Addressing Verbal Memory Weaknesses to Assist Students with Mathematical Learning Difficulties

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International research demonstrates that between 6% - 8% of students face very significant and persisting mathematical learning difficulties (MD). It is still little understood why some students experience so much difficulty in mastering basic mathematical concepts and skills. This researcher will argue that verbal memory span weaknesses have been reliably associated with MD and present research with Year 2 Queensland students which confirms this finding. The paper explores how verbal memory weaknesses can constrain early maths learning, and reports an intervention designed to circumvent these restrictions with a low-attaining Year 4 student.

International research studies in a number of countries have reported an incidence of 6% - 8% of students facing significant and persisting learning difficulties in mathematics to the extent that the students have been described as having a mathematical learning disability (Geary, 2004; Shalev et al., 2000). These students have shown an outstanding difficulty in learning basic arithmetic facts, "the major feature differentiating students with and without learning disabilities" (Ginsburg, 1997). Why is it that students of at least average intelligence, with demonstrable reasoning ability, are unable to either memorise or develop efficient strategies for computing basic arithmetic facts?

The question is puzzling because studies of normal mathematical development have suggested that the developmental shift from object counting, through to verbal thinking strategies and eventually recall or retrieval-based strategies to solve basic additions is based on an adaptive drive to save mental effort (Siegler & Shrager, 1984; Siegler, 1996). In his strategy choice model, Siegler has demonstrated convincingly that while children use a variety of strategies to solve addition problems, their choices of strategy are determined by efficiency of problem solution. Certainly, in describing the spontaneous emergence of the count-on strategy in 50% of 4 year olds, Resnick (1983) argued that the significant transition from the sum or count-all strategy to count-on strategies is an interpretative challenge because this procedure does not develop in any obvious way from the overt count-all strategies. The count-on strategy both reflects an important conceptual advance in understanding the cardinal aspect of numbers (Fuson, 1988; Steffe & Cobb, 1988) and marks a distinct step towards increasingly less effortful solutions of basic addition problems (Wright, Martland, Stafford, & Stanger, 2002).

Clearly this adaptive progression does not occur for all students (Gervasoni, 2005). Wright, Ellemor-Collins, and Lewis (2007) have described the ongoing difficulties that 3rd- to 4th-grade low-attaining students have in moving from inefficient count-by-ones strategies to a range of derived facts and recall strategies. An important consequence of a persisting reliance on count-by-ones strategies is that students may fail to develop a concept of numbers as composite units which underlies a facile and flexible ability to decompose and recompose numbers to solve both basic and more complex mental computations (Fuson, 1988; Steffe & Cobb, 1988; Gray & Tall, 1994).

In trying to explain this impasse, one cognitive characteristic that has been consistently associated with mathematical learning difficulties is a weakness in working memory capacity (Geary et al., 1991; Siegel & Ryan, 1989; Swanson & Siegel, 2001). Baddeley's (1986) working memory model, stressing the time-limited duration of information stored and processed in this short term memory system, provides a useful framework for considering failures of memorisation. Geary (2004) concluded that even in simple arithmetic, the contribution of cognitive mechanisms to the problem-solving characteristics of MD students was not fully understood, and there was a need for future research to develop intervention techniques for these students.

This paper reports two facets of the author's research designed to examine these questions: (1) a comparison of the verbal memory spans of children of low-attaining and normally-achieving Year 2 students, (2) an intervention designed for a low-attaining Year 4 student which took account of his low verbal memory span and the cognitive memory load of solving basic additions.

Method

1. Verbal Memory Span

As part of a larger study exploring early indicators of mathematical learning difficulties, the researcher undertook a comprehensive assessment battery of the mathematical, memory and processing skills of 68 students in three Year 2 classes in metropolitan Brisbane, Queensland. The children were aged from 6.4 to 7.8 years (mean 7.1 years). There were 30 girls and 38 boys.

A subset (n = 17) of students was identified as at risk of early mathematical learning difficulties, determined by a state-wide administered diagnostic process called The Year 2 Diagnostic Net – Numeracy (Education Queensland, 1997). The Year 2 Net (Numeracy) assesses students' competency on key indicators along a developmental continuum, in the areas of Counting and Patterning, Number Concepts and Numeration, Operations and Computation, and Working Mathematically. Of particular relevance to this paper, students are expected to use the count-on strategy effectively and to have mastered some basic addition facts by the middle of Year 2. Ten of the Net students were identified by their class and learning support teachers as needing intensive learning support (Intensive Net). This paper presents *t*-test comparisons between the normally achieving and Net students on one measure of verbal memory, the Digit Span task. The mean Highest Forwards and Highest Backwards spans refer to the longest span of digits a student could repeat accurately forwards or backwards respectively on a single trial.

