Developing Pedagogies in Teacher Education to Support Novice Teachers' Ability to Enact Ambitious Instruction

Elham Kazemi
University of Washington
<ekazemi@u.washington.edu>

Megan Franke
UCLA
<mfranke@ucla.edu>

Magdalene Lampert
University of Michigan
<mlampert@umich.edu>

What might it take to support novice teachers to develop the commitment and the capacity to enact ambitious mathematics instruction? In this paper, we describe our experimentation with pedagogies in teacher education to develop novice teachers' competence in eliciting, responding to, and advancing students' mathematical thinking. Our efforts centre on the use instructional activities as a tool in supporting the learning and doing of features of ambitious instruction. We explain our use of guided public rehearsal as a pedagogical practice for helping novices practice the interactive contingent nature of classroom teaching in a way that supports their direct interaction with children.

This paper describes the aims of a collaborative effort[1] to support novices[2] to teach mathematics. We will discuss our efforts to re-imagine and develop our practice for supporting mathematics instruction. To accomplish this, we will describe what we mean by preparing novice teachers for ambitious practice, explain why we have chosen to use instructional activities as the unit of practice to engage novice teachers, and describe our use of public guided rehearsals in teacher education courses. We end this paper with challenges we have faced thus far in developing our pedagogies.

Our collaboration began because of a nagging dissatisfaction with how we prepare teachers to teach ambitiously. Ambitious teaching requires that teachers teach *in response to what students do* as they engage in problem solving performances, all while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions (Kilpatrick et al., 2001; Lampert, 2001; Newman & Associates, 1996). At the heart of our dissatisfaction was our observation that while we had experienced much success in developing teachers' ability to analyze depictions of practice, we had a long way to go in improving teachers' ability to use such knowledge judiciously in their direct interaction with students.

Background and Framing

Mathematics educators have excelled at centring professional education at all levels on the study of artefacts of practice (e.g., student work), lesson planning, and analysing classroom video (e.g., Borko, Jacobs, Eiteljorg, & Pittman, 2008; Cobb, Dean, & Zhao, 2006; Fosnot & Dolk, 2001; Lampert & Ball, 1998; Kazemi & Franke, 2004; Lewis, Perry, & Murata, 2006; Schifter, Bastable, & Russell, 1999; Sherin & Han, 2004; Wilson & Berne, 1999). There is a well-established tradition of helping teachers make

---

[1] Other members of our research team are Heather Beasley and Hala Ghousseini at the University of Michigan and Angela Chan at UCLA. Adrian Cunard, Teresa Dunleavy, Allison Hintz, Megan Kelley-Petersen, Becca Lewis, Helen Thouless and Maria Zavala have supported the development and documentation of our research at the University of Washington.

[2] At the same time that we have worked with novice teachers, we have also adapted the ideas for use in school-based professional development. In this paper, we focus on describing the work as it is embedded in teacher preparation programs.

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1).

sense of children's mathematical thinking. In Pam Grossman and colleagues' terms, mathematics teacher educators have a rich repertoire of pedagogies of investigation to develop prospective teachers' ability to analyze and critique practice (Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson 2009; Grossman & McDonald, 2008). What remains to be developed is what they call pedagogies of enactment through which novice teachers are supported in actually doing the practice of teaching within the context of methods courses. We recognized that our coursework may have improved novice teachers' knowledge of students and content, but we were not going far enough in supporting novices to use that knowledge while interacting with students in challenging and diverse settings. We have been inspired by ideas that creating a pedagogy for teacher education to help teachers develop the performative aspects of teaching through carefully selected and specified instructional activities will create stronger, more skilled teachers (see Grossman et al., 2009; Grossman & McDonald, 2008, Lampert & Graziani, 2009). The future viability of professional teacher preparation requires that we systematically pursue appropriate ways to develop, fine-tune, and coach novice teachers' performance across settings. These activities must find their way into university coursework rather than be relegated solely to field placements (Lampert & Graziani, 2009). Our hypothesis is that organizing professional education in mathematics education around core instructional activities and building links from the activities to student outcomes will enable us to support ambitious teaching.

