Eliciting Growth in Teachers' Proportional Reasoning: Measuring the Impact of a Professional Development Program

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Proportional reasoning is required to operate in many mathematical domains in the middle years' curriculum. It is also a major connecting theme across both mathematics and science. Working together, middle years teachers have the potential to promote students' proportional reasoning through integrated learning experiences. However, building awareness of the connections between these two curriculum domains is an important first step. This paper reports on one aspect of a large project exploring the connections between mathematics and science curriculum. In this paper, first steps to eliciting teachers' pedagogical content knowledge in relation to proportional reasoning are discussed.

Background

Proportional reasoning is the key to understanding and operating in many domains in the mathematics curriculum in the middle years of schooling (typically Years 4-9). Fractions, percentages, ratios, decimals, scale, algebra, probability require proportional reasoning. Proportional reasoning means being able to understand the multiplicative relationship inherent in situations of comparison (Behr, Harel, Post, & Lesh, 1992). The study of ratio is the foundation upon which situations of comparison can be formalised, as a ratio, in its barest form describes a situation in comparative terms. For example, if a container of juice is made up of 2 cups of concentrated juice and 5 cups of water, then a container triple the size of the original container will require triple the amounts of concentrate and water (that is, 6 cups of concentrated juice and 15 cups of water) to ensure the same taste is attained. Proportional thinking and reasoning is knowing the multiplicative relationship between the base ratio and the proportional situation to which it is applied. Further, proportional reasoning is also dependent upon sound foundations of associated topics, particularly multiplication and division (Vergnaud, 1983), fractions (English & Halford, 1995) and fractional concepts of order and equivalence (Behr, et al., 1992). Although understanding of ratio and proportion is intertwined with many mathematical topics, the essence of proportional reasoning is the understanding of the multiplicative structure of proportional situations (Behr, et al., 1992).

Not only is proportional reasoning fundamental to many mathematics topics that students study in the middle years of schooling, it also underpins many middle school science topics (e.g., density, speed, acceleration, force, molarity, machines). The topics of ratio and proportion are typically studied in mathematics classes, and ratio and proportion in fact, have been described as the cornerstone of middle years' mathematics curriculum (Lesh, Post, & Behr, 1988). However, research has consistently highlighted students' difficulties with proportion and proportion-related tasks and applications (e.g., Behr, Harel, Post, & Lesh, 1992; Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Lo & Watanabe, 1997), which means that many students will struggle with topics within both the middle years mathematics and science curriculum due to their lack of understanding of ratio and proportion.

Proportional reasoning, as a major connecting theme across mathematics and science, suggests the potential of integrated teaching, a key principle of middle years' reform philosophy (e.g., Pendergast & Bahr, 2005). The potential is apparent for mathematics and science teachers to mutually support each other in developing rich learning experiences to promote students' proportional reasoning skills.

The Context

A current research project being undertaken by the authors brings together middle years' mathematics and science teachers around this important topic, with the intention to provide an opportunity for teachers to explore the proportional reasoning linkages between topics in both mathematics and science, and to create, implement and evaluate innovative and engaging learning experiences to assist students promote and connect essential mathematics and science knowledge. One of the main aims of this project is to promote teachers' mathematics and science content knowledge around new curriculum in these disciplines, as well as their knowledge for teaching (pedagogical content knowledge). A targeted program of professional learning is being incorporated within the teacher meetings, in which new ideas and ways of teaching to connect essential learnings in mathematics and science are shared with teachers.

In this project, the impact upon teachers' understanding of proportional reasoning is of major interest to the researchers. A current trend in evaluating the impact of professional development programs for teachers has been the creation of instruments designed to measure teachers' knowledge for teaching (e.g., Baker & Chick, 2006; Hill & Ball, 2004; Watson, Beswick, Caney, & Skalicky, 2005/6), based specifically around pedagogical content knowledge (Shulman, 1986). Such research studies have provided suggestions for the development of pen and paper surveys and questionnaires to gather data on teacher professional learning. One of the key research questions in this project, and the focus of this paper, is as follows:

How might teachers' proportional reasoning and knowledge of related key ideas be profiled, and how does this knowledge relate to their classroom practice?

