CAS Enabled Devices as Provocative Agents in the Process of Mathematical Modelling

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This paper considers the potential of Computer Algebra Systems (CAS) to enhance the processes associated with mathematical modelling and application tasks. In doing so, the role of technology in the cyclical development of mathematical models is considered in relation to current literature in this area. The analysis of data drawn from a one year study of three different secondary school classrooms indicates that CAS enabled technologies have a role to play as *provocateurs* of productive student-student-teacher interaction in both small group and whole class settings.

Background

This paper investigates the nature of the potential partnership between two powerful thinking tools – mathematical modelling and computer algebra systems (CAS) – in teaching and learning mathematics in the senior secondary school. Mathematical modelling – formulating a mathematical representation of a real world situation, using mathematics to derive results, and interpreting the results in terms of the given situation – is a significant element of the senior mathematics syllabuses in Queensland, Australia and appears, as applications of mathematics, in the curriculum documents of most other Australian states. As CAS enabled technologies are developing increasing acceptance in mainstream mathematics instruction there is need to explore and understand the synergies that might be developed between these technologies and current curriculum objectives. This paper reports on initial findings of a project designed to explore the potential of CAS enabled technologies to enhance mathematical modelling activities.

CAS and School Mathematics

While there is significant research centred around solving contextualized problems through the use of the multiple representational facilities offered by mathematically enabled technologies (e.g., Doerr & Zangor, 2000; Huntley, Rasmussen, Villarubi, Santong & Fey, 2000; Yerushalmy, 2000) and substantive argument to support the use of CAS to enhance the process of mathematical modelling (Kissane, 1999, 2001; Thomas, 2001), typically, the potential of modelling and CAS have been considered separately in mathematics education research. Further confounding this issue, there is still "very little is known about the issues which arise when teachers use CAS in their classrooms" (Thomas, Monaghan, & Pierce, 2004).

At present applications of CAS in Australian secondary school classrooms are largely limited to developing students' understanding of abstract mathematical concepts and this appears to be reflected in research designs in this area. For example, a recent review of Australian research into calculators and computer algebra systems between 2000 and 2003 (Forster, Flynn, Frid, & Sparrow, 2004) did not include a single study which focused on the role of CAS enabled technologies in mathematical modelling or the solution of contextualized problems.

There is, however, an emerging literature directed towards mathematical modelling within the context of CAS active learning environments. Thomas (2001), for example, claims CAS capable digital devices are uniquely able to support the process of mathematical modelling by enabling exploration, representation and analysis of authentic data in ways that cannot be achieved with pencil and paper, or with standard graphics calculator technology. Thus, CAS has the potential to provide access to more sophisticated life-related problems. However, because research has usually lagged behind implementation of CAS active curricula (Zbiek, 2003), there remains much for researchers and teachers to do before we understand how to best use CAS to enhance students' capabilities in the area of mathematical modelling.

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The Role of Technology in the Process of Mathematical Modelling

Based on a three year longitudinal case study of a class of students studying mathematics in technologically rich environment Galbraith, Renshaw, Goos, and Geiger (2003) provide a description of the role of technology in the process of working with applications of mathematics and mathematical modelling.

