Developing a Productive Discourse Community in the Mathematics Classroom

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Current reforms in mathematics education strongly advocate the development of mathematical learning communities in which students have opportunities to engage in productive mathematical discourse. In this paper, I address how a teacher used interactional strategies in order to facilitate the participation of students in mathematical discourse. I outline the specific pedagogical strategies the teacher used to shift students' patterns of participation from passive listeners engaging in non-productive disputational talk to engaging in collaborative interaction and productive mathematical discourse.

Developing student communication of productive mathematical reasoning has become a key objective for teachers in Western mathematics classrooms of the 21st century (Walshaw & Anthony, 2008). The pedagogical intent is that students are involved in learning communities in which all participants have opportunities to engage in productive mathematical discourse (Manoucheri & St John, 2006). In New Zealand, teachers are challenged to develop classrooms as "learning environments that foster learning conversations and learning partnerships and where challenges, feedback and support are readily available" (Ministry of Education, 2007, p. 24). Similarly, in the U.S.A the Standards (National Council of Teachers of Mathematics, 2000) promote the centrality of teachers supporting student communication of mathematical ideas and reasoning. The teacher's role is pivotal. Importantly, she must address how her students interact collaboratively. In concert with her students and through direct pedagogical actions she must co-construct a range of social and socio-mathematical norms in the classrooms. These norms are important in that they shape how the learners (the students and the teacher) participate and communicate in collaborative mathematical dialogue to explain and justify their reasoning. Developing mathematical learning communities which promote interactive mathematical talk is challenging for many teachers and their students, particularly because they may not have previously experienced learning and teaching in such classrooms. As a result teachers need many models to support these recent shifts in mathematics classrooms. The purpose of this paper is to examine and outline the interactional strategies a teacher used to construct a classroom climate in which her young students learnt and used productive mathematical discourse to examine and explain their reasoning. Of specific focus will be the social and socio-mathematical norms she developed.

Teachers take a significant role in guiding the development of mathematical discourse in the classroom and ensuring all students actively engage in it. Of importance are the social and socio-mathematical norms they co-construct with their students. The social norms regulate and maintain the environment of inquiry and guide the quality of discourse while the socio-mathematical norms regulate mathematical argumentation and influence mathematical learning opportunities (Kazemi, 1998). The teacher participates in, orchestrates and facilitates the classroom discourse (McCrone, 2005). In the role of facilitator the teacher leads shifts in the discourse, ensuring that it is conceptually focused, and reflective. Kazemi (1998) illustrated how discourse promoting conceptual reasoning was achieved through specific pedagogical actions. These included pressing students to provide conceptually focused justification for mathematical actions, questioning in

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sustained exchanges, and facilitating student examination of similarities and differences across multiple strategies. Likewise, teachers hold a key position in questioning and probing mathematical thinking. Wood and McNeal (2003) illustrated the significant role of teacher-led questions and prompts in shifting students from explaining mathematical solution strategies to justifying and defending them within collaborative dialogue.

Engaging in effective collaborative interaction requires all members of the classroom community to be active and critically constructive participants. For many students this means a shift away from the more traditional role of passive receivers of instruction. Whilst this may not be an easy transition to achieve, McCrone (2005) provided insight into effective pedagogical practices a teacher used to shift 5th grade students' participation in discourse from parallel conversations characterised by a lack of active listening to that of critical active participants. The teacher explicitly modelled active listening, reflecting, and responding carefully to the ideas of others. She initiated explicit discussions to emphasise the importance of active reflection and participation in mathematical discussions. When confident that these norms were well established she gradually modified her role to become a facilitator of the mathematical dialogue. She used revoicing during student offered solutions as she encouraged other students to engage with and respond directly to each others' ideas. Revoicing is a key pedagogical strategy used by teachers to position students in the interactive dialogue. Through teacher revoicing students learn to take a specific stance in the dialogue and develop the skills of inquiry and mathematical argumentation as they defend or challenge ideas (O'Connor & Michaels, 1996). Alternatively teachers use revoicing to clarify reasoning, highlight specific aspects of the mathematical thinking, or extend, rephrase, and further develop it (Lampert & Cobb, 2003).