The Study

In this project, a Background Teacher Survey (BTS) was designed along similar lines to Hill and Ball's (2004) instrument for measuring teacher knowledge for teaching mathematics. As emphasised by Hill and Ball (2004), knowledge for teaching cannot be equated to one's level of content knowledge, but encompasses knowledge of students, knowledge of how students learn, and knowledge of strategies for improving learning. The survey also drew upon Shulman's (1986) descriptions of four elements of knowledge for teaching: subject matter knowledge, pedagogical-content knowledge, curricular knowledge and knowledge of students as learners. The survey contained classroom scenarios and asked teachers to comment on given student responses.

The time required for teachers to adequately review and reflect upon given students' responses was a major factor in considering the number of scenarios presented. For each scenario, between four and five student responses were developed. If teachers spent approximately 5 minutes analysing and responding to each of the given student responses, each scenario would require approximately 20-25 minutes minimum for analysis. Also, the content of the scenarios was a further factor in development of the scenarios. Because this project is about the connections between mathematics and science curriculum in the middle years of schooling, the authors were keen to develop scenarios that equally favoured each curriculum area. Taking the number of student responses and content of the scenarios into consideration, the resulting survey included two classroom scenarios, one about making mixtures (recipe) and one about density (floating balloon baskets). Even though both topics equally fall within the science curriculum, the first scenario was regarded as more common-place in a mathematics classroom and the second as more linked to a science classroom. The given number of student responses was five and four respectively for the two tasks.

The issue of asking teachers to undertake a 'test' of their knowledge for teaching was a further major factor in the development of the survey. This issue was identified by Watson, Beswick, Caney, and Skalicky (2005/06) who reported on the reluctance of teachers to complete a pen and paper profile as a component of a mathematics teacher professional development project, and has also been highlighted by Hill, Sleep, Lewis, and Ball (2007) as a major limitation of pen and paper tests for teachers. Early tests of teacher knowledge, as summarised by Hill et al. (2007), were unashamed tests of mathematics content, with more recent tests designed to assess content as well as pedagogy, often in the form of a specific mathematics topic presented with a series of approaches to teaching the topic, or a student's erroneous response to a mathematics item with a series of ideas for teaching to assist the student overcome his/her misconception (e.g., Hill, Schilling, & Ball, 2004). Such multi-choice items can result in respondents locating the 'best' response through a process of elimination which provides little insight into their pedagogical content knowledge. An alternative approach was taken by Watson, Beswick, Caney, and Skalicky (2005/06) with the presentation of a classroom task (e.g., 90% of 40) with teachers required to (a) list as many appropriate and inappropriate ways students might solve the task, and (b) describe how the task might be used in the classroom.

Taking these issues into consideration, we wanted to move away from multiple choice items that forced teachers to look for the ‘best’ thinking from a list of given responses, and we also wanted teachers spend time thinking specifically about students’ proportional reasoning. Hence, our survey provided teachers with specific responses from students to direct their analysis. We were hoping that provision of specific student responses to given classroom scenarios rather than a multi-choice format would also lessen the potential of the survey being directly interpreted by teachers as a ‘test’ of teacher knowledge. In the instructions for completing the survey, we informed teachers that we hoped the classroom scenarios would raise issues around the teaching of proportional reasoning that are useful for the project, and that their comments and ideas would help us in our planning for the rest of this project.

Despite these considerations, during construction of the survey there was a considerable feeling of being impertinent in asking teachers to display their knowledge for teaching, particularly as teachers in Australia do not have a tradition of being formally assessed on their mathematics knowledge and understanding of teaching. The resulting survey is entitled innocuously as the Background Teacher Survey and specifics of the survey are presented in the next section.

