The Effect of Reform-Oriented and Other Mathematics Curricula on Students' College Mathematics Placement Test Scores

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This study examined the college mathematics placement exam results of 1,277 students learning from nine secondary mathematics curricula and two Advanced Placement (AP) mathematics programs in 25 different high schools in the United States. The results suggest that students learning from several traditional mathematics programs and AP Calculus significantly outperformed students learning from the reform-oriented mathematics program, Core-Plus Mathematics Project, on algebra manipulation and calculus readiness questions. Prior mathematics achievement, course completion, and gender also significantly influenced mathematics placement scores.

Students cross a number of divides as part of their schooling in different countries around the world. The divide between secondary and postsecondary education, which the majority of students in the United States encounter at some point on their educational journeys, is one of the most difficult for students to cross for several reasons. First, postsecondary education places greater demands on students, which can result in a decrease in their mathematics achievement or a dampening of their disposition towards mathematics (Smith & Star, 2007). Second, many of these students are required to complete college mathematics placement exams, which determine the mathematics courses in which they will enrol when they begin their postsecondary education. These exams are a form of high stakes assessment (Wilson, 2007) in that they have serious consequences for students. For instance, if students do not perform well, they may be required to take remedial mathematics courses, which will increase the time required for students to earn a college degree. Despite the import of these exams we are only beginning to understand how different secondary mathematics programs prepare students for these assessments.

These placement exams are comprised primarily of algebra manipulation tasks (Sattler, 1999). The nature of these exams is important as a number of students are learning from reform-oriented mathematics curricula based upon reform documents developed in the United States (e.g., National Council of Teachers of Mathematics [NCTM], 1989). These programs include features such as real-world contexts as the source of mathematical ideas, technology in the form of calculators or computers, and the incorporation of several different mathematics subject areas during each year of study (e.g., discrete mathematics). Some evidence suggests that students learning from one of these reform-oriented mathematics programs, Core-Plus Mathematics Project (CPMP) (Coxford et al., 1998), do not fare as well on algebra manipulation questions as students learning from other commercially developed mathematics programs[1] (Schoen & Hirsch, 2003a).

The CPMP program is designed for secondary mathematics students, ages 13-18. There are four courses, each of which contains units on algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. Each lesson consists of a launch, an investigation, a share and summarise, and an apply section. In the launch, the

[1] These are programs that focus on one mathematical area during each year of student in high school and typically contain fewer innovative elements than reform-oriented programs.

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students read about a real-world situation, think about and answer several questions about the context and the mathematics embedded within it and discuss these answers as a large group. In the investigation, the students work in groups on questions related to the launch in order to rediscover important mathematical ideas and procedures while the teacher asks questions and gives hints. During the share and summarise the students work on a set of questions which are intended to summarise the important ideas in the investigation. The teacher facilitates a discussion around the answers to these questions to assist the students in understanding the main ideas of the investigation. Students work on questions in the apply portion of the class either on their own or with their classmates related to the investigation's mathematical focus.

Due to these findings, several research studies have begun investigating how CPMP students perform on college mathematics placement exams. Schoen and Hirsch (2003a) followed CPMP students and students who had experienced a commercially developed curriculum from two schools within the same school district into a university setting. Schoen and Hirsch found no differences in terms of the grades that these two groups of students earned in their first mathematics courses. They also found no difference in the number of students who had enrolled in college calculus. However, none of these differences were tested for statistical significance.

Davis and Shih (2007) examined students learning from CPMP and the University of Chicago School Mathematics Project[2] (UCSMP) (Usiskin, 1986) in two high schools located in the same school district. Four different groups were identified: three years of CPMP, four years of CPMP, at least four years of CPMP and UCSMP, and at least four years of UCSMP. The group who had learned from four years of CPMP comprised the comparison group for all statistical tests. These groups were used since the CPMP developers included more symbolic manipulation during the fourth year of the program. Thus it was possible that students who had learned from the fourth course of the curriculum would perform significantly better than students who had learned from the first three years of the program. Students who had studied four or five years from UCSMP outperformed CPMP students who had studied from the curriculum for four years on both the algebra placement exam ($N = 192$, $t = 2.296$, $p < 0.05$) and the calculus readiness exam ($N = 93$, $t = 4.701$, $p < 0.01$). Students who had studied from a mix of UCSMP and CPMP outperformed the four-year CPMP students ($N = 93$, $t = 1.980$, $p < 0.05$) on the calculus placement examination. In all of these comparisons, gender and prior mathematics achievement were controlled. However, these results came from a dual choice school. In some of these schools, students of higher perceived ability are persuaded by teachers to take UCSMP while students of lower ability enrol in CPMP. Thus the UCSMP classrooms reflected a different set of expectations that were not considered in our model, but can influence student achievement (Oakes, 1985).