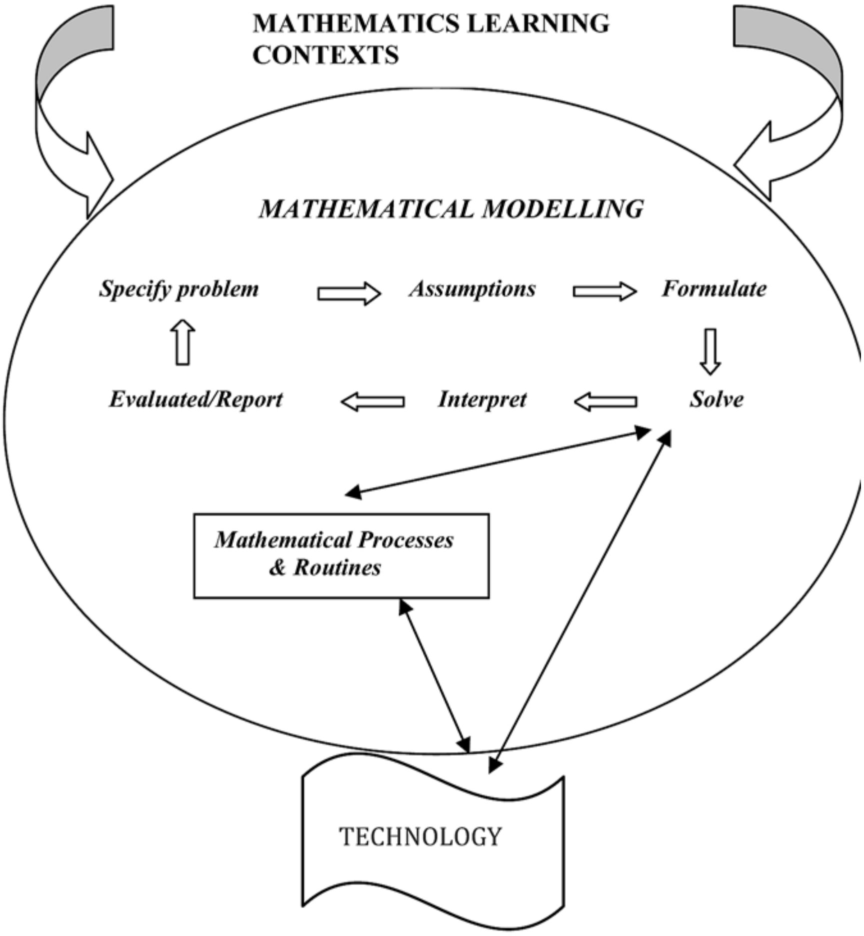

Figure 1. Some technological and mathematical interrelationships from Galbraith, Renshaw, Goos, & Geiger (2003, p. 114)

In this description, illustrated in Figure 1 above, mathematical routines and processes, students and technology are engaged in partnership during the *Solve* phase of a problem, which follows from the abstraction of a problem from its contextualised state into a mathematical model. This view identifies the conceptualization of a mathematical model as an exclusively human activity while the act of finding a solution to the abstracted model can be enhanced via the incorporation of technology. Thus technology is seen as a tool used to interact with mathematical ideas only after a mathematical model is developed rather than as a tool for exploration and development of a model or its validation as a reliable representation of a life related situation.

Context of the Study

The data described below is sourced from a 12 month study of the use of CAS enabled technologies in authentic senior secondary classroom settings. Three cohorts of students (1 Year 12 group and 2 Year 11 groups) from different schools were observed on three different occasions each for periods of time ranging from 45 minutes through to 90 minutes. All classes were studying Mathematics B, a subject that includes substantial elements of calculus and statistics. The schools included one government school and two non-government colleges. Data collection instruments included observational field notes, video and audio recording of small groups of students working on specified tasks, and video and audio recording of episodes of whole class activity. In addition, individual student and teacher interviews were conducted after each class session in order to ascertain their perceptions of the benefits offered by CAS enabled technologies to learning mathematics in general and specifically to working on mathematical modelling tasks.

On the majority of occasions, students and teachers worked on tasks that incorporated some element of mathematical modelling. While the three teachers' experience with the use of the Nspire handhelds, and CAS in general, varied from novice to expert, all were experienced users of other technologies (e.g., graphing calculators, spreadsheets, statistical packages) as tools to teach mathematics. Students' experience in the use of technology to learn mathematics also varied across the three classes with one group having very limited exposure and the other two groups with extensive previous use. None of the students had used the Nspire handhelds before the beginning of the year in which the study was situated.

Each class was equipped with a set of Texas Instruments Nspire handheld devices (at least one for each student and the teacher) and one licence for software that mirrored the facilities of the Nspire handheld device. These technologies possessed all of the features of a typical graphing calculator, such as function and graph plotting modules, but also included a CAS capability that is highly integrated with other calculator facilities. Other features included a fully functional spreadsheet (again with CAS integrated capability) and a well developed feedback mechanism for reporting on input errors. The data analysed in the two vignettes following are based on observational field notes of whole class activity and audio and video recordings of students and teachers working together in both small group and whole class settings.

