Challenging Mathematical Conversations

Jill Cheeseman
Monash University
Jill.Cheeseman@education.monash.edu.au

One finding from a larger investigation of teacher-child interactions that challenged young children to probe their mathematical understandings is reported in this paper. It is the interlinked nature of the conversational exchanges between teacher and child in mathematics lessons in the early years of primary school. These conversations serve to support, extend and challenge children's thinking. One detailed example of such mathematical conversations, the story of Jordan, is presented to illustrate the phenomenon.

Children construct mathematical ideas in the course of their interactions with their teacher and classmates (McNeal, 2001; Mercer, 1996). Interactions in whole class settings have been studied (Kyriacou & Issitt, 2008; Mercer, 1996; Mercer & Littleton, 2007); however, little is known about the interactions between teacher and child in one-to-one conversations during the mathematics lessons of young children.

The behaviours of highly effective teachers as they conversed with young children in mathematics lessons were investigated in a detailed study (Cheeseman, 2009). Of particular interest what was described by Alexander (2005) as dialogic teaching, the sort of exchanges that Kyriacou and Issitt (2008) found only to a minor extent in their review of research in the United Kingdom. They observed that some teachers made use of extended dialogue, asked children to explain or justify their method, and asked follow-up questions. They also found that teachers engaged in one-to-one private dialogue when children were working individually on set tasks. However, they noted that "surprisingly little research is reported on the dialogue during such interactions" (p. 8). This is the gap in the literature that my larger study addressed. The practice of four teachers was closely examined, analysed, and described. One of the findings of the study is reported in this paper – the interlinked strings of conversations that teachers conducted as they challenged children to probe their mathematical thinking.

Brown and McIntyre (1993) attempted to describe and theorise the "professional craft knowledge" of teachers. They said: "While we recognise that there are those with mastery of some aspects of teaching, we have no coherent account of what they are masters of or how they achieve what they achieve" (p. 13).

Ainley and Luntley (2005) argued that expert teachers possess knowledge that is "contextualised and attention-dependent" (p.74). A pilot study by these researchers examined what teachers attended to in the "relative invisibility of minute-by-minute practices in the classroom" (p.74). At the time of writing, their work was still in its developmental stages. However, exactly what "master teachers" or "expert teachers" do as they interact within the classroom continues to be of interest to researchers.

Background

Highly effective teachers were identified and studied in the Early Numeracy Research Project (Clarke, et al., 2002). Common characteristics of the teachers' practices were described. One category of behaviours that emerged from the data concerned the creation of classroom learning communities. As a member of the original research team, I was of the view that the challenging mathematical conversations between teachers and children

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1). Palmerston North, NZ: MERGA.

were a key to the learning gains the teachers could help young children achieve. To investigate in greater depth the nature of classroom conversations between teachers and children, the practices of the teachers were examined with a focus on the conversations (Cheeseman, 2009).

Much has been written about mathematical discourse, talk, questioning, and listening in the mathematics classroom. However, the ways in which teachers actually go about constructing and developing exchanges is not so well documented. There has also been a call for more of what Mercer (2000) termed dialogic teaching. This style of teaching is compatible with a social constructivist theory of learning that holds that learning occurs or is constructed in a social situation where meaning is made between the participants (Wood, 2001).

Methodology

Complementary accounts methodology was described and used by Clarke (2001). Data collection procedures that constructed "integrated data sets" combining videotape and interview data were used, the reflective voice of participant teachers and students in the data set were included, and a research team to carry out a multi-faceted analysis of classroom data was utilised. While the methodology adopted for this study differed from that of Clarke (2001), similar fundamental techniques were used including: videotaping the whole mathematics lesson, audio taping participants' reconstructions of classroom events, and analysing the multiple data sets. The points of difference in the present study were:

- the focus of the study – the teacher's interactions involving negotiations of mathematical meaning with children,
- the age of the student participants - children were in the first three years of primary schooling, and
- the use of video-stimulated recall to prompt children 5 to 7 years of age to reflect on their mathematical thinking.

These adaptations to the original complementary accounts methodology proved useful in capturing the complexity of early primary mathematics classroom interactions. Each teacher's classroom was observed and data were collected over several "ordinary" mathematics lessons on consecutive days.

