How Group Composition Can Influence Opportunities for Spontaneous Learning

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Classroom video, and video-stimulated interviews of small group work, in a Grade 5/6 classroom are used to show ways group composition can influence learning opportunities. Vygotsky's (1933/1966; 1978) learning theory on the spontaneous creation of knowledge as compared to the guidance of an expert other frames this group analysis. Illustrations from two groups show how opportunities to spontaneously create new knowledge can be limited or enhanced by psychological factors associated with the inclination to explore that have been linked to resilience in the form of optimism (Seligman, 1995, Williams, 2003). This study contributes to our knowledge on forming groups to promote deep learning. It raises questions about other ways in which learning may be influenced by optimistic orientation and about building this personal characteristic to enable deep learning.

Introduction

During my years as secondary mathematics teacher in a rural area, without access to research findings (1970-1992), I experimented to improve students' learning opportunities. I found students seemed to learn more when new topics commenced with work in groups on unfamiliar problems rather than by initially learning rules and procedures, then memorising and practising. I now know that Skemp (1976) wrote about this deep learning at least ten years previously, and that research into the deep understandings that can develop when students' struggle to develop new mathematical ideas together was well underway (Cobb, Wood, Yackel, & McNeal, 1992; Wood & Yackel, 1990). To improve student opportunities to learn in my classes in the nineteen eighties, I observed group dynamics, and trialled group compositions to find what seemed to work. One of the things I found: "The need for a positive group member who can overcome negative influences in that group" is elaborated and theoretically underpinned through the present study.

I now know, what I described as 'positive' was an 'inclination to explore' (Williams, 2003), and one type of 'negative' was the result of 'not being inclined to explore' or wanting to remain within the confines of what was known rather than explore unfamiliar mathematical territory. Other types of negative influences such as 'inclined to engage in off-task activity' rather than focus mathematically are not explored in this study. My research question is: "Does the relative inclination of group members to explore new mathematical ideas influence group learning opportunities? And, if so, in what ways?

Theoretical Framework

The theory framing this study (Vygotsky, 1933/1966, 1978; Seligman, 1995; Williams, 2007a) is associated with 'spontaneous learning' in comparison to learning under the guidance of an 'expert other' (Vygotsky, 1978; Wood & Yackel, 1990). It includes psychological factors that influence whether a student is likely to undertake spontaneous learning (Seligman, 1995; Williams, 2005). Spontaneous learning occurs when a student or group create their own Zone of Proximal Development (ZPD) or overlapping zones with the assistance of 'cognitive tools'. The ZPD is the distance between what the student presently knows and what they can learn under the guidance of an 'expert other' (Vygotsky, 1978). Vygotsky did not state that this expert other was needed for learning to occur. He used this concept of an expert other to show what a child had the potential to learn at a given time. Vygotsky (1933/1966, 1978) recognized that people could create new knowledge. A personal email communication from James Wertsch articulates this:

> On the one hand, Vygotsky clearly did emphasize the influence of existing cultural and social forces in the development of individual mental functioning in the child. On the other hand … Vygotsky talked about how one uses cultural tools such as toys when playing and in the process creates ONE'S OWN zone of proximal development. (Wertsch in Williams 2005, Appendix 0.3).

Spontaneous learning (Williams, 2007a; Wood, Hjalmarson, & Williams, in press) can occur when students discover mathematical complexities in a task and decide that they want to explore them. In doing so, students ask themselves a question about this complexity, creating a mathematical challenge, overcome by exploring unfamiliar mathematics to develop new conceptual knowledge. Conceptual tools that could be used to support new learning include language, symbols, diagrams, concrete aids and technology, used as tools to think and to assist communication with peers (see for example, Williams, 2005). Deep learning can occur as a result (Williams, 2007a, 2005, 2000). This fits with Davydov's (1990) findings that students who mentally reorganise knowledge can develop new conceptual understandings. Various pedagogies have been developed which have led to instances of spontaneous learning (e.g., Cobb, Wood, Yackel, & McNeal, 1992; Dreyfus & Tsamir, 2004; Wood, Hjalmarson, & Williams, in press).

Some students are willing to explore unfamiliar mathematical ideas to develop new conceptual knowledge (e.g., Dreyfus & Tsamir, 2004; Williams, submitted) whilst others want to remain within the confines of what they have been taught (Anthony, 1996). This inclination to explore (or not) is the psychological characteristic upon which this study focuses.