Harwell et al. (in press) researched the first college mathematics course enrolment and first college mathematics course grades of 1,667 students learning from reform-oriented mathematics programs including CPMP and commercially developed mathematics programs in 85 high schools. They found that students learning from reform-oriented programs were more likely to enrol in a less difficult class than students who had

[2] The UCSMP program is aligned with reform-oriented mathematics programs such as CPMP, but also contains elements that are more traditional (e.g., a focus on a sequence of algebra-geometry-advanced algebra-precalculus).

experienced commercially developed mathematics programs (Coefficient = 0.717, $p < 0.01$). More specifically, students learning from commercially developed mathematics programs were nearly twice as likely to enrol in calculus as a first college mathematics course than students learning from other programs. There were, however, no differences in terms of the grades that students earned in their respective courses. This study possesses the same design flaw as Davis and Shih (2007) as reform-oriented students were drawn from dual choice high schools. Combining different mathematics programs as was done in Harwell et al. is also problematic in that reform-oriented programs differ from one another. Additionally, combining curricula in this way can lessen the effect of a particular program (National Research Council, 2007).

Hill and Parker (2006) compared CPMP students with a large group of students learning from other programs on their first mathematics college course enrolment at Michigan State University in the United States. They found that CPMP students were more than twice as likely to enrol in what they described as remedial mathematics programs ($N = 3{,}314$, $z = 3.97$, $p < 0.0001$). These are classes that cover content that is considered to be equivalent to the mathematics that students would encounter in high school. This is an important finding as research by Adelman (2006) suggests that as students increase their remedial course-taking their chances of completing a bachelor's degree decrease. Hill and Parker also examined the grades that CPMP students received in different entry-level college mathematics courses. CPMP students outperformed the comparison group in college calculus ($N = 311$, $z = -2.05$, $p < 0.05$). The comparison group, on the other hand, outperformed CPMP students in college algebra ($N = 458$, $z = 2.39$, $p < 0.05$) and remedial mathematics courses ($N = 311$, $z = 2.11$, $p < 0.05$). It is important to note that Hill and Parker did not have access to students' high school course transcripts, thus they could not be sure of the courses that students actually completed in high school.

Schoen and Hirsch (2003b) examined students who had studied from algebra, geometry, advanced algebra, and Precalculus with students who had studied from all four CPMP courses on a college mathematics placement exam consisting of beginning algebra, intermediate algebra, and calculus readiness sections. There were no statistically significant differences on the beginning algebra and intermediate algebra subtests. On the calculus readiness subtest, the CPMP students scored significantly higher than the comparison group ($N = 341$, $t = 4.93$, $p < 0.01$). Both groups of students were of higher ability as measured by standardised tests, but the study design did not include other student-level variables (e.g., gender), which is linked with student achievement (Harwell et al., in press).

The studies above are important in that they clarify how students learning from CPMP and other programs perform at the postsecondary level, but a growing number of students in the United States are also taking courses within the Advanced Placement program. The Advanced Placement (AP) program is administered by the College Board in the United States and consists of a set of courses in different subject areas, which give students an opportunity to earn college credit while attending high school. Research on this program typically lumps these different courses together and has shown that they have a positive influence on student achievement at the postsecondary level as seen in higher grades (Geiser & Santelices, 2004). However, research of this nature has not focused exclusively on the influence of AP Calculus AB or AP Statistics on postsecondary achievement.

This study was designed to answer two questions. (1) What is the influence of CPMP when compared with other secondary mathematics programs on combined algebra and

calculus readiness college placement exams when controlling for student characteristics (e.g., gender)? (2) What is the influence of AP Calculus AB or AP Statistics on combined algebra and calculus readiness college placement exams when controlling for secondary mathematics course taking and student characteristics?

