The Role of Information Graphics in Mathematical Proficiency

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There is scant research on the role of graphics in students' mathematical performance. This paper distinguishes between the contextual and informational roles of graphics and provides an overview of the types of information graphics. It also presents findings from a new mathematics instrument that has been used to quantitatively and qualitatively assess students' performance on information graphics. Key findings using this instrument have provided insights into age, gender and item effects on performance, and difficulties that students experience interpreting graphics.

The development of a mathematically literate populace is a key goal for educators:

> (Mathematical literacy) is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2003)

Hence, students need to become proficient in interpreting *information graphics* (e.g., graphs, tables, maps) because such graphics are used to manage, communicate, and analyse information (Harris, 1996). Information graphics are distinct from contextual graphics in that they represent mathematical information that supplements rather than complements the text or symbolic expression. For example, in Figure 1, the picture of the scales *complements* the text but contributes no mathematical information. In contrast in Figure 2, the picture of the scales *supplements* the text and is essential to the solution. The purpose of this paper is to provide an overview of information graphics in mathematics and to outline how students' proficiency with graphics can be measured.


Jan bought a new set of scales for $39. How much change did she receive from $50?

Figure 1. A contextual graphic.

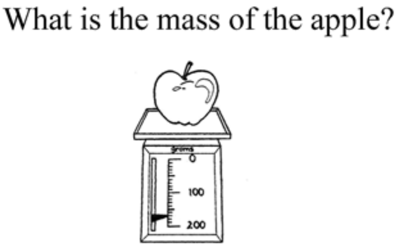
What is the mass of the apple?

Figure 2. An information graphic (Queensland School Curriculum Council, 2001b, p. 31).

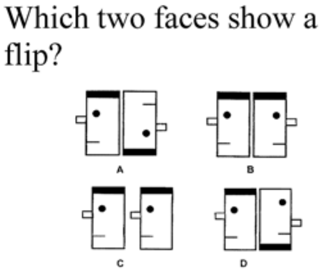
Which two faces show a flip?

Figure 3. A retinal list graphic (Queensland School Curriculum Council, 2001a, p. 13).

Information Graphics and Mathematics

Information graphics is a burgeoning field with over 4000 graphics in common use (Harris, 1996). In mathematics, these graphics variously convey *quantitative, ordinal* and *nominal* information through a range of perceptual elements (Mackinlay, 1999). These elements are position, length, angle, slope, area, volume, density, colour saturation, colour hue, texture, connection, containment, and shape (Cleveland & McGill, 1984). In mathematics, information graphics can be classified into six graphical languages which have unique spatial structures based on their perceptual elements and the encoding techniques that represent information (Mackinlay, 1999) (See Table 1). For example, Figure 2 is an Axis item because information is encoded by the placement of a mark on a vertical axis. Figure 3 is a Retinal-list item that uses orientation. Further graphic examples can be found elsewhere (Diezmann, Lowrie, & Kozak, 2007; Diezmann & Lowrie, 2006; Lowrie & Diezmann, 2007). Excluding the Miscellaneous languages, the other five languages have substantial similarities within their categories. For example, although there are different kinds of maps, maps typically convey information about the location of landmarks and the distance between locations. In contrast, Miscellaneous languages are broad and undefined and include for example pie charts and calendars.

Table 1

Descriptions of Graphical Languages and Encoding Techniques

Axis Languages (e.g., horizontal and vertical axes	A single-position encodes information by the placement of a mark on an axis.
Opposed-position Languages (e.g., line chart, bar chart, plot chart)	Information is encoded by a marked set that is positioned between two axes.
Retinal-list Languages (e.g., Graphics featuring colour, shape, size, saturation, texture, orientation)	Retinal properties are used to encode information. These marks are not dependent on position.
Map Languages (e.g., road map, topographic map)	Information is encoded through the spatial location of the marks.
Connection Languages (e.g., tree, acyclic graph, network)	Information is encoded by a set of node objects with a set of link objects.
Miscellaneous Languages (e.g., pie chart, Venn diagram)	Information is encoded with additional graphical techniques (e.g., angle, containment).