The data were treated in cycles of analysis in an iterative process. Researchers in mathematics education have reported similar approaches to the analysis of their data. Groves and Doig (1998) reported viewing videotapes many times, interspersing transcription and summarising with viewing. Cobb (1995) described analysis of data that "involved a continual movement between particular episodes and potentially general conjectures" (p. 35). Each of these descriptions has some resonances with the process used in this study. The iterative process used here involved many viewings of the data, alternating between a "close up" focus and a "standing back" to gain some perspective on the data, and raising and testing conjectures about the findings. Lampert (1990) characterised the "zig-zag" path of mathematical activity from conjectures to proofs by "revising conclusions and revising assumptions in the process of coming to know" (p.30).

The starting point was the video record of one lesson, and the data for each teacher were analysed prior to beginning the analysis of data from a different teacher. In this way a picture was built of the phenomena under examination in isolated sessions and it evolved to an analysis of a collection of related sessions (Lesh & Lehrer, 2000). A descriptive summary of the practices of each teacher was written, examples of the phenomena as evidenced in that classroom were detailed, and features of each specific interaction were listed. In this way events that had initially received equal attention gradually evolved so that some received special attention as exemplary or illuminating. Following an analysis and reporting of each lesson's data for one teacher, patterns and similarities were sought in the behaviours observed. After the data from all four teachers' were analysed, a cross-case analysis was conducted.

Cycles of analysis were used to telescope or look more intensely and in detail at the data. They began by giving attention to entire videotaped lessons and ended with a deeper and more time consuming focus on a relatively small number of specially selected events. Individual portrayals of teachers' practices with regard to the phenomena under investigation were developed from observations, videotapes, and interviews. As Stake (1994) noted, "it may be the case's own story, but it is the researcher's dressing of the story [with] the aim of finding the story that best represents the case" (p. 240).

Interlinked strings of mathematical conversations

A detailed analysis of the challenging mathematical conversations undertaken by four teachers showed that each teacher created interlinked conversations with children during their lessons. Each teacher had a different style and pattern of interaction, and yet each of them developed strings of mathematical conversations with a handful of children each day. The teacher whose practice is used as an example in this paper, focused principally on one or two children, having four or five different exchanges with each of them. During the same lesson she also engaged another four children in conversations that consisted of two or three exchanges. These conversations were in addition to one-off exchanges that happened with many other children during the lesson.

Challenging Mathematical Thinking: The Story of Jordan

The story of Jordan comes from a mathematics lesson "Sue" taught with children in their first year of school. The problem of the day was: "A teacher asked the class to go up on the stage for assembly. She asked half of the class to sit and half of the class to stand. How might that look?" The 5-6 year-old children were asked to draw something on a large piece of paper to show their solution to the problem.

Jordan was selected during the initial setting up of the lesson to show half of a piece of string. As he adjusted the ends of the string to make them match, Sue asked him to explain to the children why he was doing that. He replied that he wanted to get it "just the same".

Later Sue called a group of four children to work with her on the floor. She told them that they were going to use materials to work the problem out. Sue had collected a container of two-coloured counters which she tipped onto the floor. Patrick and Jordan decided that they wanted something different so they were sent off to get the materials that they were going to use. Soon they returned with a container of plastic dinosaurs. Patrick demonstrated to Sue how the 'sitting down' group would be tipped over onto its side while the 'standing up' group would be upright dinosaurs. Sue then restated the problem and left them to get on with finding a solution.

Jordan was slow to get organised and Sue was quite insistent that he show her what he planned to do. Once he became engrossed with the task she turned her attention to others in the group. From time to time she asked for an explanation or justification of Jordan's thinking about what he was doing. He had placed two rows of dinosaurs on the paper, one row consisted of nine figures standing up and the other was of nine lying down. When he was asked what he had, he replied that he had half standing and half sitting. Sue pressed him for how he knew that. He said that they were “equal”. Sue said, “What’s equal?” and got another dinosaur and put it at the end of the standing row. Jordan said that wouldn’t be half because one was longer. It was clear what Sue was probing for. Was Jordan looking at the length of the two rows or the number of objects in each row? Could Jordan conserve number? Did Sue know something about Jordan’s number understandings that made her keep questioning him? She still pressed for an explanation of why it was half. Then she was immediately satisfied when he explained that the numbers have to be the same.

