contexts, thus making the need for the term self-efficacy highly questionable. We can speak rather of task or context-specific confidence.

How and why do education researchers measure task-specific or context-specific mathematics self-efficacy or confidence? Journal entries, and written or verbal responses to open questions are frequently used to gather qualitative data. In order to quantify and/or monitor mathematics attitudes and confidences however, most researchers use Likert-style questionnaires. For example, asking students to indicate a level of confidence for each task performed provides a valuable indicator/warning of areas in which a student is over or under confident. Aggregated over a topic, indicators of that kind provide a measure of self-efficacy for that set of tasks. Such task-specific measures may not be broad indicators of mathematics self-confidence and feelings, however. Most commonly, confidences are assessed by means of Likert-style self-response scales that invite learners' responses to a set of statements/items. The items are designed to tap underlying factors/constructs that have been chosen for investigation by the researchers, and may vary from context to context.

The shortcomings of self-report instruments are widely known and reported. Importantly, it is widely acknowledged that the wording of items is open to different interpretations by respondents and researchers. Equally importantly, researchers know that respondents may consciously or unconsciously answer with bias or not be frank. Nevertheless, self-report scales remain the most common means of measuring self-concepts because they are phenomenological by nature, and because such instruments enable timely and easy data capture. Clearly, such data must be interpreted with awareness of the shortcomings, and reporting appropriately.

The online ETS and Buros research instrument databases contain some that are not Likert-style. All are of the types noted in Keith and Bracken's (1996) review of self-concept instruments: namely *checklists, Q-sorts and free response questions.* Free response instruments invite respondents to complete partial statements or answer open questions. These do not provide quantitative measures, but offer a rich breadth of views. Data of this kind are often used to develop items for self-report scales. Checklists are useful for gaining a broad qualitative description of the respondent who indicates which items he believes apply to him from a list of descriptors. Q-sorts may be used to establish the level of agreement with each descriptor - the respondent sorts the descriptors into piles that are most and least like him. By far the majority of instruments are Likert-style, however, and hence this study explores and reconciles such instruments via the work done by four sets of research teams.

Analysis of Four Current Sets of Research Instruments

The instruments chosen for this study constitute a substantial body of work in this field by researchers from the cognate disciplines of psychology, mathematics, and education, namely, Galbraith and Haines (2000), Pierce, Stacey, and Barkatsis (2006), Tapia and Marsh (2004), Fogarty, Cretchley, Harman, Ellerton, and Konki (2001). All are self-report scales that invite Likert-style responses, all have been subjected to substantial trial and analysis, and all are currently in use for research into secondary or tertiary mathematics learning.

These instruments advance the early work of Fennema and Sherman (1976) who developed the field significantly. Their nine scales comprised 108 self-report items and measured what they called *confidence in learning mathematics, mathematic anxiety; motivation for challenge in mathematics (effectance motivation), mathematics usefulness, attitude towards success in mathematics, mathematics as a male domain, mother/ father support* (2 scales), and *teacher support*. Because *Anxiety* data correlate strongly with *Confidence,* the *Anxiety* scale was dropped. Over three decades, researchers have further refined those scales, but what factors are now targeted as important in mathematics learning?

- Galbraith and Haines developed instruments for use with undergraduates. Three of these are called *mathematics confidence*, *mathematics motivation,* and *mathematics engagement* scales. Computer attitude scales were developed at the same time.
- Fogarty, Cretchley, Harman, Ellerton, and Konki analysed a *mathematics attitudes* instrument designed for use with undergraduates, calling it *mathematics confidence* after the dominant factor. Technology attitude scales were also developed.
- Tapia and Marsh developed what they termed *self-confidence, value of mathematics, enjoyment of mathematics,* and *motivation* scales.
- Pierce, Stacey, and Barkatsas developed *mathematics confidence, behavioural engagement* and *affective engagement* scales for use at school level.