The need for effective pedagogical strategies extends beyond the structuring of student participation in collaborative dialogue when a teacher is present to include students' independent interaction in small problem solving groups. The important work of Mercer (2000) investigated student talk when working in small groups. The results of this study showed that students commonly used three different forms of talk—exploratory, disputational, and cumulative talk. These types of talk involved different levels of critical but constructive engagement with the reasoning of others. Disputational and cumulative talk, they characterised as unproductive. When students engaged in disputational talk they focused on self-defence and holding control rather than trying to reach joint agreement. In the use of cumulative talk they avoided questions and argument which resulted in a lack of evaluative examination of reasoning. Exploratory talk was described by these researchers as a productive form of talk in which the students explored and critically examined shared reasoning. An important finding was that exploratory talk required specific teacher attention, intervention, and scaffolding.

The theoretical stance of this study draws on the emergent perspective of Cobb (1995). From this socio-constructivist learning perspective, Piagetian and Vygotskian notions of cognitive development connect the person, cultural, and social factors. Therefore, the learning of mathematics is considered as both an individual constructive process and also a social process involving the social negotiation of meaning. Language within this frame is considered to hold both a communicative/cultural and psychological function. It provides a tool for thinking together and jointly creating knowledge and understanding.

Method

This paper reports on episodes drawn from a larger research study involving a 3-month classroom teaching experiment (Cobb, 2000) situated in an inquiry classroom environment.

The study was conducted at a New Zealand urban primary school and involved 25 students between the ages of 9-11 years. The students were from pre-dominantly middle home socio-economic backgrounds and represented a range of ethnic backgrounds.

Collaborative teaching design experiment (Cobb, 2000) was used and this supported a teacher-researcher partnership and the development of a trajectory which focused on developing algebraic reasoning. Although the algebraic reasoning is not the focus of this paper the development of social and socio-mathematical norms within an inquiry environment were integral to supporting student development of early algebraic concepts. During the teaching experiment data was generated and collected through classroom artefacts, participant observations and video-recorded observations.

The findings were developed through on-going and retrospective data analysis. On-going data analysis shaped the study as the researcher and teacher collaboratively examined the classroom practices and modified the participatory practices to develop an inquiry learning climate. Retrospective data analysis used a grounded approach identifying categories, codes, patterns and themes. These were used to develop the findings of the classroom case study.

Results and Discussion

Collaborative interaction in productive mathematical discourse requires that students take the role of active listeners and participants. Initially many students viewed their role as passive listeners. For example, the teacher asked the students to describe their role in class discussions and typically a student responded:

Mike: Sit quietly and listen to what they are saying and don't interrupt.

In this early phase of the study, class discussions were characterised by unproductive silence. Student initiated question asking was limited to teacher modelled questions copied in a way which did not relate to the students' need to sense-make:

Hamish: Could you have done it any other way…?

During small group work, many students engaged in disputational talk. They focused on self-defence and holding control rather than reaching joint agreement. For example, when questioned about his solution strategy, a student responded with:

Peter: Because I felt like it.

In another instance, a group examining true and false number sentences used disputational talk and so failed to reach consensus:

Rani: If you plus 3 to equal that.

Matthew: No you can't do that.

Rachel: Why?

Matthew: Because if you do then it's changing the whole thing.

Zhou: I'm getting even more confused.

Through our collaborative discussion and reflections on the current existing interaction patterns the teacher and I acknowledged that the social norms needed to be addressed. All students needed to engage in interactive dialogue. Specific teacher actions led to shifts in the nature of students' participation. Students moved to become more critical active participants who engaged in productive discourse.

Structuring Norms which Engaged the Students in Mathematical Inquiry

The teacher initiated and maintained on-going discussions related to student responsibilities in small groups. She introduced a code of working together that involved only one pen and one piece of paper for the group to use. She also required that *all* group members be able to share back a group selected strategy solution. The students were reminded of their responsibility to each other but also their own responsibility to actively engage, question, and individually sense-make:

Teacher: You have to help and you have to understand, everyone in your group needs to understand the strategy. It is not good enough if it is only one person you need to try and help the rest of your group understand it.

Ruby: You have to ask questions if you don't understand.

Teacher: Exactly you don't just sit there and hope that others will explain it to you. You need to ask questions yourself.

The discussions emphasised the need for individual and group accountability. At the same time, active listening and questioning were emphasised:

Teacher: Your job in maths is to actually think about what other people are saying and whether or not you are agreeing. Think about is there a question I need to ask as she goes along.

