Mixing Colours: An ICT Tool Based on a Semiotic Framework for Mathematical Meaning-Making about Ratio and Fractions

Andy Yeh
University of Southern Queensland
<yeh@usq.edu.au>

Rod Nason
Queensland University of Technology
<r.nason@qut.edu.au>

This paper reports on the research and development of an ICT tool to facilitate the learning of ratio and fractions by adult prisoners. The design of the ICT tool was informed by a semiotic framework for mathematical meaning-making. The ICT tool thus employed multiple semiotic resources including topological, typological, and social-actional resources. The results showed that individual semiotic resource could only represent part of the mathematical concept, while at the same time it might signify something else to create a misconception. When multiple semiotic resources were utilised the mathematical ideas could be better learnt.

Much research has recognised and been advocating the importance of multiple knowledge representations in mathematics education (Adiguzel, Akpinar, & Association for Educational Communications and Technology, 2004; Ainsworth, Bibby, & Wood, 2002; Alagic & Palenz, 2006; Kendal & Stacey, 2003; Patterson & Norwood, 2004; Porzio, 1999; Reed & Jazo, 2002; Schuyten & Dekeyser, 2007; Siegler & Opfer, 2003). ICTs such as computers and graphic calculators have been widely utilised to provide multiple knowledge representations of mathematical concepts. However, among the use of multiple knowledge representations (e.g., text, numbers, icons, objects, graphs, and animations etc.), there has often been a lack of a theoretical framework to inform the use of different representations and to explain the effects these representations have upon the learning of mathematics. Therefore, a semiotic framework for mathematical meaning-making (Lemke, 2001; Yeh & Nason, 2004a) was adopted in this study.

The Semiotic Framework

Representations of mathematical concepts have been classified differently in different contexts. For example, in learning algebra and computer algebra system (CAS) context, representations have been classified into three categories: numerical, graphical, and symbolic (Kendal & Stacey, 2003). Moreno (2002) classified representations into visual, verbal, and symbolic for using interactive multimedia in learning addition and subtraction. For teaching number in early childhood, Payne and Rathmell (1975) developed a teaching model that classified representations into object, language, and symbol. Papert (1993) also categorised knowledge representations into action, image, and symbol. These different types of representations are all essential and central to the learning of mathematics. However, there are other representations such as colour, sound, and animations etc. that could be good representations for certain mathematical ideas yet to be classified and utilised. In searching for a unified and inclusive theoretical framework for knowledge representations, the researchers have developed a semiotic framework.

Originated from the study of linguistics, modern semiotics has evolved to view everything as a “language”, where the “language” does not only refer to spoken or written language, but also include dance, gestures, fashion, rituals of primitive tribes, music, sculptures, visual pictorial imageries, dreams, and so forth (Oliveira & Baranauskas, 2000). Semiotics is generally regarded as the study of signs and sign-using behaviour. It is also perceived as a single unified system for meaning-making.

Peirce (1839-1914) developed a semiotic triad consisting of three constructs: sign, object, and interpretant (Figure 1), and argued that all cognition is irreducibly triad. Cunningham (1992) elaborated that a sign mediates between the object and its interpretant. The sign, however, is not the object itself. He pointed out that a sign is only an incomplete representation of the object. A sign can only represent certain aspects of the object and in addition, it has aspects that are not relevant to the object. Therefore, a sign has a certain meaning emission capacity, and will be interpreted differently by different individuals (interpretant).

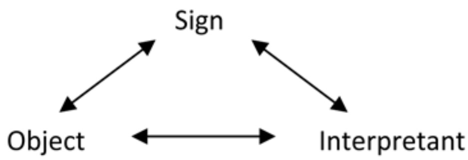

Figure 1. Semiotic triad.

Lemke (2001) made an explicit classification of mathematical signs (representations) into typological and topological resources. *Typological resources* are those which *convey meanings by their distinguishable type*. They are discrete, qualitatively distinctive, and are but not limited to natural languages and symbols. *Topological resources* on the other hand, *convey meanings by their variations in degrees*. They are continuous, quantitatively different, and are but not limited to visual graphics representations. Lemke (2001) stated that in general, mathematical expressions are constructed by typological systems of signs, but the values of mathematical expressions can in general vary by degree within the topology of the real number. He pointed out that students often have a great deal of trouble typically in understanding functional notation (typology) and its meaning in terms of quantitative co-variation (topology). However, they can be greatly aided by employing the topological strategies such as graphs and other visual representations. For example, when asked to order the size or ratio of a given set of fractions 13/19, 11/17, 4/6, 9/13, there is no simple way to tell from these typological representations except by performing the divisions and change to decimal forms. But their relationships can be easily understood if a graph or diagram of those ratios is visually presented.