This inclination to explore fits with Seligman's (1995) construct of explanatory style (Williams, 2003). Explanatory style includes perception of and response to successes and failures. An 'optimistic' child (Seligman, 1995) perceives failure as 'temporary', 'specific', and 'external', and success as 'permanent', 'pervasive', and 'personal'. Inclination to explore is associated with optimism because exploring what is unknown (present failure) is consistent with the perception that 'not knowing' is temporary and 'knowing' can result from personal effort [Failure as Temporary; Success as Personal]. Perceiving success as pervasive is attributing success to a characteristic of self "I succeeded, I am smart". Perceiving failure as specific involves examining failure to see what could be changed to increase the likelihood of future success (instead of perceiving the failure as pervasive or relating to a characteristic of self: "I failed, I am stupid"). Optimistic students look for ways to overcome problems they encounter [failure as temporary] by examining what can be altered to increase chances of succeeding [failure as specific]. They perceive that personal effort can lead to success (Williams, 2003, submitted, 2008).

The interview dialogue and classroom activity of a struggling student (Dean) in a previous study (Williams, 2005) is used to illustrate an optimistic orientation and elaborate on the dimensions of optimism. Dean persevered in spite of the failures he encountered:

> Cause the first time I do stuff um I get a bit *stressed* [quiet laugh at himself] … I always don't get it at *first*. (Dean's interview, p. 285)

His use of "at first" indicates he perceived failure as temporary. His perception of success as personal is illustrated in the following quote:

> I write it down in my book and then when he's talking [about] something that I have already known then I just look over it again. (Dean's interview, p. 285)

Rather than seeing himself as stupid for failing to understand how to juxtapose angles to find their sum [failure as pervasive], he studied the teacher's activity and found how this differed from his own [failure as specific]:

> I didn't know where the corners [angles] went- he [teacher] told me you put [them] facing in but … I was doing it all different- I was facing them out and up (p. 281)

As Dean was still unable to execute the procedure he looked for another way [failure as temporary, success as personal] rather than waiting helplessly. He altered what he attended to and found his own way to achieve success [failure as temporary, specific, personal]. Students like Dean who perceive 'failure' to understand as temporary and able to be overcome through effort by analysing the situation to see what can be changed to increase chances for success (learning more) are 'inclined to explore' / optimistic / resilient. When problem solving in mathematics, they cope with adversities associated with encountering failures before success. This study examines how this influences learning opportunities.

Research Design

This study is part of a broader study of the role of optimism in collaborative problem solving in Grade 5/6 classrooms in an Australian government school. The teaching approach has been demonstrated to elicit creative thinking (see Williams, 2000, 2007b). It involves small group problem solving with interim reports to the class. Three tasks were undertaken across the school year (Tasks 1, 2, 3) for three, two, and one eighty-minute sessions respectively. To enable study of:

Interactions within groups,

Learning outcomes,

Optimism or lack of optimism of group members,

the Learners' Perspective Study (LPS) methodology (Clarke, 2006) was adapted. Three cameras simultaneously focused on six groups in the classroom. The fourth camera captured student reports. The private talk of the three groups in the foreground on the cameras was captured. Video-stimulated interviews were undertaken individually with at least four students after each lesson. To enable students from each group access to both their group and the reporting sessions in the interview, a mixed image of one group with the reporting sessions as an insert in the corner was generated, and a second group had the reporting session in the background on their group video. If members of the third group were interviewed, there was opportunity to use their group video and the video of student reports. Students used the video remote and selected the parts of the lesson that were important to them for any reason and discussed what was happening, what they were thinking, and what they were feeling. In addition, they were asked questions about whether they were good at maths, and how they thought they were going in maths, and how they made those decisions. These questions generated indicators of optimism.

For Task 1, groups were composed by the teacher using my descriptions of what made a 'good group'. The description included:

Gender balance

Similar paces of thinking (as opposed to similar performance)

A student expected to 'positively' influence the most negative member

The iterative changes to group composition from Task 1 to Task 3 were informed by my previous teaching and research knowledge of group composition and my video analysis of group interactions in previous tasks in this study. The intention was to optimise group composition to improve opportunities for spontaneous learning for class members.

Optimism or lack thereof was studied through discourse analysis (Säljö, 1999) and these analyses were triangulated with student enactment of this characteristic on lesson videos. In this study, the intention was to examine how these students enacted optimism or lack thereof and what effects this had on learning opportunities for group members. Video and interview data and photocopies of group work produced in class together provided evidence of what students had learnt, and what had influenced their opportunities to learn.

Purposeful Group Selection

Over the period in which the three tasks were undertaken, Group 1 remained the same because they developed new conceptual understandings during each task. For Task 3, Group 1 (Patrick, Gina, Eliza, Eriz) consisted of three students (Patrick, Gina, and Eliza) because Eriz was absent. All four students in this group displayed optimistic indicators.