The Background Teacher Survey

The survey BTS is a pen and paper survey containing two classroom scenarios in which students are engaged in tasks requiring proportional reasoning. A problem is posed to the students in each scenario and a series of student responses to the problem are presented. To complete the survey, teachers are required to comment on the given responses, providing reasons for why they think the students answered the way they did. The first scenario is entitled *Sticky Mess* and describes a cooking activity where students must alter a recipe to make a bigger mixture to the one given in the original recipe. The second scenario is entitled *Ballooning* and describes students making hot-air balloons using balloons and baskets of various sizes and recording those that fly and those that do not, with class results presented as a data set and the mass and volume of the teacher’s untrialled balloon given. After teachers have commented on the given students’ responses to each scenario, they are asked to complete the following three questions in relation to the Ballooning scenario:

a) Place the students’ responses in order of increasing quality.

b) Explain the criteria you used for your ranking.

c) Do you see this as a mathematics or a science lesson (or both?). What types of concepts potentially could be developed through such a lesson? Please elaborate.

Participants

Fourteen teachers from six different schools completed the BTS both at the start of the year and at the end of the year. All teachers were teachers of students in the middle years of schooling, but came from a broad range of schools and teaching situations. Some teachers taught secondary mathematics classes only, some taught secondary science classes, some taught both secondary mathematics and science classes, and some were primary teachers teaching both mathematics and science. The schools include single-sex schools, low socio-economic status schools, high socio-economic status schools, Catholic schools, Independent schools, and State schools. The teaching experience of participating teachers ranges from less than one year’s teaching experience to over 25 years teaching experience.

Survey Administration

On the first day of the meeting with teachers, the BTS was distributed. The teachers were asked to individually work through the presented scenarios, making notes and writing comments as requested without sharing their ideas with anyone else. Teachers were assured that their comments on the surveys would only be read by project personnel, and their surveys would be de-identified by replacing their name with a code. It was suggested that teachers should aim to spend about 35-40 minutes on the survey.

Surprisingly, all teachers approached the survey in a positive manner with no verbal objections raised or further questions asked. The teachers completed the survey in silence and appeared to take a serious approach to completing the survey individually and comprehensively. All teachers managed to complete the survey in the allotted time.

At the end of the first year of the project, and after the teachers had attended 5 full professional development days and 4 afternoon workshops throughout the year, the teachers completed the BTS a second time. The teachers approached the survey in the same manner as noted in the first administration. Several teachers who did not complete the first survey but who were present for the second survey completed the survey for the first time, but their responses are not included in this data set.

Results

The first item, *Sticky Mess,* provided teachers with five student responses, only one of which was correct. The *Sticky Mess* scenario is that a recipe requires 4 cups of sugar and 10 cups of flour, but a larger amount is made with 6 cups of sugar and students must determine how many cups of flour are required for the new mix. Duane suggests that the answer is 12 cups, noting that an increase of 2 cups of sugar will require an extra 2 cups of flour. Eva suggests that the answer is 15 because you need 2½ times as much flour as sugar. Ivy suggests that the answer is 18 because you double 4 and add 2. Tyrone says that you need 12 because you need 6 more cups of flour than sugar, and Teresa sets up a proportional equation, (incorrectly) solves for x and decides that the answer is 2.4.

The fourteen teachers in the study diligently responded to each of the students' answers and identified that Eva was the only student who correctly gave the right answer and that all other students were focusing on other things. Several teachers congratulated the students on actually providing a response ("the student has a least provided a reason for their answer"; "the student's response is encouraging because he is making connections and seeing patterns in the quantities of ingredients"), and some teachers made other comments about things that could be done in the classroom ("I would suggest a cooking lesson"; "I'd like to let the student make the mix using both recipes, then compare them") and other comments ("obviously doesn't help cook in the kitchen or help mix concrete"). Teacher responses on the second survey provided a much more focused analysis of students' responses and directly responded to students' understanding of ratio. For the students who clearly used an additive strategy (Duane and Tyrone), the teachers couched their discussion using the word 'addition'. In the second survey, all teachers repeatedly referred to 'additive thinking' and 'multiplicative thinking' in relation to students' responses.