All four teams clearly target what they term *mathematics confidence, motivation, and engagement*. But their instruments differ substantially in wording and length, and they tap mathematics self-concepts, feelings, and beliefs about learning preferences and behaviour under different labels. To reveal the nature and range of the underlying factors tapped by the scales, I classified the items in these instruments into groups that tap similar factors. This strategy reveals the emphases placed by the different researchers, and the nature of their mathematics confidence, motivation, and engagement scales. The classification is subjective to some degree, but the items group readily into ten factors: *innate talent and other mathematics self-concepts (including learning confidence or self-efficacy), anxiety, interest, enjoyment, intellectual stimulation, reward for effort, diligence, valuing mathematics, willingness to do mathematics, and approaches to learning mathematics.*

To facilitate comparison of the statements, Table 1 offers my grouping of the scale items into these factors. Items from the teams' Mathematics Confidence scales are labeled MC, those from Mathematics Motivation scales are labeled MM, and those from Mathematics Engagement labeled ME. Affective Engagement items are labeled AE, and Behavioural Engagement items BE. A few items could be classified in more than one group. Three Mathematics Engagement (ME) statements group well under *diligence* along with the Behavioural Engagement items. Other ME items tap a range of approaches to learning and doing mathematics. Items from the Value scale are labeled VM.

Table 1

Grouping of Scale Items Into Factors and Labels

Factor	Item	Label
Talent, confidences and self-efficacies	*I do not have a mathematical mind.* *I am not naturally good at mathematics.* *I have a mathematical mind.* *I have a lot of confidence when it comes to mathematics.* *I am confident with mathematics.* *I find mathematics confusing.* *When I have difficulties with maths, I know I can handle them.* *I know I can handle difficulties in mathematics.* *I can get good results in mathematics.* *It takes me longer to understand maths than the average person.* *I have never felt myself able to learn mathematics.* *I have less trouble learning mathematics than other subjects.* *Having to learn difficult topics in mathematics does not worry me.* *No matter how much I study, maths is always difficult for me.* *I am able to solve mathematics problems without too much difficulty*.	MC MC MC MC MC MC MC MC MC MC MC MC MC MC MC
Anxiety	*I find mathematics frightening.* *The prospect of having to learn new maths makes me nervous.* *I am more worried about mathematics than any other subject.* *Studying mathematics makes me feel nervous.* *I am always under a terrible strain in a math class.*	MC MC MC MC MC
Interest	*I find many mathematics problems interesting and challenging.* *I can become completely absorbed doing maths problems.* *I am interested to learn new things in mathematics.*	MC MM AE
Enjoyment	*I don't understand how some people seem to enjoy spending so much time on maths problems.* *I enjoy trying to solve new mathematics problems.* *I have never been very excited about mathematics.* *I don't understand how some people can get so enthusiastic about doing maths.* *Mathematics is a subject I enjoy doing.* *Learning mathematics is enjoyable.*	MC MC MC MM MM AE
Intellectual stimulation	*I get a sense of satisfaction when I solve mathematics problems.* *I like to stick at a maths problem until I get it out.* *Having to spend a lot of time on a maths problem frustrates me.* *The challenge of understanding maths does not appeal to me.* *If something about mathematics puzzles me, I would rather be given the answer than have to work it out myself.* *If something about maths puzzles me, I find myself thinking about it afterwards.*	AE MM MM MM MM MM
Reward for effort	*Maths is a subject in which I get value for effort.* *In mathematics you get rewards for your effort.*	MC AE
Diligence	*If I make mistakes, I work until I have corrected them.* *If I can't do a problem, I keep trying different ideas.* *I concentrate hard in mathematics.* *I try to answer questions the teacher asks.* *When learning new mathematics material I make notes to help me understand and remember.* *I don't usually make time to check my own working to find and correct errors.* *I find it helpful to test understanding by attempting exercises & problems.*	BE BE BE BE ME ME ME