Group 2 (Sam, Jarrod, Wesley, Donald) was formed as a result of progressive iterations of group composition intended to compose a group in which a high performing student, Sam, would participate in spontaneous learning. To the surprise of the teacher, he did not do so in Task1. Sam's interview data showed evidence of lack of optimism, so I was not surprised. In his interview after Task 1 and 2, Sam reported that he found the tasks boring and had not learnt anything new. This fitted with evidence on the lesson videos, and with Sam's interview descriptions. Sam had an instrumental understanding of the mathematical ideas associated with Task 1 (Volumes of Cuboids, see Williams, 2007b) prior to and after Task 1. He knew to multiply length by width by height to get volume but his interview showed that unlike other students who had learnt from Task

1, he did not know why this was so. In contrast, Jarrod, Wesley, Gina, Patrick, and Eliza, could explain why this formula was relevant by referring to the rectangular prism and discussing the layers within. In general, these students were not aware of this formula until they generated it during Task 1. Sam was selected for detailed study because he was a high performing student who had not participated in spontaneous learning opportunities. Although any member of Group 1 could have illustrated optimistic activity, Patrick was selected for analysis because like Sam, he was a high performing male student.

Task 3

> Make each of the whole numbers from one to twenty inclusive using:
> - Four of the digit four and no other digits
> - Any or all of the operations and symbols
>
>   + + - - × / ÷ √ . () ²
>
> Think about how to make all the sums as fast as possible

Figure 1. Task 3: The Fours Task.

Task 3 (see Figure 1) was accessible to students with varying understandings of whole number operations because the numbers could be generated with simple operations, or through many permutations and combinations of more complex operations and symbols. During the task, it was anticipated that students would learn more about the operations and symbols and how to use them through their conversations in groups, and during the reporting sessions. Increased familiarity with these symbols was expected to increase their opportunities to create new sums. Trying to find fast ways to generate sums was expected to promote generalisation as it did for Group 1 (see below).

Task Implementation

This task spanned one eighty-minute lesson. The teacher and I team-taught with myself undertaking most of the task implementation and the teacher focusing much of the reporting session. The class undertook the following activities in the order given:

Three minutes commencing the task alone

Shared what they had done with the rest of their group

Approximately ten minutes of small group work

Two minutes preparing their report for the class

Approximately twenty minutes of group reports (1-2 minutes each)
(focused on some aspect of what the group had done or tried to do)

Another cycle of group work and reporting

Groups had a set of tiles with fours, operations, and symbols to assist their thinking and communication. Transparent tiles were used by students on an overhead projector, during reporting sessions, to enabled students to communicate in visual images and language (Ericsson & Simons, 1980). These reports were discussed after each reporting session without the class teacher or myself judging the correctness of the mathematics produced. Rather, we asked questions to stimulate further thinking in groups. Further information on this approach can be found in Williams (2007b) for this study and Williams (2000) and Barnes (2000) for other studies.

Results and Analysis

Non-Optimistic Sam and Optimistic Patrick

One of the indicators of lack of optimism Sam displayed in his interview was Success as External. He described learning for him as listening to the teacher, reading books, and searching the Internet. Unlike Patrick, he did not include self-generated knowledge, which is an indicator of optimism [Success as Personal]. Sam gave some indication that he did not examine situations to see what more he could learn. Sam stated in his interview after Task 1 that he knew everything in the lesson before hand. When asked to identify some of the things he already knew, Sam answered: "Didn't I say I knew it all … Which reports do you think I should be thinking about?" The reasons for this response are not clear-cut. He may not have looked in detail because he considered that he 'knew it all' (as indicated by his boredom), or because he considered his peers would not know more than he did about anything. This response suggests Sam wanted guidance on what to attend to because he was not inclined to go outside his present understandings and identify for himself how the presentations of others matched his own thinking. Whatever the circumstance, there was sufficient evidence to indicate he was not optimistic

In contrast, Patrick described one way he learnt was by thinking about mistakes made by others and how they could be overcome [Success as Personal]. In his interview during Task 1, he discussed a group who had made a twenty-four cube rectangular prism when they had intended to make a twelve-cube prism (Williams, 2007b). This group had not been able to correct their mistake before they reported. Patrick stated in his interview:

> You know how they got it wrong- it made me think about (pause) how they could get it *right* (pause) … it was … 2 2 6 [dimensions of rectangular prism] … they got 24 and they have to get 12 what if they changed the 6 to 3 and that would just halve it and instead of 24 they would have 12.

Although not stated explicitly Patrick appeared to have halved the number of stacks in the height rather than manipulated numbers. This fitted with the way his group had been considering different prisms as made up of flat stacks of cubes. Patrick did not see the failure to make the rectangular prism with 12 cubes as permanent. He examined the situation to see what he could change to gain success through his personal effort. He identified possible variables he could control, and adjusted them [Failure as Temporary; Failure as Specific; Success as Personal], thus demonstrating optimism.