Sue then swung Jordan’s focus to the number of objects in the whole “class”. To find how many were in the class altogether Jordan counted the standing row in twos to 8 and on from 9 to eighteen. He needed support to count beyond 12 in twos. He commented something about “it’s counting by nines”. He added that he could do it if he could count by nines. Sue asked him whether he could count on a calculator then sent him to get one. As he was walking back to his workspace on the floor he had already keyed in the strokes to have his calculator count by nines. (Sue later wrote the keystrokes he told her on his paper as 9 + = =). He sat on the floor calling out the sequence of numbers he was generating on the calculator display 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108. He was rocking back and forth and smiling to himself. Sue was smiling too then she said “so if you could count by nines you could do it, or you could get the calculator to count by nines, couldn’t you?” Next she challenged Jordan to write down what he had done. Having worked with several other children, Sue came back to Jordan. He had put out four rows of dinosaurs by that time and knew that there were 36 of them on the floor. Sue asked him to run through what he had done and to say how many groups of nine he had begun with. She emphasised the groups of nine by recording what he told her. She then encouraged Jordan to think about his extension of the original problem, where he had two rows of people sitting and two rows standing. Sue recorded, in symbols that made sense to him, the solution that Jordan had found using materials (Figure 1).

There was a little comment by Jordan when Sue was drawing the stick figures sitting and standing. It had the familiarity of a child sitting beside his mother as she drew him a picture, Jordan sat close and smiled at Sue:

Jordan: You’re really good at that.

Sue: I’m not really; I don’t quite know what I’ m doing with this leg. What will I do with that one? Anyway he’s sitting isn’t he?

Jordan: Mm.

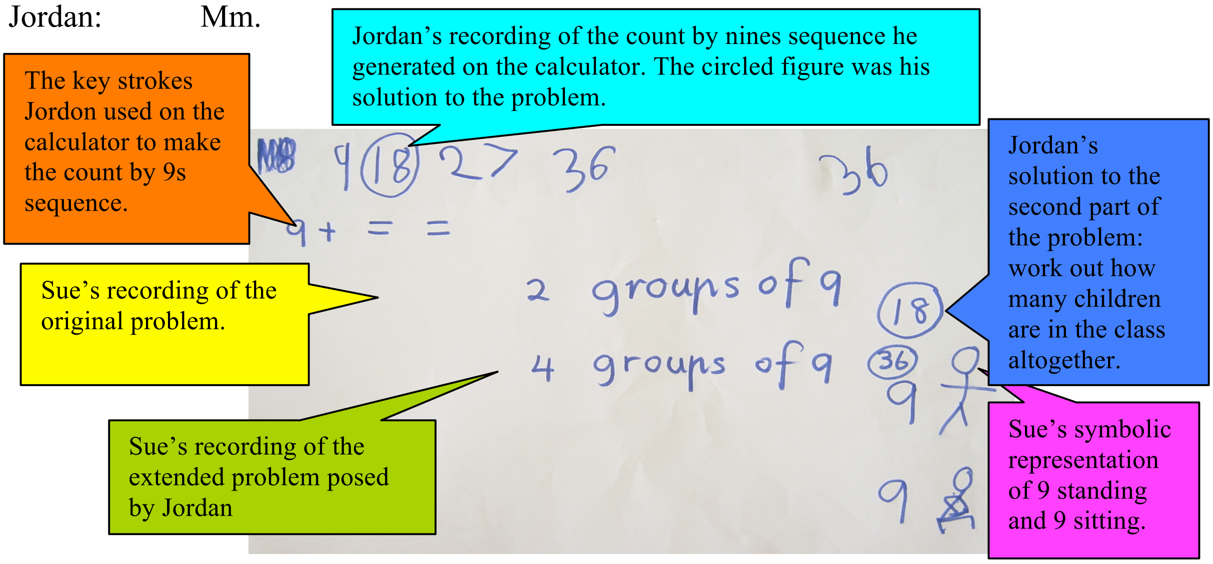

Figure 1. The front of Jordan’s sheet of paper.

Jordan proved to Sue that there were 18 in the whole class. He used the calculator to skip count by nines. Sue then issued another challenge: “If you would like to, have a go at counting by nines and see if you can see a pattern”.