Valuing mathematics	*Mathematics is important in everyday life.* *Mathematics is one of the most important subjects for people to study.* *High school math courses would be very helpful no matter what I decide to study.*	VM VM VM
Willingness to do mathematics	*I would like to avoid using mathematics in college.* *I am willing to take more than the required amount of mathematics.* *I plan to take as much mathematics as I can during my education.*	MM MM MM
Approaches to learning mathematics	*I prefer to work on my own than in a group.* *I like to revise topics all at once rather than space out my study.* *I prefer to work with symbols (algebra) than with pictures (diagrams& graphs).* *When studying mathematics I try to link new ideas to knowledge I already have.* *I find working through examples less effective than memorizing given material.*	ME ME ME ME ME

Findings

The four sets of instruments differ substantially in number of items, style of wording, and scope. Two sets balance positive and negative statements to counter bias and careless response, and they tap a few factors with broad scales. For example, Fogarty et al's single 11-item scale assesses self-concepts, confidence, and motivation. The other two teams tap fewer factors, each with just 3 or 4 statements, a strategy favoured for children's scales.

Mathematics Confidence and Self-Concepts

The four *mathematics confidence* scales (so-called) are substantially different, varying from narrow to broad. But all four position broad mathematics self-concepts as central.

- All four scales tap a range of self-beliefs about *ability to do and learn mathematics*.
- Three include self-beliefs about *talent* or innate ability.
- Three include feelings of *anxiety.*
- Two tap mathematics *learning self-concepts*.
- The broadest scale includes *interest, enjoyment* and *excitement,* which others tap under what they call *motivation* or *affective engagement*.

Other Motivations to Do Mathematics

The *mathematics motivation* scales (so-called) are also substantially different.

- Galbraith et al's *mathematics motivation* scale taps the following factors: interest in mathematics, enjoyment of mathematics, and intellectual stimulation (including elements that Pierce et al term *affective engagement)*.
- Pierce et al's affective engagement scale taps enjoyment, interest, intellectual stimulation, which Galbraith tap under motivation. It also taps reward for effort, which Galbraith includes under mathematics confidence. The behavioral engagement scale taps diligence, which may also be a motivational factor.
- Fogarty *et al*'s broad *mathematics confidenc*e instrument is better termed *mathematics attitudes* because alongside self-concepts, it taps motivation factors.
- Tapia & Marsh *motivation* scale taps something different: intentions to avoid or choose mathematics.

It is not surprising that the underlying factors are grouped and termed differently by the researchers: Cretchley et al., (2000) and others have shown that self-concepts are also motivators for doing and learning mathematics. Figure 1 shows the areas of overlap in the factors tapped by the four sets of scales analysed here, labelled T, P, G, and T for brevity.

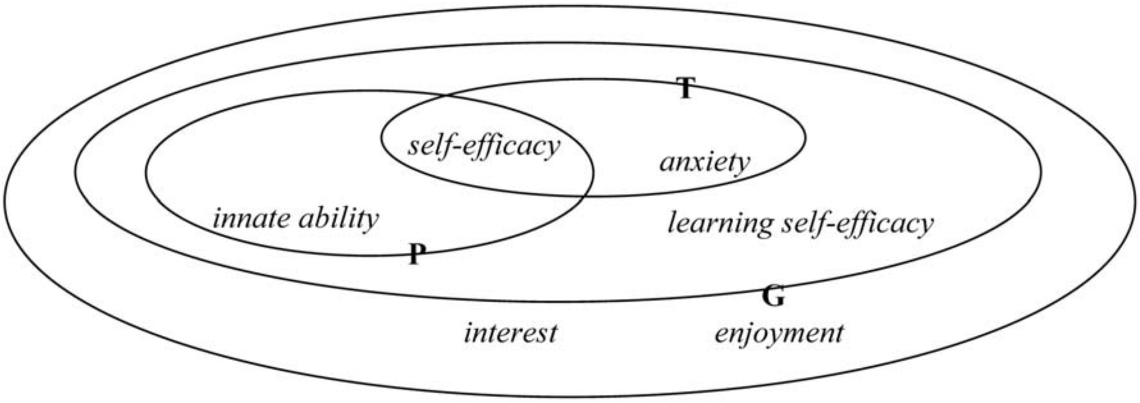

Figure 1: Venn diagram of self-beliefs tapped by four different *mathematics confidence* instruments labeled P, T, G, F.