Composition of Sam's Group

Sam was purposefully placed in a group that was expected to optimize his opportunities to undertake spontaneous learning. His group contained all boys because he was a quiet student and it was possible that interacting with girls might have limited his novel contributions on the previous two tasks. Two of the boys placed in Sam's group (Jarrod and Wesley) were high performing students who had demonstrated they could think creatively in Tasks 1 and 2. Although the other boy, Donald had dominated activity in another group and taken their thinking off track, it was considered that Jarrod would be focused enough and sufficiently dominant to keep this group on track. In other words, it was considered that Jarrod would be able to provide the positive influence necessary to focus the group. The email quote below shows an excerpt of my discussion with the teacher of Jarrod's capacity when forming groups for Task 2.

> Callum and Amit played around a lot. I think Amit would contribute if we added a serious eager boy … [like] Jarrod. … [and] Elsa might have more to contribute [in her group] if Jarrod were not dominating (Email communication from researcher to teacher).

This quote captures my analysis of Jarrod's eagerness and capacity to think creatively and my faith in him to entice a good student into creative mathematical thinking, and to bring a group back to a productive direction if ideas with mathematical flaws were presented. Jarrod was considered an ideal group member to entice Sam into creative thinking and to 'overrule' Donald if his thinking was flawed.

Activity in Task 3

Sam and Patrick's activity during individual time showed Patrick's willingness to explore unfamiliar mathematics "I went looking for hard one's first like decimals and stuff and times" and Sam's inclination to remain within the mathematics he knew. Both students generated an equal number of correct sums. Patrick generated most of his sums by retaining the underlying structures and changing the positions of operations. He progressively increased the number of unfamiliar symbols and operations he used (see Williams, submitted, for more information). Sam generated his sums quickly, stopped early, covered his work and waited. Although Sam's number fact recall was faster than Patrick's, the sums Sam generated, and his less sustained use of patterns to generate them, and the way he stopped when there was still more time left suggested he was unable to proceed. Unlike Patrick, Sam did not progressively increase the number of harder operations he used and did not try decimals or brackets. Sam was not inclined to explore.

During group activity, Sam listened to the strategies for finding sums reported by Jarrod and Wesley then instead of engaging in group activity as expected, he spent several minutes extending his list of sums by using the mathematics Jarrod and Wesley had described. He did not extend their ideas. Once he had completed all the sums he could, Sam monopolized the remaining time by explaining to Donald how to find the answers to the sums rather than engaging in exploratory activity. The types of creative thinking previously undertaken by Jarrod and Wesley did not occur in this group. When Wesley stated it was not possible to use the decimal point, this was not discussed; Sam just included it in the presentation without justification. No new ideas were generated.

Patrick contributed to the development of new ideas in his group in various ways. For example, when Gina generated a sum and Eliza queried it, Patrick looked for what could be changed so they did not have to start again "Put something in the middle like a plus or something" [Failure as Specific]. He was the first to begin to package parts of sums as mathematical objects. For example, he put his hand over - 4 + 4 at the end of a sum and moved them away slightly "We don't really need these … they cancel each other out". His ideas formed the foundations of some of the insights developed by this group including:

> -4 + 4 can be used if wanting a small answer;
>
> Brackets can change the size of the answer; and
>
> The order in which operations are applied can change the answer.

These are 'big ideas'.

Discussion and Conclusions

These cases differed in learning outcomes and indicated that the relative optimism of group members influenced available learning opportunities. Where all students were optimistic, rich learning occurred, but limited learning occurred when a non-optimistic student focused group activity on mathematics familiar to optimistic students who had previously created new knowledge. This study raises questions about whether another group composition could have led to Sam creating new ideas. As three different groupings were tried and none were successful, it seems likely that factors other than group composition may need to be changed for a non-optimistic student like Sam to recognise that learning can be self generated. Although further research is required, these findings provide a starting point for identifying what to attend to and what types of group compositions to examine to explore how to form groups to increase learning opportunities. This study did not illustrate a 'positive' student overcoming the influences of a negative student. Study of such group interactions could be fruitful (if they exist and my teaching experience suggests they do). Longitudinal research is required to study students like Sam over numerous tasks to see if he does finally develop optimism and to identify factors that contributed to this. Most importantly though, given this study shows optimism can increase successful problem solving activity, we must build the optimism of students that are not yet optimistic? This is the focus of my present research (Williams, 2007, 2008).

Acknowledgements

The study was funded by a Deakin University Central Research Grant and data collection and analysis was undertaken through the International Centre for Classroom Research.

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