Student attention was drawn to those students who modelled appropriate behaviour. For example, Zhou made a recording error, and when Josie, a member of Zhou's group, stepped in to progress his explanation, the teacher affirmed the way in which Josie modelled group responsibility:

Teacher: Thank you Josie for helping to clarify there. Can you see what she did then? That's what I mean, get the help, the support from your group.

In this way, class and group discussions were used to advance the instructional agenda and as a means for the teacher to model and encourage productive mathematical behaviour.

In this classroom, when the students worked in problem solving groups, the emphasis was placed on developing collaborative agreement. Our review of the initial set of videoed lessons drew our attention to how the students were interpreting this as always needing to agree. To shift the students towards engaging in mathematical argumentation the teacher directly discussed with the class how to disagree within discussions:

Teacher: What if you don't agree?

Mike: If you don't agree ask them why…why did you do that?

Teacher: You can say I'm not sure about that, I'm not convinced by that part there. Can you convince me?... it's not just sitting there and tuning out while the other people are talking, you actually need to be involved and engaged in what is happening. So if you disagree and are not sure of that part, you actually need to say that.

In following lessons, the teacher sought other opportunities to affirm students' right to disagree:

Teacher: Good on you Bridget, that takes someone brave to say they are not entirely convinced.

In the evolving social learning climate the teacher explicitly modelled that errors were an opportunity to explore and extend reasoning. For example, during a class discussion about the properties of zero, a student made a conjecture:

Gareth: H times zero plus Z equals X.

The teacher intervened and without indicating her own position or whether the conjecture was correct asked all students to think carefully about what was being explained. At the same time she gave the student explaining time to re-conceptualise erroneous thinking:

Teacher: Talk to the person next to you. Do you agree with this statement H times zero plus Z equals X? You need to convince us why you agree or disagree.

Then, following extended discussion time she asked the explainer to re-evaluate and re-explain. As we see from the response Gareth modified his initial conjecture:

Gareth: We worked out there had to be two Z's, one after the equal sign because H times zero equals zero… the Z should be on both sides of the equal sign. H times zero plus Z equals Z.

Thus, errors become reflective tools which provided the students with opportunities to both recognise errors and re-conceptualise their reasoning.

Increasing the Press on Social and Sociomathematical Norms

Toward the end of the first month of the study we observed that the students were readily participating in interactive dialogue which focused closely on the mathematical reasoning of their peers. Our analysis showed that the students had begun to regulate what they accepted as an appropriate mathematical argument. The teacher maintained the press through providing reflective space to allow the students to explore and analyse similarities and differences between mathematical explanations. For example, Rachel shared an algebraic number sentence as a solution strategy:

Rachel: [writes 9 + ■ = A] Nine plus square equals A.

At this point, the teacher invited the students to provide alternative solution strategies:

Teacher: Does anyone else have a different way of representing that problem? Okay Matthew.

Matthew: [writes 9 + ● = B] Nine plus circle equals B.

The teacher then asked the students to examine and compare the algebraic number sentences:

Teacher: I want everyone to look at that and I want you to think has Matthew shown us a different way or is it similar to a way that is already there?

She then provided space for the students to think and discuss each sentence with others seated near them:

Teacher: Matthew do you think that is similar or different to the one that is already there?

Matthew: Similar.

Then she increased the press for further explanatory justification:

Teacher: Why is it similar?

Matthew: Because that [points to 9+●=B] is just another way of doing that [points to 9 + ■ = A].

Through the reflective space created by the teacher, Matthew and his fellow students were supported to make connections, analyse, and critique their own and others' arguments.

Questioning was a tool that the teacher used to develop richer argumentation. She listened closely to how students questioned and then explicitly pressed them to ask questions which structured the mathematical argument:

Teacher: What I have noticed often is that people are asking the question can you explain that in a different way? Now that isn't always helpful and sometimes we just use it because we don't know what else to ask so what are some other questions that we might have to ask during this session?

Bridget: We had to convince people that it would work for any number including zero.

Teacher: Great so you can use words like convince us that it would work for any number?...

Through placing an explicit focus on question-asking the students' subsequent use of questioning shifted toward drawing justification and the provision of conceptually focused reasoning as evident in the videoed lessons.