After the children had worked on solutions to the problem for about 40 minutes the class was given notice to finish and to pack up so that they could be ready to share their ideas. Three children, including Jordan, were asked to be ready to tell what they had done. The first child to share was Nicholas. By asking leading questions, Sue led him through a succinct description of his solution – six standing and six sitting was 12 altogether. She asked him to justify his solution in terms of the problem then focused on the mental computation strategy that he used to find the total. She described how he was originally going to “count all” of the people he had drawn but she had challenged him to do something else. Nicholas then said that he remembered when they were using dice and he knew that 6 and 6 was 12. Sue connected that to a previous time when they had found double letters and introduced “doubles” in numbers. She asked Jordan what you get when you double 9. He responded immediately with 18.

Brianna was asked to talk about her work especially to say how she changed her original solution and why. She originally had 4 children standing and 5 sitting. In asking Brianna to explain her thinking Sue seemed to be emphasising the need for two halves to be equivalent in number. She was also praising the correction of flawed thinking.

Then Sue asked Jordan to report to the class. She set up the report by describing the fact that Jordan had dinosaurs and asked Jordan to describe what he did to start with. He talked about how he had nine sitting and nine standing and as he was doing so he read his paper and picked up his calculator and pressed the keystrokes to generate a count by nines pattern. Sue stopped him there to state the solution.

Then using the thinking that Jordan did as an extension to the problem of the day, Sue capitalised on the opportunity to have the class look at the sequence of numbers generated by counting by nines and to identify and predict some patterns. Children were then gathered around Sue to look at what Jordan found and to check what he had done. First Jordan generated the sequence of numbers he had produced by getting a calculator to skip count by nines. Then Sue challenged the children to explain why he had written the pattern as: 9, 8, 7, 6, 5, 4, 3, 2, 1 (Figure 2). Lexie described Jordan’s figures as “the numbers on the end”.

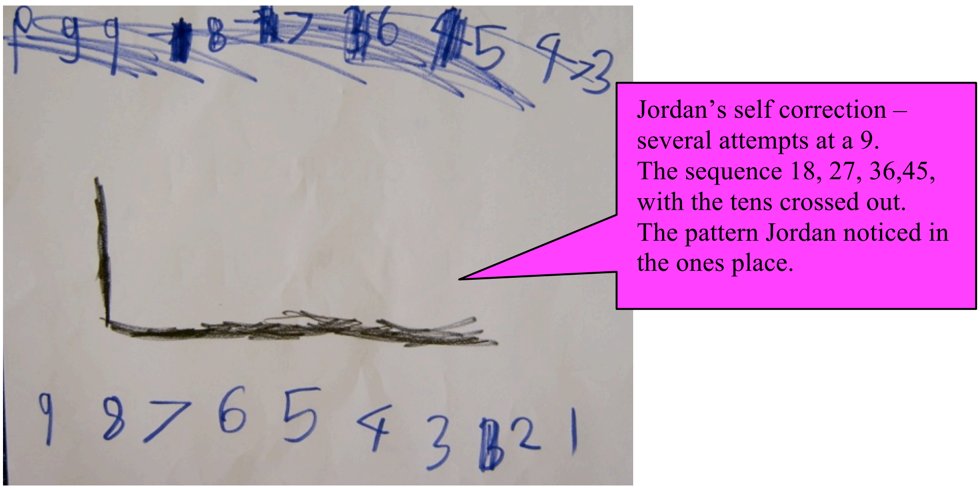

Figure 2. The back of Jordan’s sheet of paper.

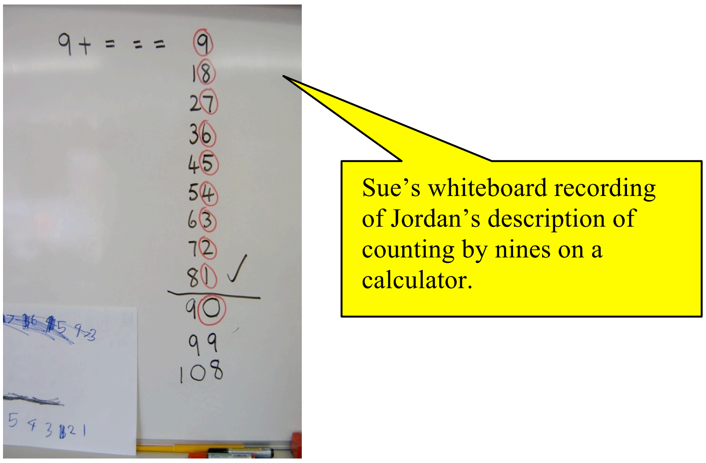

Figure 3. Jordan’s work and Sue’s use of the pattern he had found.