Results

There were significant differences in verbal memory capacity of the Net, Intensive Net and normally achieving students, as measured on the Digit Span task (Table 1). While the mean Highest Forwards span for the normally achieving (NA) students was 4.95 ($SD = .925$), the Net students showed a significantly lower mean of 4.29 ($SD = .686$): $t(58) = -2.659$, $p < 0.05$, and Intensive Net an even lower mean of 4.0 ($SD = .471$): $t(51) = -3.150$, $p < 0.01$. There were no significant differences in the mean Highest Backwards span results of the groups.

If we compare these results with data reported from the WISC-III standardisation samples (Wechsler, 1991), the mean for the Year 2 NA students is consistent with the mean Highest Forwards span of WISC-III 7 year-olds (mean = 4.98, $SD = 1.03$). In contrast, the results for the Intensive Net and Net students, with means of 4.0 and 4.29 respectively, are well below the mean Highest Forwards span of WISC-III 6-year olds of 4.73, $SD = 0.94$ (Finnane, 2006a).

Table 1

Mean Highest Forwards and Highest Backwards Span on the Digit Span Tasks as a Function of Net Status

Span	**[a]Intensive Net students (n = 9)**	**Net students (n = 16)**	**Normally achieving students (n = 43)**
Mean Highest Forwards span	4.00	4.29	4.95
SD	(.471)	(.686)	(.925)
Mean Highest Backwards span	2.90	3.00	3.09
SD	(.738)	(.612)	(.648)
Mean Digit Span SS	8.50	9.00	10.6
SD	(1.841)	(1.837)	(2.546)

Note. [a]Intensive Net students are a subset of the Net students.

Method

2. Case Study – Sam

Sam (pseudonym) was an 8-year old boy in Year 4 at the study school, referred to the researcher by his Learning Support teacher as he was facing difficulties in all areas of learning mathematics. Sam's experienced class teacher was puzzled as she was unable to teach Sam multi-digit subtraction with MAB blocks: "He just couldn't get it at all". Sam also faced significant difficulties in learning to read, but his teacher noted his good oral presentation skills and aural comprehension. Previous cognitive assessment had shown that Sam had above average verbal and non-verbal reasoning capacity and good visual sequential memory. However, he had visual-motor coordination problems as measured on the Coding test (9th percentile) of the Wechsler Intelligence Scale for Children – Third edition (WISC-III). Sam showed poor verbal memory with a Highest Forwards span of 4 at the age of 8 years on Digit Span, consistent with the span shown by the Intensive Net students in Year 2 (Table 1).

Assessment of Sam's mathematical abilities on a range of assessment tasks showed that he had very poor number sense, poor counting skills to 100, oral and written teen/ty confusions, and marked difficulty reading and writing operator symbols. He had been unable to master any basic arithmetic facts apart from the small Doubles facts.

The researcher carried out an intervention once weekly with Sam designed to build his number and quantity sense, to clarify his understanding of base-10 structure and address his teen/ty errors, to facilitate mastery of basic arithmetic facts and assist him to carry out addition and subtraction with regrouping. This paper presents only the phase of intervention designed to teach Sam the Ten Facts for the light that it sheds on the way in which low verbal working memory capacity can constrain learning and to raise for discussion the benefits of activities which can overcome these limitations. The intervention consisted of the following phases:

- Ten Fact combinations were modelled with a Ten Frame, with Sam having the opportunity to make all the combinations with two sets of different coloured counters.
- Ten Facts were reinforced with a card game *Ten Snap*. Each player was dealt half of the pack of cards and placed a card in turn upon a central pile. The game was played according to the rules of familiar Snap games with each player aiming to "snap" when a pair of consecutive playing cards combined to make ten. Players could "snap" the 10 card when it appeared, to reinforce the facts 10 + 0 = 10; 0 + 10 = 10, and this was one of the most enjoyable moments of the game for Sam.
- *Ten Snap* was demonstrated to Sam's father to play at home, combined with daily practice of Ten Facts. Sam was given practice sheets of written Ten Facts in horizontal format (1 + _ = 10; 4 + _ = 10 etc.) with ten facts per page.
- Progress was assessed by recording Sam's speed of response on the Ten Facts and his accuracy in producing combinations of ten on a *Make 10* task (Finnane, 2006b). On this task students are given a blank page with the instruction to make the number 10 in as many different ways as they could (see Fig. 2).