In summary, these current researchers into affect have placed emphasis on mathematics self-concepts and other motivations for doing mathematics.

- Self-concept factors targeted for research are *talent, confidences*, *self-efficacies* and *anxieties*.
- Other motivation factors targeted for research are *interest, enjoyment, intellectual stimulation*, *reward for effort, valuing mathematics,* and *diligence*.
- Other factors targeted for research are willingness to study mathematics, and approaches to learning mathematics.

Summary and Discussion

While valuable work on affect in mathematics learning has been done with qualitative data, few studies have taken on the difficult task of quantifying and monitoring key affective factors, and assessing their role in mathematics learning. Difficulties with terminology and measurement make research difficult and hinder interpretation of the literature. The field needs accessible terminology and research instruments. To advance this work, this study investigated the factors currently targeted for research into mathematics learning by four experienced research teams, and their research instruments. Analysis revealed two primary areas of research interest: *mathematics self-concepts* and *intrinsic motivations to do mathematics*. Secondary areas targeted by some of these researchers were *willingness to study mathematics* and *approaches to learning*. The underlying factors investigated in these two primary areas are as follows:

- self-concept factors: mathematics talent, confidence, self-efficacy, anxiety.
- other motivational factors: interest, enjoyment, intellectual stimulation, reward for effort, valuing mathematics, diligence.

Terminology

The investigation revealed that scale labels like *confidence, motivation, engagement* are used differently and often too briefly for clarity. In particular, different levels of specificity in the ways in which confidence and self-efficacy have been measured may explain the variations reported in the literature to date (Carmichael & Taylor, 2005). The terms *self-concept, confidence,* and *self-efficacy* need clarifying, and much in the literature (Bandura, 2005; Marsh & Hattie, 1996) supports using these terms as below, general to specific.

- *Self-concepts* refer to the full range of self-beliefs about abilities and potentials to do and learn mathematics, from broad and innate to very specific;
- *Self-confidence* (usually termed just *confidence*) refers to self-beliefs about abilities to do and learn mathematics in some context, not necessarily generally. Hence a learner may be confident within one area of mathematics, but perhaps not another.
- *Self-efficacy* refers to self-beliefs about the abilities to perform specific tasks, in line with Bandura's position that its measurement be closely task-specific. Hence, a student may have high level of self-efficacy for factorizing a quadratic polynomial, but a low level for a cubic.

Proposals for Research and Measurement

The items used by these four research teams provide a selection from which scales can be built for research into *mathematics self-concepts* and a range of *intrinsic motivations* to do mathematics. Ideally, however, self-report measures of this kind should be supplemented with data of other types, some of which are listed in this report. More specific recommendations are as follows:

- Scales measuring *willingness to study further mathematics* need to be developed. Tapia and Marsh's *motivation* scale comprises just three items tapping this construct.
- Research and theory position beliefs about innate talent as largely entrenched, confidences as less so, and self-efficacies as highly contextual. Hence, for monitoring self-concepts over a learning intervention, a broad *confidence* instrument is recommended. For identifying which tasks are associated with a lack of confidence or *self-efficacy*, a question should be asked for each task.
- Monitoring levels of mathematics self-concept and/or intrinsic motivation is clearly different to identifying the dominant factors for an individual or sample, which may vary with context. A broad scale tapping the factors identified here, *interest, enjoyment, intellectual stimulation*, *reward for effort, valuing mathematics,* and perhaps *diligence*, will address a valued range.

Research into mathematics attitudes and their impact on learning needs to extend way beyond naïve tests of their correlation with performance. Research has already shown the latter are variable and contextual. Deeper understanding of the nature and depth of learners' attitudes to mathematics informs course development and classroom practices.

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