The second scenario *Ballooning* provided four student responses showing varying levels of data analysis. The first student, Josh, looked only at data from one balloon that was similar in mass and volume to the teacher's balloon. Josh stated that he thought the teacher's balloon would not fly because the located balloon didn't fly. The second student, Jess, reorganised the data table showing lightest to heaviest mass and, ignoring one balloon that did not fit the pattern, reasoned that the teacher's balloon would not fly because, in general, light balloons fly but heavy ones do not. The third student, Jamie, drew a graph to show the two values (mass and volume) of each balloon. Drawing a line between the balloons that could fly and those that did not, reasoned that the teacher's balloon would be located above the line and hence would probably fly. The fourth student, Jim, included an extra column in the given data table to show the ratio of the mass to the volume of each balloon and reordered the table to show ratio in ascending order. Jim reasoned that balloons with ratio greater than 1.2 did not fly and that because the ratio of the mass to volume of the teacher's balloon was 1.2, it should fly because another balloon in the data set also had a ratio of 1.2 and it could fly.

The teachers' responses to the Ballooning task were not as focused as the Sticky Mess, either in the first or the second survey. Ballooning is a task that is a specific exploration of density and, even though there were 6 teachers of secondary school science in the group, none of the teachers used this word in discussing the students' responses in the first survey. The use of the word density was peppered throughout the second survey, but not by all teachers. Most teachers mentioned that two of the students' responses showed a means of exploring the relationship between volume and mass and that both these factors affect flight. All teachers identified the flaw in the first student's (Josh) response (finding a balloon with dimensions similar to the teacher's) and the limitations of the second student's (Jess) response (focusing on mass only) where anomalous data was dismissed, but commented on the value of reorganising the data table in increasing mass for ease of analysis. The third students' (Jamie) response (plotting each balloon on a mass/volume graph) was described as an "innovative approach", as showing "excellent reasoning and manipulation of data", and “a useful starting point”, but spontaneous graphing was questioned: “I couldn’t see a student opting to plot a graph voluntarily”. Several teachers commented on the appropriateness of the line of best fit drawn by Jamie, suggesting that a different line could give a different conclusion: “if the line of best fit was drawn slightly raised it would give a different answer”; “is this a good (valid) line of best fit? Maybe the line would be more sloped and sit above teacher point”; “there are actually a number of ways she could have drawn this line”. The fourth student’s (Jim) strategy was generally applauded by all teachers in both the first and second survey. All teachers commented that Jim specifically considered the relationship between the two variables to support his hypothesis of whether the teacher’s balloon would fly or not: “his idea is logical”; “this is the right way to go about it”; “conclusions drawn from supporting data”. Two teachers specifically questioned whether such a student would take such an approach to data analysis: “Student must have had experiences with density and ratios to think to do what he did here”, and “Where did you get this kid? This is higher level thinking as he can see a relationship between the two variables”.

Following analysis of each student’s response to the Ballooning task, the teachers were asked to rank order the students’ responses in terms of increasing quality. On both the first and second survey, all teachers except one, ranked Josh lowest (who focused on only one other balloon like the teacher’s balloon), Jess second (who reordered the table in increasing mass and concluded that mass determines ability to fly), Jamie third (who plotted a graph), and Jim highest (who calculated ratio of mass : volume). In explaining their ranking, all teachers emphasised the logical nature of Jim’s approach and his search for a relationship upon which to base his conclusion. Several teachers mentioned the graph as a potential valuable strategy, more frequently in the second survey than the first, but also mentioned errors in the line of best fit and how alternative conclusions could have been drawn. The dissident voice rated Jamie as the student with “no idea” and Jess as “almost there”, thus suggesting that the graph was a very flawed approach, but that focusing on mass only was a useful approach.