Sue then orchestrated a scenario where the children were able to predict the next number in the pattern and she even challenged them to search for a pattern in the tens column as well (Figure 3).

The children had been concentrating on mathematics for a long time and there were a few minutes till recess so Sue said that they could have a few quiet minutes to do what they wanted, to eat some “brain food” (fruit and vegetable snacks that they have close at hand) or to play with a calculator to do what Jordan did. It was very interesting to see that at least half of the class got a calculator to investigate “Jordan’s numbers”.

A timeline of Sue’s interactions with Jordan in Lesson 2 can be seen in Figure 4 It reveals that Sue interacted with Jordan on ten separate occasions and on two of these occasions she spent an extended period of time engaged in conversation with him. The interactions with Jordan can be seen to take a general form: raise a challenging question requiring thought and action, leave the child to do some mathematics, review and raise a new challenge, leave the child to do more mathematics, require a report of mathematical thinking.

Features of Sue’s challenging behaviour with Jordan

During the exchange with Jordan, Sue:

- asked Jordan to demonstrate half of a continuous quantity;
- required him to model the problem;
- had him explain his solution;
- expected that he justify that solution;
- asked him to generalise the concept of half of a discrete quantity;
- stimulated him to consider the “whole”;
- requested that he calculate the sum of two equal sets;
- expected the transfer his previous experience and knowledge of skip counting on a calculator to the current problem;
- prompted him to link his skip counting by 9 to “groups of 9” i.e. connect repeated addition to emerging multiplicative thinking;
- asked him to write and read 2-digit numbers;
- challenged him to search for pattern in number – the nines sequence; and
- expected him to describe what he did and what he found to other children.