Lemke (2001) continued to add insights into his typological and topological dichotomy. He argued that mathematics is also a system of related social practices; a system of ways of doing things. He then reworded the dichotomy into *actional-typological* and *actional-topological* resources for mathematical meaning-making. The "actional" modifier imposed a strong and powerful message linking mathematics to social discourse and real-world, authentic applications. Yeh and Nason (2004a) employed Lemke's semiotic framework to inform their design of a computer 3D microworld named VRMath (Yeh & Nason, 2004b) for learning 3D geometry. They found that computer environments are rich in providing these actional-typological and actional-topological resources. Moreover, they identified that the social-actional component of semiotic resources played a critical role in offering opportunities (e.g., negotiation) and legitimacy (i.e., the purpose of learning) in the mathematical meaning-making processes. Therefore, they suggested that a semiotic framework informed ICT learning environment should provide three types of semiotic resource namely: ***typological***, ***topological***, and ***social-actional*** resources.

The Context

Many adult learners in prisons do not have numeracy levels necessary to gain access to many jobs and vocational education programs (Australian National Training Authority, 2001). To assist with the rehabilitation and increase future life skills of adult inmates, the Enhancing Numeracy In Prisons Project (ENIPP) was established. The overall goal of this project was to develop and evaluate an integrated numeracy program utilising latest advances in (1) Critical Numeracy, Mathematics Education, Indigenous Education, and Information and Communication Technologies in Education (ICTE) theory and research, and (2) Information and Communication Technologies (ICTs).

The participants in this research study were a cohort of inmates enrolled in the education program at a male's correctional centre in eastern Australia. These adult inmates were identified to have some basic number facts skills but very little knowledge and understanding about ratio and fractions (National Reporting System Numeracy Level 1-2). In a pre-interview with the participants, the researchers noted that some participants were involved in the vocational training workshops where they were mixing paints. One particular inmate also spoke about his experience in mixing paints for painting his car. This gave the researchers an initial idea of a social-actional and a topological semiotic resource (i.e., colours) that could be designed and utilised for learning ratio and fractions.

Two and four computers were provided in the two classrooms. These adult learners were able to access these computers during class sessions. There was no networking between computers. During teaching and learning sessions, two to three participants were asked to work together on one computer.

Mixing Colours: The ICT Tool

Informed by the semiotic framework, the researcher designed the mixing colours software (Figure 2), which enabled the users to mix five primary colours (i.e., white, red, yellow, blue, and black) in different ratios for a variety of colours.

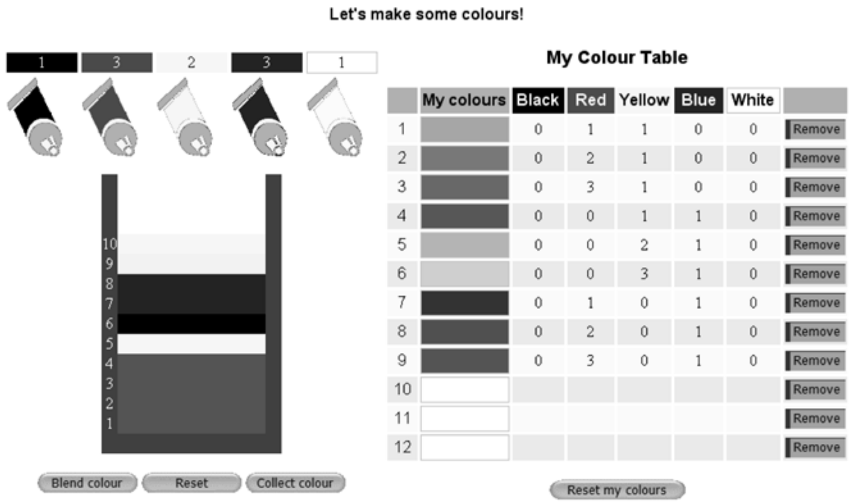

Figure 2. The Mixing Colours software.

Consistent with the semiotic framework, this ICT tool has employed topological, typological, and social-actional resources. For topological resources, the colour spectrum is utilised as the main topological resource. The gradient of colours has the meaning capacity about ratio. For example, the more reddish in the orange series colours means a bigger proportion of red colour in the mix. The container can also be seen as another topological resource because it varies in size to represent the whole. Typological resources include the colour paste icons, texts, numbers, and the colour table. Social-actional resources include the actions of Blend colour, Collect colour, Reset colour, and Remove colour. In regard to the social aspect, a task to mix paints for painting the interior walls of a house was designed to be explored with this ICT tool. Authentic materials such as the colour wheel, colour catalogue and brochure from paint shops were provided with some worksheets.

The Results

The teaching and learning sessions began with the introduction of the task: painting the interior walls of a house. A rainbow colour wheel (Figure 3) was presented to the participants and three colouring schemes were introduced:

1. A monochromatic scheme: Based on one colour, but in many shades and different hues, from dark to light.
2. An analogous scheme: Using the adjacent colours on the colour wheel.
3. A complementary scheme: Using the colours opposite each other on the colour wheel.

Figure 3. The colour wheel.