Our observational data showed that some students needed additional scaffolding to engage in dialogue premised in inquiry and argument. The teacher would carefully observe student engagement and when needed reposition students using revoicing to ensure that all students in the group took a stance. For example, as the students discussed the commutative law one stated:

Rachel: It didn't work with everything.

Teacher: So what did it work with?

Rachel: Pluses

At this point, the teacher revoiced the statement positioning her to further explain and justify her stance:

Teacher: So you're saying it only worked with the plus or it worked with plus?

Rachel: It worked with plus.

As a result of teacher guidance the students gained confidence in their ability to take a stance and reach consensus through mathematical argumentation and justification.

As the study progressed, in the large group setting and in small problem solving groups, the students readily engaged in exploratory talk as a way of investigating and critically examining their shared reasoning. For example, as a group examined a functional relationship problem, they interacted and explored ideas. Initially, a student made an error:

Josie: It's fourteen.

Another student, Steve, disagreed providing mathematical reasoning for his disagreement:

Steve: No it is thirteen because you are adding two each time. It doesn't work because if you are adding two on each time and it is odd numbers it can't be fourteen because it's an even number.

Josie listened carefully to Steve's argument and subsequently used his reasoning to explain how her explicit generalisation was linked to the geometric model:

Josie: [points to the vertical line] There is always one in the middle. It is always an uneven number because there is always one in the middle for that line there.

The requirement that students develop communal agreement and understanding within a culture in which they could expect justification of claims and a press for collaborative interaction increased the levels of exploratory talk. This is illustrated during small group problem solving when the teacher asked the students to identify what changed and stayed the same in an equation:

Bridget: Five.

Another student in the group revoiced Bridget's answer and pressed her to justify:

Heath: So you think five. Why do you think five is the main number?

When Bridget was unable to provide justification, Heath stepped in:

Heath: I think it's because that's how much you get paid.

Another example illustrates how the students had appropriated the teacher's many models of questioning and probing until convinced. The students actively engaged in productive exploratory dialogue as another student explained his solution strategy:

Heath: You add three to each table then the plusing two bit…

A student asked a question focused on eliciting justification:

Josie: Why isn't it two fives added together?

At this point, other students stepped in to collaboratively develop the explanation and provide justification while Josie continued to press for further justification:

Matthew: [points to the model] Because you couldn't put one there.

Josie: But each table is meant to have five.

Heath: Yeah but on one table it's five, it starts off with five but then you…

Hayden: You can't sit someone right in the middle of the table. They can't sit here [points to the middle of the model] because they'd be on the table… the people on the edge always had to move out again.

These examples illustrate the students developing use of collaborative interaction and productive mathematical discourse. In the classroom learning community, students were both pressed and supported by their peers to develop mathematical explanation and justification in accordance with what had been established as acceptable mathematical arguments within the learning community.

Conclusion and Implications

This study sought to explore how a teacher used interactional strategies to facilitate her students to engage in collaborative interaction and productive mathematical discourse. The teacher in this study took a key role in facilitating the establishment of norms which guided student interaction within her classroom. Through pedagogical actions she guided effective discourse and supported student engagement in collaborative interaction. Similar to the findings of other researchers (e.g., Kazemi, 1998; Manoucheri & St John, 2006; McCrone, 2005) specific pedagogical actions which focused on scaffolding the social and sociomathematical norms, active listening and questioning, engaging in agreement or disagreement were key elements in the development of a classroom environment which included collaborative interaction and productive mathematical discourse.

Evident in this study were the changing participation patterns which began with what Mercer (2000) and Mercer and his colleagues (Wegerif, Mercer, & Dawes, 1999) termed disputational or non-evaluative cumulative talk. These unproductive patterns of talk were characterised by the students taking a passive role or engaging in disputes. This paper highlighted the shifts in participation and interaction following specific pedagogical actions which aimed to develop productive exploratory discourse, collaborative interaction and shift focus from explanation to explanatory justification within mathematical argumentation. Clearly evident in this study was the significant role the teacher had in shifting the students towards greater use and facility with argumentation and justification. Of importance in the study was the immediate enactment of community accepted social norms which provided the foundation for communally agreed socio-mathematical norms.

Implications of this study suggest that shifting student participation to engage in collaborative interaction and productive mathematical discourse requires specific attention from the teacher both to the participation structures and the specific nature of the discourse. Facilitating change in the students' interaction was a lengthy process. Further research is required to validate the findings of this study due to the small sample of participants involved.

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