Two Vignettes Where CAS Promotes Contention

The observation related to technology use and mathematical modelling reported above stands in contrast to classroom observations in a current study which aims to investigate the role of CAS based technologies in enhancing the process of modelling in upper secondary mathematics programs. The two vignettes reported below come from classrooms in two different schools – one a government school and the other a private college. In both cases the teachers challenged students to make use of CAS based technology, Texas Instrument's Nspire handheld devices, as an aid to working with problems that required skills related to some aspect of mathematical modelling.

Vignette 1

The vignette described below took place in a Year 12 (final year of secondary school) mathematics classroom where students were investigating the nature of population decay towards extinction. These students were using TI-Nspire CAS handheld technology which had been introduced to them only a few weeks earlier. During one lesson observation the teacher set the students the following question:

When will a population of 50 000 bacteria become extinct if the decay rate is 4% per day?

One pair of students developed an initial exponential model for the population y at any time x and equated this to zero - - in the belief that the solution to this equation would give them the number of periods, and hence the time, it would take for the population to become extinct. When they entered this equation into their handhelds, however, the device unexpectedly responded with a *false* message, as illustrated in Figure 2 below.

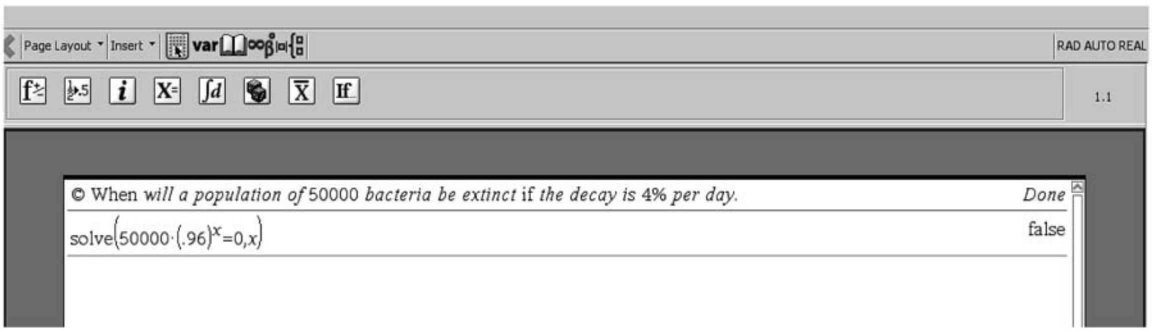

Figure 2. Nspire display for the problem $y = 50000 \times (0.96)^x$

The students were initially concerned that this response had been generated because they had made a mistake with the syntax of their command. They re-entered the instruction several times and tried a number of variations to the structure of the command but did not consider that there was anything at fault with the parameters they had entered. When the students asked their teacher for assistance, he looked at the display and stated that there was nothing wrong with the technical side of what they had done but they should think harder about their assumptions.

After further consideration, and no progress, the teacher directed the problem to the whole class. One student indicated that the difficulty being experienced was because "you can't have an exponential equal to zero". This resulted in a whole class discussion of the original pair of students' assumption that extinction meant a population of zero. The discussion identified the difficulty as equating an exponential model to zero and then considered the possible alternatives. Eventually the class adapted the original assumption to accommodate the limitations of the abstracted model by accepting the position that extinction was "any number less than one". Students then made this adjustment to their entries on the handheld and a satisfactory result was returned.

Vignette 2

In the second classroom students were asked to work on the following task.

The CSIRO has been monitoring the rate at which Carbon Dioxide is produced in a section of the Darling River. Over a 20 day period they recoded the rate of CO2 production in the river. The averages of these measurements appear in the table below.

The CO2 concentration [CO2] of the water is of concern because an excessive difference between the [CO2] at night and the [CO2] used during the day through photosynthesis can result in algal blooms which then results in oxygen deprivation and death of the resulting animal population and sunlight deprivation leading to death of the plant life and the subsequent death of that section of the river.