Results

Sam quickly mastered the Ten Facts with a mean response speed of 1.8 seconds per fact after 3 weeks. This was in striking contrast to his pretest times of 4.5 seconds per fact. Response times between 0.96 and 1.6 seconds were sustained over a 6-month period (Fig. 1).

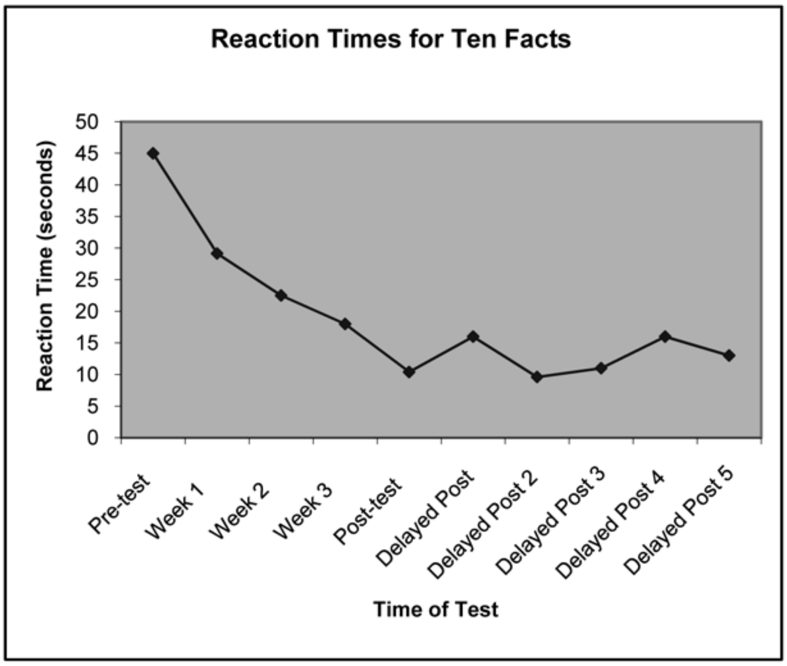

Figure 1. Sam's total response times on Ten Facts (10 facts per page)

Discussion

Why was the intervention successful? After a long failure in mastering addition facts, Sam's success could be partly due to visualizing the combinations on the Ten Frame. Importantly, *Ten Snap* kept Sam's focus on the solution of 10 as a constant whole, while he searched for the limited number of combinations, or parts. In discussing MD students' failure to learn arithmetic facts, Geary suggested an explanation whereby for slow counters and students with low working memory capacity, the addends could decay in working memory by the time they had finished carrying out their counting strategy (Geary et al., 1991). This meant a loss of the necessary association between the addends and solution, which gradually builds up associative strength in long term memory leading to recall (Siegler & Shrager, 1984). *Ten Snap* facilitates the associations with the solution 10 by actively focussing attention and promoting recognition memory of the facts. The fun aspect of the game and lack of penalty for missing a combination acts to minimize anxiety, a factor known to impact on functional working memory capacity (Ashcraft & Kirk, 2001). *Make 10* provided a different, easy opportunity to consolidate the connections.

Another salient feature of the intervention which was facilitative for Sam could be the absence of any need to produce written operator symbols in either the *Ten Snap* game or Ten Facts sheets. This is suggested by Sam's asking "Would it be all right to leave out the plus sign?" when he was completing the *Make 10* task (see Fig 2). From Sam's confusion over operator symbols and demonstrated difficulties on the Coding test in reproducing the plus sign, we might assume that this request was made to minimise cognitive load and visual motor effort. Sam did not minimise the importance of the meaning of operator symbols, but chose to insert them only when necessary to convey the different operation involved for example, changing from addition combinations to the multiplication 1 x 10 = 10 (Fig. 2).

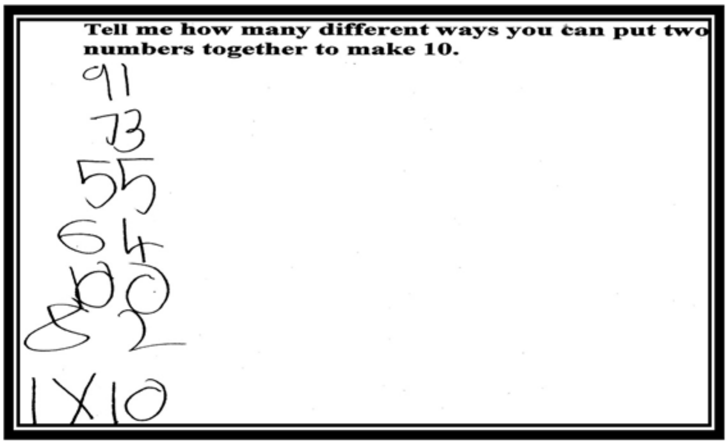

Figure 2. Sam's solutions on the Make 10 task when he asked if he could leave out the plus sign