After the participants had chosen their colour scheme, the colour catalogues consisting of a variety of colours for different colour schemes were given for the participants to practice the colour mixing in the ICT tool. Most participants were very much motivated in this colour mixing activity. They spent a few hours to play colour mixing with adding, blending, collecting, and resetting actions. Some of the participants actually learnt how to derive the secondary colours such as mixing yellow and blue to get green. Few participants didn't bother to remember the formula for secondary colours but kept using trial and error to match the colours in the catalogues. For monochromatic scheme colours, most participants were able to utilise black or white to change the darkness or brightness. During the activity, proportional language such as "more red" and "more blue" was often heard from the participants as they instructed and suggested to each other. In many cases, this ICT tool was unable to create a perfect match of colour to the catalogue colour. The participants (of the same computer) had to negotiate to agree with the closest colour, and then record the colour formula (i.e., the ratio of each primary colour) to the colour table or on a worksheet. It was noted that in this initial activity, most participants' attention was focused on the topological sign of colour variations. It seemed that the participants were able to make sense of different ratios from the colour variations. It was not until the next session that interference of individual signs was revealed.

1:1≠2:2, The Interference of Topological Sign

In the next session, Rob (pseudonym) was proud to show his workbook to the researchers for he had created and recorded 100 different colours. The researchers commended his work and then invited him to recreate some of the results in the ICT tool. Rob went on and recreated those results on screen (Figure 4) as requested by the researchers.

My Colour Table

	My colours	Black	Red	Yellow	Blue	White	
1		0	1	1	0	0	Remove
2		0	2	1	0	0	Remove
3		0	2	2	0	0	Remove
4		0	1	1	0	1	Remove

Figure 4. Colour table recreated.

The researcher asked "Are colour 1 and colour 3 the same?" Rob looked at the screen and replied firmly "No, they are different colours". The researcher then suggested removing the colour 2 from the colour table so colour 1 and colour 3 sat next to each other. When colour 2 was removed, the researcher asked again "Are the two colours the same?" Rob looked and thought for a while, and then he said "They are different colours". The researcher then identified another similar ratio with three colours as 1:1:1 and 2:2:2 in Rob's workbook, and asked Rob to recreate colours again. However, even the two colours were sitting next to each other; Rob still claimed that they looked different. The researcher then asked Rob to look at the numbers showing 1:1 and 2:2, and pointed out that the ratios are the same. Despite the effort to explain, Rob still insisted that the colours looked different to him.

The Mysterious 4th, the Interference of Typological Sign

Luke, Ben, and Cam (pseudonyms) were working together to mix colours for the kitchen walls. They matched a few colours from the colour catalogue and collected those colours to the colour table (Figure 5).

My Colour Table

	My colours	Black	Red	Yellow	Blue	White	
1		0	1	2	2	2	Remove
2		0	1	4	3	3	Remove
3		0	1	3	3	3	Remove
4							Remove

Figure 5. Colours for kitchen.

The researchers questioned "What portion is red colour in colour 1? and how about the other colours? Can you write down the fractions of individual colours in colour 1?" Luke was confident. He wrote down quickly the fractions on a paper as:

	Red	Yellow	Blue	White
Colour 1	1/4	2/4	2/4	2/4

Surprised, the researchers then asked Luke to also write down the fractions for colour 2 and 3. Luke continued to write down fractions as:

Colour 2	1/4	4/4	3/4	3/4
Colour 3	1/4	3/4	3/4	3/4

Ben and Cam seemed to agree with what Luke wrote: all the denominators are four. The researchers soon realised that the whole had been lost. Instead, the whole has been linked to the number of the primary colour used. Because there were four primary colours used (i.e., red, yellow, blue, and white), it gave a wrong sign for the denominator. The researchers then pointed out the container with the number showing the whole of the mix. The three participants were then able to recognise the whole and make sense of the idea about fractions.

Discussion

The two episodes above confirmed and illustrated the incomplete nature of signs to represent the object (mathematical concepts). Depending on different interpretants, a sign may also carry meanings that are not relevant to the object. This applies to both topological and typological signs. Even the social-actional resources, whilst they can reinforce correct ideas to facilitate knowledge building, they might also reinforce incorrect ideas.

Notably, topological resources tend to attract the attention of learners first. Topological resources allow a certain degree of error or uncertainty. In typological resources, accuracy must be met, which could be a source of being perceived as cold, abstract and difficult. However, over-reliance on topological resources can also impede the development of deep understanding of mathematical structures. This is evident in Rob's example, in which he strongly believed in his vision and totally ignored the meaning of the ratio notation.

The importance of multiple representations or the use of multiple semiotic resources is also confirmed in this study. The results showed that when multiple semiotic resources were utilised, the mathematical ideas or concepts could be better learnt. The future design of this ICT tool should also focus on how to minimise the interference of signs, and maximise the meaning emission capacity of signs.

Conclusion

This study has presented a semiotic account of knowledge representations and mathematics as a meaning-making endeavour. This paper will now conclude with two important implications derived from this study:

1. Viewing mathematical knowledge representations from a semiotic perspective is more inclusive than other existing classifications of representations in the research literature. The topological (meaning by degree) and typological (meaning by kind) resources have encompassed most material processes. And with the social-actional resources linking history, culture, and real-world applications, learning mathematics would be most meaningful to learners.
2. The semiotic framework is essential for the design of ICT tools for learning mathematics. ICT tools that are informed by the semiotic framework will seek out to utilise any possible meaning-making resources across typological, topological and social-actional resources. This will also lead to more creative and innovative design of ICT tools.

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