Methodology

A total of 1,277 students learning mathematics in 25 high schools and graduating from 2000-2005 were included in this study. These students were from the upper Midwest portion of the United States and were predominately white. They were somewhat similar to the larger population of college students in the United States as the majority was female. The 25 high schools were located in rural and suburban areas. The schools were also of varying size with the largest having 2,400 students and the smallest just 400 students in grades 9-12 (ages 14-18). Students were enrolled as freshmen at Northern University[3] beginning in 2000. Eight schools used the CPMP program, 12 schools used the UCSMP program, and the last five schools used commercially developed mathematics programs from Prentice Hall, Addison Wesley, and McDougal Litell or used a combination of different programs such as UCSMP along with other programs or teacher-developed activities. These included UCSMP with teacher worksheet supplementation (UCSMP/WS) and UCSMP with trigonometry and college algebra (UCSMP/Tr/CA). One school was transitioning from UCSMP to a McDougal Litell program with students studying from both programs hence we coded them as UCSMP/ML.

Recent research on student achievement in the United States suggests that gender (Harwell et al., in press), ethnicity (Lubienski & Crockett, 2007), socio-economic status (Lubienksi & Crockett, 2007), and prior mathematics achievement (Harwell et al., in press) are important predictors of student achievement. Thus, these independent variables or their proxies (e.g., ACT Mathematics subtest for prior achievement) were used in this study.

Implementation is an important issue with regard to examining the impact of a particular curriculum. Indeed, in a recent review of six commercially developed programs and 13 reform-oriented programs, 52% of the curriculum comparison studies that met the committee criteria to be included in the review did not record any information on how the programs were implemented (National Research Council, 2007). As a minimum, the National Research Council recommended that information on course completion and professional development be gathered. We were able to gather course completion information on the nine different mathematics programs in this study by contacting the mathematics department chairs at each of the high schools, but professional development data proved to be too difficult to obtain. In order to develop course completion codes for students learning from a different program, we found the sum of the chapters appearing across all courses of the set of textbooks used at a school. For example, CPMP contains a total of 21 chapters across the first three courses. The last course contains 10 chapters, but we only included 8 of these since the developers recommend that students who are interested in the mathematical sciences study these chapters. Thus CPMP was conceptualised as having 29 total chapters. Students who completed the first three courses had completed 21 out of 29 chapters for a 72% course completion rate.

In addition, six out of eight of the schools using CPMP were field test schools which previous studies have assumed use the curriculum in ways that are more aligned with the

[3] Pseudonym.

developers' ideas about the curriculum (Schoen & Hirsch, 2003a, 2003b). In addition, we received information from some of the schools that while they had adopted a program like UCSMP, teachers made regular supplementations to the curriculum (UCSMP/WS). In these cases, this lack of fidelity to the program was coded as a separate mathematics program for analysis purposes.

Two dependent measures were used in this study: combined algebra placement test scores and calculus readiness subtest scores. Students at Northern University are asked to complete one of two different mathematics placement tests. The test that students take is based upon their mathematics backgrounds and ACT mathematics subtest. If students score below 21 on the ACT mathematics subtest and have taken two or three high school mathematics courses they are asked to take the algebra placement exam. This exam contains three sections: basic skills (20 points), beginning algebra (15 points), and intermediate algebra (15 points). Students who have a stronger high school mathematics background complete the calculus placement test which also has three sections: beginning algebra (15 points), intermediate algebra (15 points), and calculus readiness (20 points). All questions are multiple choice and students are allowed to use graphing calculators without symbolic manipulation capabilities on both tests.

The beginning algebra and intermediate algebra subtests appear on both placement exams, thus we made these two tests into a combined algebra assessment worth 30 points. Since these subtests appear on both assessments the combined algebra test had a larger sample size than calculus readiness. Altogether the combined algebra test covers different symbolic manipulation skills ranging from finding a linear equation in slope-intercept form to factoring a quadratic expression. The calculus readiness subtest consists of 20 multiple choice questions and includes symbolic manipulation, trigonometry, functions, and beginning calculus ideas like the area under a simple curve. The sample size for this question was smaller as it appears on only one of the placement exams.

The data analysed in this study were hierarchical in nature, with students nested within schools. Because the amount of variability beyond the student level on the calculus readiness subtest was less than 7%, we followed Lee's (2000) recommendation to use multiple linear regression. Using the same criterion, we used hierarchical linear modelling (HLM) in analysing algebra placement test scores (Raudenbush & Bryk, 2002). The use of HLM required us to combine students learning from UCSMP with those that had learned from this program and others (e.g., UCSMP/ML). No curricula were combined in the calculus readiness subtest analysis. All curricula variables were compared against CPMP, white students were compared against non-white students, and male students were compared against female students. A significance level of 0.05 was used for all statistical tests.