Reading and retrieving the meaning of the operator symbols from long term memory was clearly a persisting difficulty for Sam. Later in the intervention, when asked to complete a sheet of 10 Plus problems, starting with the items 10 + 2 = _ and 10 + 8 = _, Sam asked:

S: Are these like 20 and 80?

R: not 20 and 80, just have a look there

S: oh oh.

R: what were you thinking of?

S: the times tables. Oh these are easy, I can do these ones. I don't need help. I don't need help!

R: ready, set, go!

Drawing symbols remained an ongoing concern for Sam and he used the same compensatory strategy later in the intervention to assist achieving his own goal. To demonstrate that he had taught himself the nines times tables, Sam said that he would write them out (Figure 3). As he prepared himself, telling me that he would write out up to the 12 x 9 fact, Sam explained that he would take a step to avoid getting confused.

S: I'm gonna do it up to 12

R: can you?

S: and the last one is 108

R: ok

S: now I'm ready. I just need to do the dots.

R: what are the dots?

S: **just so I don't get mixed up.** Ok, so 18, this one's 27, and 36, 5 times 9 is 45, 6 times 9 is 54, 7 times 9 is 63, 8 times 9 is 72, 9 times 9 is 81, 10 times 9 is 90, 11 times 9 is 99, 12 times 9 is 108, are all of them right?

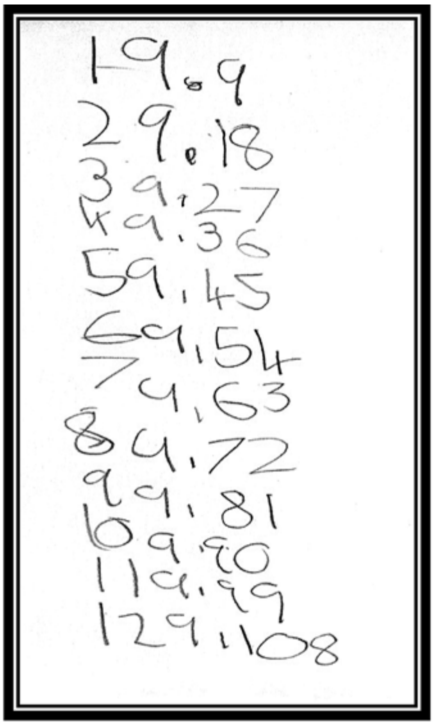

Figure 3. Sam's 9x table with minimal use of symbol

The beneficial effects of Sam automatising the Ten Facts had not only given him the confidence to set himself a goal at his grade level in mastering the 9 times tables, but led to a spontaneous interest in partitioning numbers which had not been apparent earlier on the *Make 10* activity. Sam showed excitement when he suddenly thought of a new way to respond to what was now a routine task:

> S: I did my ten facts but I did it a new way. I did three of them. 3 plus 3 plus 4 equals 10. 4 plus 4 plus 2 equals 10. 5 plus 2 plus 2 plus 1 equals 10, and that was all of them. And instead of doing the easy way, I did a hard way.
>
> R: And do you want to tell us the easy way, what do you mean by that?
>
> S: Well, it's the easy way to do it - like 9 plus 1, but I wanted to try to do it a hard way.

Recalling basic facts which had been all but impossible for Sam at the beginning of Year 4 had become "the easy way". Sam's perception of known facts as "easy" resonates well with information processing models of automatised responses as involving minimal if any cognitive resources.

At a later date, Sam showed that his partitioning skills were not restricted to the Ten Facts which he had automatised, but had a broader application. After he had hesitated at 8+8 when working on large Doubles facts, the researcher had suggested that Sam could check when he felt unsure by using an up to ten strategy, that is, 8 + 8 is the same as (8 + 2) + 6 or 10 + 6. The aim here was to illustrate to Sam another use of his known Ten Facts and to encourage partitioning. Sam was interested and responded by asking "*Could I do 16 another way?*" He made several interesting combinations, with obvious enjoyment (Fig. 4).