From experience it is known that a difference of greater than 5% between the [CO2] of a water sample at night and the [CO2] during the day can signal an algal bloom is imminent.

Table 1

Rate of CO_2 Production versus Time

Time in Hours	Rate of CO_2 Production
0	0
1	−0.042
2	−0.044
3	−0.041
4	−0.039
5	−0.038
6	−0.035
7	−0.03
8	−0.026
9	−0.023
10	−0.02
11	−0.008
12	0
13	0.054
14	0.045
15	0.04
16	0.035
17	0.03
18	0.027
19	0.023
20	0.02
21	0.015
22	0.012
23	0.005
24	0

Is there cause for concern by the CSIRO researchers?
Identify any assumptions and the limitations of your mathematical model.

Students were expected to build a mathematical model, initially by inspecting a scatterplot of the data which in turn helped them determine the general form of function that would best fit the data. This general form was then to be adapted to fit the specific data presented in the question and the resulting equation used to answer the questions at the end of the task. Students had studied strategies for determining if a particular function type was most suited to a data set. Most recently, students were introduced to a technique where ln versus ln plots of data sets was used to determine if a power function was an appropriate basis on which to build a mathematical model. This appears to have influenced the actions of two students as the transcript below indicates.

Researcher: So you are up to now where you are building the model are you?

Student 1: Well we worked out a plan of what we are going to do, we are just putting it on paper.

Researcher: So do you want to tell me what the plan is?

Student 1: The plan is to do the LOG LOG plot of both the data see if they are modeled by a power function. We have previously seen that the …

Researcher: So that is something you have learnt to do over time? When ever you see data look like that you check to see if it's a power function of Log/Log and if it is what was the graph look like of a Log/Log?

These students experienced problems with this approach, however, as the technique involves, in this case, finding the natural logarithm of 0.

Student 1: 0.44 zero… (entering information into calculator). Don't tell me I have done something wrong. Dammit. Mummbles … Start at zero is it possible to do a power aggression? I don't think so!

This comment was in response to the display which resulted when the students attempted to find the natural logarithm of both *Time* and *CO2 output* data using the spreadsheet facility of their handheld device (Figure 3 below). Students were surprised by the outputs they received for both sets of calculations, that is, the #UNDEF against the 0 entry in the *Time* column and the lack of any entries in the *CO2* column. In addition, an error message was produced indicating results where not available in the current settings of the handheld device.

After a little more thought students realised where the problem lay.

Student 1: L N time is going to be equal to the L N of actually time. Tim … oh is that undefined cause it's zero?

Student 2: Yep.

Student 1: Right now if I go back to my graph … Enter

Student 2: If you try zero fit, it will just go crazy.

Students eventually identified the problem with their approach and realised their initial assumption, that is, the best model for whole data set as power function, was at fault. Eventually, they realise it was best to model the data with two separate functions.

Student 1: So we have fitted a linear model for the top data and then we fitted a power function to the bottom data given we take the absolute value of those the question asks, the difference greater than 5% we need to look at the actual CO2 produced, now what we have got is the rate, to go back to the actual CO2 absorbed we need to integrate the model or both models and then use the percentage difference formula – predicted minus actual divided by actual or in this case night minus day divide by day x 100 to look at whether for any x or any t there is any percentage difference greater than .05.