Results

The results for the combined algebra placement exam are shown in Table 1. The findings suggest that prior mathematics achievement as measured through ACT Mathematics and high school mathematics GPA was a statistically significant predictor of students' algebra placement exam scores. The percent of the course that students completed was also a significant predictor of algebra placement. AP Calculus AB, UCSMP, and Prentice Hall statistically outperformed students learning from CPMP when other variables (e.g., student gender) were controlled.

Table 1

Combined Algebra Placement Results $n_{Level\ 1} = 833;\ n_{Level\ 2} = 25$

Fixed Effect	Coefficient
Mean Algebra Placement	13.37***
Gender	0.013
White	0.54
ACT Mathematics	0.64***
High School Math GPA	1.63***
Parental Income	0.00
Course Completion	0.07***
AP Statistics	-0.72
AP Calculus AB	2.98***
UCSMP	2.63**
Prentice Hall	3.43*
Addison Wesley	1.68

*$p < 0.05$. **$p < 0.01$. ***$p < 0.001$.

The results for the calculus readiness subtest bore similarities to the findings for the algebra placement exam as seen in Table 2. That is, prior achievement proxies, course completion, and AP Calculus AB were again significant predictors. Students learning from UCSMP and Prentice Hall outperformed CPMP students on the calculus readiness exam. There was a also a gender effect in favour of male students.

Table 2

Calculus Readiness Subtest Results

	Coefficients (dependent variable – adjusted mean achievement)[a]				
	B	*SE B*	β	*t*	Prob. level
Constant	-20.998	1.652		-12.709	0.000
Gender	0.957	0.325	0.099	2.947	0.003
Ethnicity	-0.986	0.820	-0.039	-1.202	0.230
ACT Math	0.659	0.065	0.397	10.149	0.000
HS Math GPA	1.440	0.303	0.180	4.758	0.000
Course Completion	0.089	0.015	0.222	5.870	0.000
Parental Income	0.000	0.000	0.025	0.784	0.434
AP Statistics	-0.202	0.589	-0.011	-0.343	0.732
AP Calculus AB	3.354	0.422	0.294	7.955	0.000
UCSMP/Tr/CA	-0.258	2.229	-0.004	-0.116	0.908
UCSMP/ML	-1.163	0.896	-0.043	-1.297	0.195
McDougal Litell	-2.667	1.618	-0.054	-1.649	0.100
Addison Wesley	-0.711	0.733	-0.034	-0.970	0.332
UCSMP/WS	-0.209	0.930	-0.008	-0.225	0.822
UCSMP	1.011	0.409	0.105	2.472	0.014
Prentice Hall	2.275	0.792	0.107	2.874	0.004
Prentice Hall/Trig	-0.188	1.048	-0.006	-0.180	0.858

[a]Adjusted $R^2 = 0.58$.

Discussion

The findings suggest that students learning from CPMP in these eight schools appear to be at a disadvantage on the combined algebra placement test and calculus readiness subtest at Northern University when compared to students learning from UCSMP or Prentice Hall. As students' scores drop on these placement exams they are more likely to be required to take remedial mathematics courses that do not count towards a college degree. For instance, the cut-off on the combined algebra placement test is 12. If students score below this number they are strongly encouraged to take a remedial mathematics course at Northern University while students scoring above this value may take a college algebra course, which counts towards several different college majors.

Some mathematics programs appeared to be more amenable to mixing with other textbook programs. Students learning from UCSMP outperformed students learning from CPMP on the calculus readiness subtest when other factors such as gender were controlled. However, when UCSMP was supplemented with teacher developed worksheets or other textbooks it did not significantly predict student placement test achievement. Programs like Prentice Hall, on the other hand, could be combined with textbooks developed by different textbook authors. Future research needs to more carefully examine this issue.