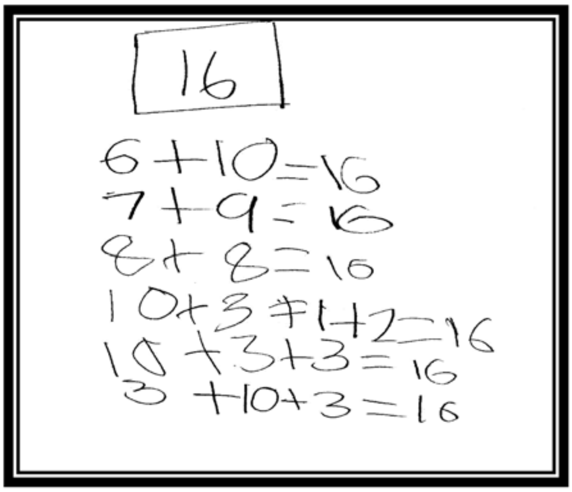

Figure 4. Sam's self-initiated combinations to make 16

It was notable that Sam took pleasure in representing not only combinations that reflected his previous success in solving 9 Plus, 10 Plus, and large Doubles problems, but also combinations that reflected his emerging interest in partitioning numbers, and in turn his conceptual understanding of numbers as composite units.

Conclusion

Baddeley's (1986) working memory model offers a helpful framework for understanding and addressing surprising constraints to early mathematical problem solving in children with otherwise good reasoning abilities. While their normally achieving peers move along a predictable trajectory of increasingly more efficient strategies for solving basic additions, culminating in effortless retrieval of basic addition facts, students with MD typically remain caught in a cycle of persisting slow, effortful, and error-prone count-by-one strategies.

Research with 6- to 7-year old students caught in the Queensland Year 2 Net demonstrated that the students needing intensive intervention had significantly lower verbal memory spans than students achieving expected mathematical milestones. This may have left them vulnerable to continuing to depend on their fingers to keep track of their counting, unable to manage the cognitively demanding double count verbal strategies in working memory.

An intervention designed for a Year 4 student showing poor counting fluency, low memory span and minimal known facts resulted in sustained memorisation of Ten Facts. In turn, automatisation led to increased student autonomy in number exploration and spontaneous goal setting for further basic fact mastery. The activities chosen focussed his attention on part-whole relationships, and acknowledged the complex retrievals necessary in carrying out what may appear to be simple additions. Future intervention research needs to further identify and address the multiple areas where MD students may be facing retrieval difficulties.

References

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety and performance. *Journal of Experimental Psychology: General*, 130(2), 224-237.

Baddeley, A. D. (1986). *Working memory*. Oxford: Oxford University Press

Education Queensland (1997). *Queensland Year 2 Diagnostic Net*. Brisbane: Education Queensland.

Finnane, M. (2006a). Verbal memory span limitations as a factor in early mathematical learning difficulties. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education* (Vol 3, pp. 3-81). Prague, PME.

Finnane, M. (2006b). *The role of fluency in mathematical development: Factors associated with early learning difficulties in mathematics*. Unpublished doctoral thesis, University of Queensland.

Fuson, K. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27*(5), 787-797.

Gervasoni, A. (2005). The diverse learning needs of young children who were selected for an intervention program. In H. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of International Group for the Psychology of Mathematics Education* (Vol 3, pp. 33-40). Melbourne, PME.

Ginsburg, H. (1997). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities, 30*, 20-33.

Gray , E. & Tall, D. O. (1994). Duality, ambiguity and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education*, *25(2),* 116-140.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. Ginsburg (Ed.), *The development of mathematical thinking* (pp.109-151). New York: Academic Press.

Shalev, R .S., Auerbach, J., Manor, O, & Gross-Tsur, V. (2000). Developmental dyscalculia: Prevalence and prognosis. *European Child and Adolescent Psychiatry, 9* (6), 1158-1164.

Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60*, 973-980.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229-293). Hillsdale, NJ: Lawrence Erlbaum.

Steffe, L. P. & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Swanson, H. L., & Siegel, L. S. (2001). Learning disabilities as a working memory deficit. *Issues in Education, 7*(1), 1-48.

Wechsler, D. (1991). *Manual for the Wechsler Intelligence scale for Children* – Third edition. San Antonio, TX: The Psychological Corporation.

Wright, R. J., Ellemore-Collins, D., & Lewis, G. (2007). Developing pedagogical tools for intervention: Approach, methodology, and an experimental framework. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 843-852). Adelaide, MERGA.

Wright, R. J., Martland, J., Stafford, A. & Stanger, G. (2002). *Teaching number: Advancing children's skills and strategies*. London: Paul Chapman.