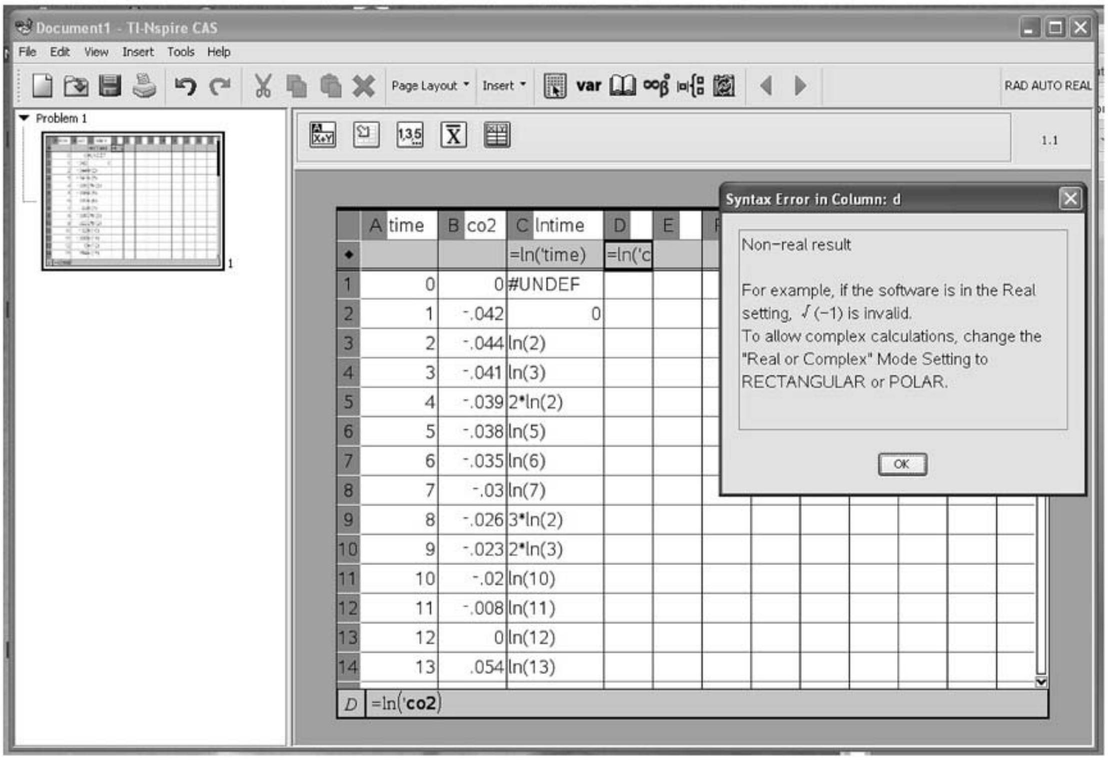

Figure 3. Nspire display of spreadsheet for natural logarithm of time and CO2 output data.

Follow-up Whole Class Discussion

In each of the vignettes described above, teachers found other students in their classes were experiencing the same or similar blockages to their progress and used this feedback as the centrepoint for a whole class discussion in which the problematic issue was explored and then resolved. It is important to note, also, that during both small group and whole class discussions students themselves contributed to the improvement of knowledge and understanding. Technology has therefore played a role in catalysing student participation in their own learning through small group and more public interactions with the teacher and their peers.

Discussion and Conclusion

In contrast to the role attributed to technology in mathematical modelling by Galbraith, Renshaw, Goos, and Geiger (2003), in the episodes described above, the electronic output forced students to revaluate fundamental assumptions they had made within the context of the described problems. This means that technology related activity takes place during the *assumptions* phase rather than only at the *solve* juncture outlined in Figure 1. Consequently, this assigns a role to technology in the conceptualisation of the model rather than simply as a tool which is used to solve a mathematical problem after it has been abstracted – a position more consistent with a view of technology as a *partner* or *extension-of-self* in technology enhanced mathematical activity (Goos, Galbraith, Renshaw, & Geiger, 2003).

The unexpected output on the handheld devices in both vignettes acted to *provoke* students to rethink their original assumptions and to make adjustments to their initial approaches solving these problems. Students were forced to reshape their early thinking to satisfy the demands of both the context and the limitations of their abstracted model. Thus technology, in this case, has fulfilled a more interactive role than simply that of a powerful computational tool.

These *provocations* also represent opportunities for teachers to gain an awareness of students' misconceptions and then to provide appropriate scaffolding in order to move the students forward in their understanding of the issue that was proving problematic. As reported above, both teachers used the consternation generated by the error messages recorded on students' handhelds to structure a forum in which student-student-teacher interaction played an important role in resolving the issue of concern.

The role of CAS based technologies as a *provocateur*, as reported in this paper, of productive student-student-teacher interaction in both small group and whole class settings is therefore an area worthy of further research in relation to mathematical modelling and the field of technologically rich learning environments.

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