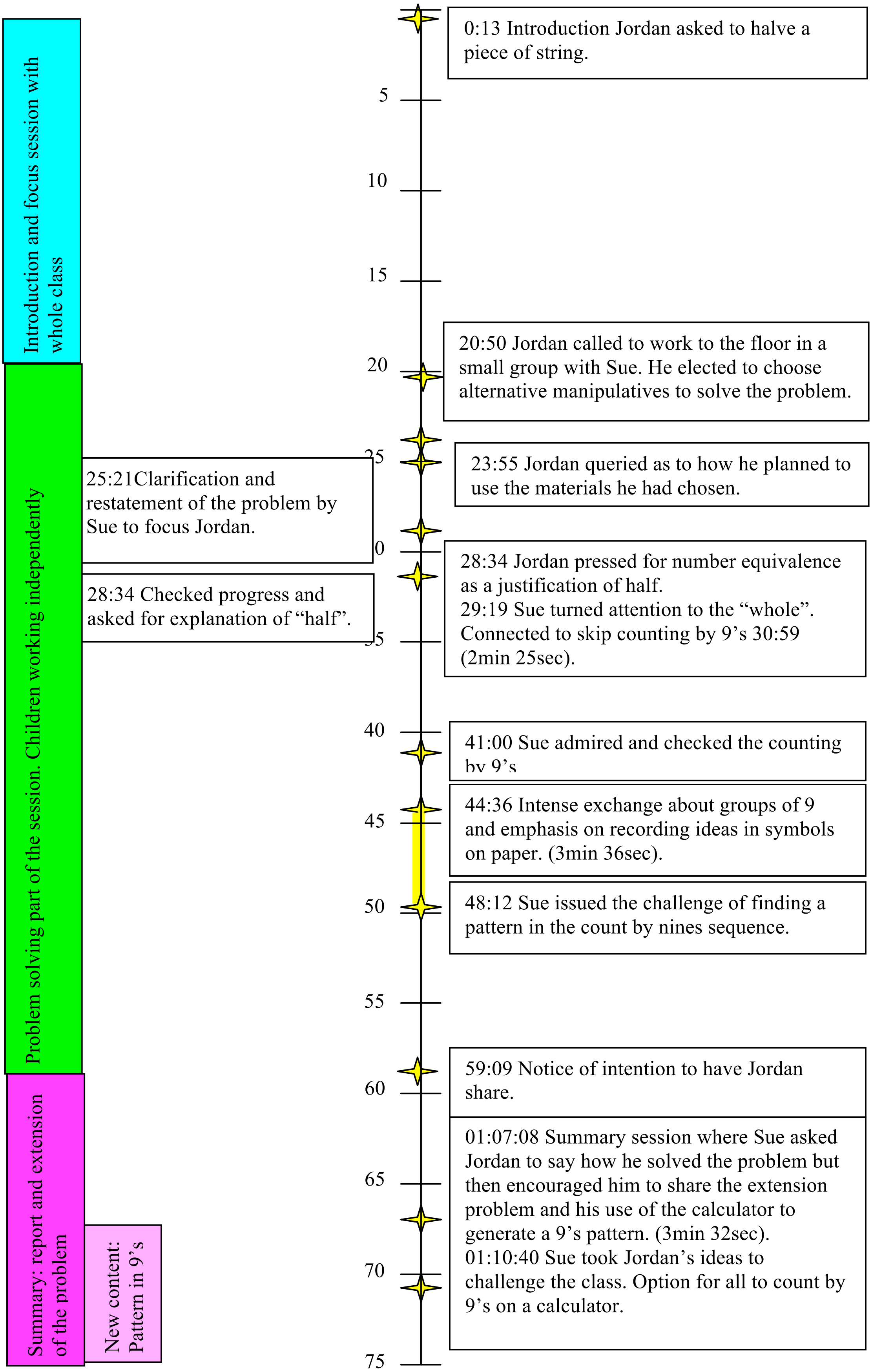

Figure 4. Timeline of Sue's interactions with Jordan during the lesson.

In conclusion

The exchanges with Jordan illustrate the interlinked mathematical conversations teachers have with young children during mathematics lessons. While Sue's particular style of interaction may be unique, each of the teachers in the study (Cheeseman, 2009) conducted strings of interlinked interactions with their children. As can be seen from the features of the conversations with Jordan, it is during such exchanges that children are called on to demonstrate, model, explain, calculate, justify, generalise, transfer, connect, and describe their mathematical thinking. It is clear that such opportunities to challenge children to think mathematically are an important component of the practice of highly effective teachers of mathematics with young children.

References

Ainley, J., & Luntley, M. (2005). The role of attention in classroom practice: Developing a methodology. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce & A. Roche (Eds.), *Building connections: Research, theory and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 73-80). Melbourne: MERGA.

Alexander, R. (2005). *Towards dialogic teaching: Rethinking classroom talk*. Rose Hill, North Yorkshire, UK: Dialogos.

Brown, S., & McIntyre, D. (1993). *Making sense of teaching*. Buckingham, UK: Open University Press.

Cheeseman, J. (2009). *Challenging children to think: An investigation of the behaviours of highly effective teachers that stimulate children to probe their mathematical understandings.* Unpublished PhD Thesis, Monash University, Melbourne.

Clarke, D. M., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., et al. (2002). *Early Numeracy Research Project: Final report, February 2002*. Fitzroy, Victoria: Australian Catholic University, Mathematics Teaching and Learning Centre.

Clarke, D. J. (2001). Complementary accounts methodology. In D. J. Clarke (Ed.), *Perspective on practice and meaning in mathematics and science classrooms* (pp. 13-32). Dordrecht: Kluwer.

Cobb, P. (1995). Mathematical learning and small-group interaction: Four case studies. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 25-129). Hillsdale, NJ: Lawrence Erlbaum.

Groves, S., & Doig, B. (1998). The nature and role of discussion in mathematics: Three elementary teachers' beliefs and practice. *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (pp. 17-24). Stellenbosch, South Africa: PME.

Kyriacou, C., & Issitt, J. (2008). *What characterises effective teacher-initiated teacher-pupil dialogue to promote conceptual understanding in mathematics lessons in England in Key Stages 2 and 3: A systematic review*. London: EPPI-Centre, Social Science Research Unit, Institute of Education, University of London.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analysis of conceptual change. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum.

McNeal, B. (2001). Making sense of mathematics teaching in real contexts. In T. Wood, B. S. Nelson & J. Wakefield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and Instruction, 6*(4), 359-377.

Mercer, N. (2000). *Words and minds: How we use language to think together*. London: Routledge.

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking*. London: Routledge.

Stake, R., E (1994). Case study research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research*. London: Sage.

Wood, T., Nelson, B., & Warfield, J. (2001). *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.