Assessing Proficiency with Information Graphics

Typically, students' ability to decode particular types of graphics is not the focus in mathematical tests. However because knowledge of graphics impacts on mathematical performance (Baker, 2001), we constructed an instrument to assess students' knowledge of graphics. The Graphical Languages in Mathematics [GLIM] Test is a 36-item multiple choice test that was designed to investigate students' knowledge of each of the six graphical languages. The test was composed of mathematical items containing information graphics that were sourced from state, national and international tests that have been administered to students in their final three years of primary school. The purpose of using previously published items was to ensure that items (a) were representative of what students were expected to be able to do in the upper primary years, and (b) had been subject to rigorous quality control in item development. We compiled a database of possible test items from a large range of published tests, excluded contextual items, and classified remaining items according to the graphical languages. Due to the limited Connection items in existing mathematics tests, content free Connection items were included from published science tests for the same age group. In total, we identified 58 suitable items for trialling. This set of items was variously trialled with primary-aged children ($N = 796$) in order to select items that: (a) required substantial levels of graphical interpretation, (b) required minimal mathematics content knowledge, (c) had low linguistic demand, (d) conformed to reliability and validity measures, and (e) varied in complexity. A total of six items were selected for each graphical language and arranged according to difficulty based on students' performance on the trial. Our final selection of 36 items was validated by two experienced primary teachers. These items were then arranged so that every sixth item belonged to the same graphical language. For example, Items 1, 7, 13, 19, 25 and 31 are Axis language items in ascending order of difficulty. Figure 2 is Item 31 without the multiple choice answers. We have successfully used the GLIM test over a 3-year period in both mass testing and interview situations. A description of the administration of the GLIM test under these two conditions and some key findings follow.

The GLIM Test in Mass Testing Situations

The multiple-choice GLIM test can be administered in mass testing situations (approximately 45-60 minutes). The items are scored 1 and 0 for correct and incorrect responses respectively. Students' performance is calculated on each language subtest and the overall test. The maximum scores for the subtests and whole test are 6 and 36 respectively. We have used the GLIM test to monitor primary students' performance on interpreting information graphics over a 3-year period through annual administrations of this test. The participants in the 3-year mass testing study were 327 students (Female = 148, Male = 169) from nine primary schools in two states. In the first year of the study, students were approximately 9- 10-year-olds. Three points of interest have emerged from the various analyses of students' results during this project. First, although many of the information graphics are not explicitly taught to primary students, their performance was significantly higher year by year on each graphical language. Second, the analyses reveal gender differences in favour of boys over time. These first two points are discussed in Lowrie (this symposium). Third, after accounting for gender, spatial ability was a contributing factor to students' success (Lowrie & Diezmann, 2007).

The GLIM Test in Interview Situations

We have also interviewed students on items from the GLIM test. The interview students were sourced from different schools to the mass testing cohort. For pragmatic reasons, we interviewed students on 12 different items each year for three years. In the first year of the study, students were interviewed on the easiest pair of items from each graphical language. Students were approximately 9- 10-year-olds. In the second and third years the same students were interviewed on the moderately difficult items and the most difficult items respectively. In each year, the students completed one pair of items at a time from the same graphical language and then explained their responses. Students were encouraged to justify their thinking but no scaffolding was provided by the researcher.

Our qualitative analyses revealed three key issues of interest. First, some students have incorrect conceptions of graphics. For example, some students interpret a structured number line (Axis language) as a counting model rather than a measurement model (Diezmann & Lowrie, 2006). Second, students' conception of a graphic is manifest in how they use it. For example, students who hold a measurement conception of a number line justify their responses to the identity of a missing number in terms of points of reference, relative proximity to given numbers and directionality (Diezmann & Lowrie, 2006). Third, high and low performers interpret graphics differently. For example, high performers used multiple cues from the graphics and were more knowledgeable about everyday graphics (e.g., calendar) than low performers (Diezmann et al., 2007).

Concluding Comments

The GLIM test has provided a useful instrument for gaining insight into students' performance on graphical languages and the issues impacting on their performance. Our experience on this project has increased our resolve that in the Information Age, students' ability to interpret information graphics is fundamental to their mathematical proficiency. That is, just as the mathematical and linguistic demands of an item impact on performance so too does the graphical component. This point has implications for the credibility and interpretation of new national tests (see Diezmann, this symposium). In addition, the extent to which various components of an item impact on performance has been further explored by members of our project team (see Logan & Greenlees, this symposium). (For a full set of papers relating to our research on graphical languages in mathematics see the project website http://www.csu.edu.au/research/glm/index.htm).

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