The teachers were asked to state whether they felt the Ballooning task was a science or a mathematics activity, and to list concepts that could be potentially developed through the Ballooning task. In both surveys, teachers rated the task as both a mathematics and science task, and provided a comprehensive list of concepts that could be developed through Ballooning. However, in the first survey, the word ‘density’ was only listed by 3 teachers, and data handling was mentioned repeatedly (fair testing, controlling variables, graphing). In the second survey, the word ‘density’ was mentioned by 8 teachers and the list of concepts that Ballooning could develop was more focused, with further density activities mentioned that could complement and consolidate the ballooning task.

Discussion

The data generated from the BTS raised several issues that overshadowed its capacity to evaluate teacher pedagogical content knowledge of proportional reasoning. Comparing the results of the first survey to the second survey showed only marginal differences in teachers’ proportional reasoning, although there was a noticeable difference in the preciseness of teachers’ language when they discussed each of the scenarios. It is evident that the project has provided teachers with the language to discuss students’ proportional reasoning, and the differences between additive thinking and multiplicative thinking were clearly enunciated by teachers. This finding is similar to that of Watson et al., (2005/06) who reported that their pre and post profile showed a greater use of specific language that had been used throughout the professional development program by project teachers.

The actual design of the items in the BTS was an issue in terms of data yield. As outlined previously, the items were specifically presented to focus teachers’ attention on students’ proportional reasoning skills and to favour equally science and mathematics. However, the teachers’ written discussion of each student’s response resulted in teachers repeating the strategy of the child and evaluating it in terms of its appropriateness. No teachers specifically mentioned teaching approaches to assist students develop their proportional reasoning, except in a very general sense (e.g., “need to develop their understanding of ratio”; “given them cooking classes”). The survey did not ask teachers to comment about possible teaching approaches or reasons for why the students answered the way they did, and this may have been a reason for the limited description of the students’ responses. Also, the number of responses for each scenario that teachers were required to respond to may have lead to rather stilted discussion. This suggests that the directions for teacher completion of the survey require attention. In the design of the survey, the authors were keen to move beyond multi-choice responses, although Hill & Ball (2004) stated that such items can be used to measure teacher's pedagogical content knowledge. Other researchers have combined pen and paper surveys with interviews (e.g., Chick, Pham, & Baker, 2006) to elicit greater depth of responses, or used other data sources (e.g., Watson, et al., 2005/06).

The issue of 'testing' teachers' professional knowledge was a major consideration in the design of the BTS. The format of the survey did not appear to make any teachers uncomfortable and all teachers completed the survey in a professional manner. Other researchers have highlighted the issue of 'testing' teachers and how teacher reactions to 'testing' often reflect the degree to which teachers volunteer to take the 'test' (complete the survey). For example, Hill and Ball (2004) discussed the low return rate of teacher surveys, but how incentives such as certification and teaching contracts encourage completion and return. Watson et al. (2005/06) discussed the choice that teachers have in terms of survey completion in relation to the funding body of the research project – a project funded by the employing body ensures that all teachers complete required tasks. Why did teachers in our project readily complete the first and second survey? The answer to this question may be that the survey was not perceived as a test, or that the items were designed in a manner that aligned with their professional knowledge. A further reason may be because they felt comfortable with the researchers, although this is unlikely on the first day of a professional development program. This issue of the actual design of survey items is not a trivial one, as it can impact teachers' approach to the survey and therefore quality of response. However, in the BTS, it is clear that more specific questions need to be posed to further elicit teachers' pedagogical content knowledge in terms of proportional reasoning.

Concluding Comments

This paper has reported on one aspect of a current research project exploring the connections between topics in mathematics and science through proportional reasoning. The focus of this paper was on teacher knowledge of proportional reasoning and measuring growth of teacher knowledge as a result of participating in a targeted professional development program. Analysis of data from the teacher survey suggests that further work is required on the survey items to create a useful instrument, although the necessity of combining interview and other data, including classroom observations, to determine growth of teacher professional knowledge is clearly apparent. This is a first step towards eliciting teachers' proportional reasoning across both mathematics and science.

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