This study sheds light on the influence of two AP mathematics programs on students' college mathematics placement exams. AP Calculus AB completion was a strong predictor of students' combined algebra and calculus readiness subtest scores when compared with CPMP. This underscores the importance of the AP Calculus AB program with regard to postsecondary achievement and may help to explain why students who study from AP in high school complete their bachelor degrees in less time than comparison students. AP Statistics was not statistically different than CPMP in terms of these two placement exams. This certainly could be due to the placement exams' preoccupation with algebra while this is a skill that receives less attention in AP Statistics. Also, it could be argued that these exams are designed to place students in mathematics courses, not statistics courses hence, there is a lack of alignment between the course and the placement exam.

There were a number of limitations associated with this study. A great deal of evidence suggests that teachers matter yet we had little data on this important predictor at the high school level. Also, while six out of eight of the CPMP schools were field test sites, which should have higher levels of implementation we did not have classroom observation data to record the implementation of any of these programs as this was a retrospective study. There was also an issue of quantity that was unaccounted for in our model. That is, the UCSMP program consists of five courses while CPMP has only four. Thus the findings in favour of the former program could be due to time spent studying from the curriculum. Perhaps future models could account for this difference by including a variable representing year.

Curriculum was considered a school level variable, but we had few other variables at this level included in our models (e.g., urban vs. suburban) which can influence student understanding. The placement exams used in this study came from a test bank of items developed by the Mathematical Association of America and are available to other universities. In addition, the focus on algebra manipulation in these exams and the use of graphing calculators is similar to placement exams used at other universities (Sattler, 1999). Yet it remains the case in the United States that each university can develop its own placement exam. Thus we are leery of generalizing these results to other universities in the United States.

References

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. Washington, DC: U.S. Department of Education.

Coxford, A. F., Fey, J. T., Hirsch, C. R., Schoen, H. L., Burrill, G., Hart, E. W., et al. (1998). *Contemporary mathematics in context: A unified approach (Courses 1-4)*. New York: Glencoe/McGraw-Hill.

Davis, J. D., & Shih, J. C. (2007). Secondary options and post-secondary expectation: Standards-based mathematics programs and student achievement on college mathematics placement exams. *School Science and Mathematics, 107*, 336-346.

Geiser, S., & Santelices, V. (2004). *The role of Advanced Placement and honors courses in college admissions*. Center for Studies in Higher Education, University of California at Berkeley.

Harwell, M., Post, T. R., Cutler, A., Maeda, Y., Andersen, E., Norman, K. W., et al. (in press). The preparation of students from National Science Foundation-funded and commercially developed high school mathematics curricula for their first university mathematics course. *American Educational Research Journal*.

Hill, R. O., & Parker, T. H. (2006). A study of Core-Plus students attending Michigan State University. *The American Mathematical Monthly, 113*, 905-921.

Lee, V. E. (2000). Using hierarchical linear modeling to study social contexts: The case of school effects. *Educational Psychologist, 35*, 125-141.

Lubienski, S. T., & Crockett, M. D. (2007). NAEP findings regarding race and ethnicity: Mathematics achievement, student affect, and school-home experiences. In P. Kloosterman & F. K. Lester, Jr. (Eds.), *Results and interpretations of the 2003 mathematics assessment of the National Assessment of Educational Progress* (pp. 227-260). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Research Council. (2007). *On evaluating curricula effectiveness: Judging the quality of K-12 mathematics evaluations*. Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curriculum Materials. Mathematical Sciences Education Board, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven, CT: Yale University Press.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks, CA: Sage.

Sattler, N. J. (1999, April). *Placement testing in 2000*. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, San Francisco, CA.

Schoen, H. L., & Hirsch, C. R. (2003a). The Core-Plus Mathematics Project: Perspectives and student achievement. In S. L. Senk & D. R. Thompson (Eds.), *Standards-based school mathematics curricula: What are they? What do students learn?* (pp. 311-343). Mahwah, NJ: Erlbaum.

Schoen, H. L., & Hirsch, C. R. (2003b). Responding to calls for change in high school mathematics: Implications for collegiate mathematics. *The American Mathematical Monthly, 110*(2), 109-123.

Smith, J. P., & Star, J. R. (2007). Expanding the notion of impact of K-12 Standards-based mathematics and reform calculus programs. *Journal for Research in Mathematics Education, 38*, 3-34.

Usiskin, Z. (1986). The UCSMP: Translating grades 7-12 mathematics recommendations into reality. *Educational Leadership, 4*, 430-435.

Wilson, L. D. (2007). High-stakes testing in mathematics. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1099-1110). Charlotte, NC: Information Age.