The theoretical thrust of our work entails articulating what we mean by "practice" when we say teacher education should be focused on it (Ball & Cohen, 1999; Kazemi & Hubbard, 2008; Smith, 2001). Our aim is to use what we learn from our efforts to contribute to a *theory* of learning teaching in, from, and for practice, and to contribute to improving the *practice* of mathematics teaching and the *practice* of teacher education. Social practice theory defines "a practice" as a way of doing something that derives its meaning from the shared knowledge and values of a profession (Reckwitz, 2002). Common tools and language are used in work contexts to enable the development of shared meanings about what counts as a problem and what constitutes an appropriate solution (Lampert, Boerst, & Graziani, in press). Learning a practice is also a process of becoming—identities shape and are shaped by the developing practice. (e.g., Holland, Lachiotte, Skinner, & Cain, 1998; Wenger 1998).

The statement that teaching is a complex endeavor is taken as shared among scholars and practitioners alike. No one would argue that it is straightforward work that relies simply on knowing the right things to do. More typically, our portrayals and study of teaching has shown that it is highly contingent and unpredictable, tailored to particular students, and requiring rapid online diagnoses. Lampert and Graziani (2009) explain how to make our way through the paradox created by our depictions of the complexities of teaching.

> If professional education for teaching is to make ambitious teaching more common, it seems that we would need to make several assumptions that contradict the idea that this kind of teaching is entirely context bound and independently constructed. First, we would need to assume that this kind of teaching involves stable and learnable practices and that we could specify the kind of skills and knowledge needed to do those practices (Stein et al., 2008). Second, we would need to assume that teacher educators could teach these skills and knowledge, and that novices could learn them. We need to confront this seeming contradiction between flexibility and stability in order to figure out how to build knowledge for teacher education if the goal is ambitious teaching (p.492).

Structuring the Work of Learning to Teach

Our work has entailed developing *pedagogies of practice* focused around the systematic use of strategically selected instructional activities inspired by a model used in a teacher education program in Italy studied by Lampert and Graziani (see Lampert et al., in preparation and Lampert & Graziani, 2009 for further elaboration about the rationale for using instructional activities as a tool for teacher learning). Our model consists of recurring cycles of planning, rehearsal, enactment and reflection using instructional activities as the focus of teaching.

Instructional Activities.

Instructional Activities are tasks enacted in classrooms that structure the relationship between the teacher and the students around content in ways that consistently maintain high expectations of student learning while adapting to the contingencies of particular instructional interactions (Lampert & Graziani, 2009). In order to support the kind of teaching that we aim to have novices learn, we have selected particular instructional activities that:

- make explicit the teaching moves that are implied in the kinds of cognitively demanding tasks that are found in curriculum materials available for use by novices;
- structure teacher-student interaction using these moves in relation to teaching the mathematical content that students are expected to learn in elementary school;
- enable novices to routinely enact the principles that under gird high quality mathematics teaching including:
  - engage each student in cognitively demanding mathematical activity
  - elicit and respect students' efforts to make sense of important mathematical ideas
  - use mathematical knowledge for teaching to interpret student efforts and aim for well-specified goals
- be generative of other activities by including the teaching and learning of essential teaching practices (high leverage practices) like explaining, leading a content-rich discussion, representing concepts with examples, and the like (Franke et al., 2001).

We have initiated this work with instructional activities by identifying several that develop knowledge of number and operations, topics which are central to elementary mathematics curriculum. Key mathematical ideas related to number and operations include (1) understanding the structure of the number system, (2) making meaning of operations (addition, subtraction, multiplication, and division) and their use in solving problems, and (3) using operations through computational procedures. They are adapted from a synthesis of recent research and development in the field that relates computational fluency with conceptual understanding (e.g., Carpenter et al., 1999; Chapin, O'Connor, & Anderson, 2003; Cobb et al., 1991; Fennema, et al., 1993; Fuson, et al., 1997; Fosnot & Dolk, 2001; Hiebert, et al., 1997; Gravemeijer & van Galen, 2003; NRC, 2002).

The choice to focus around particular activities is not intended to signal the importance of these activities themselves or that these are somehow the "right" ones. Rather the choice of *an* instructional activity is intended to indicate that learning in and from practice needs an instructional vehicle to focus practice in ways that enable teacher educators to teach it and novices to engage in it. Our hypothesis is that this set of IAs will serve as a productive starting place for novice teachers, enabling them to develop broadly applicable skills and knowledge. We plan to adapt these and add others through the development process. If this work proves to be successful, we hope that others will identify more activities and other domains to work toward building a theoretically and empirically grounded instructional system for elementary mathematics (Bryk, 2007;

Cohen, Raudenbusch & Ball, 2003). The four activities we are using to begin our work are described below:

Counting. The teacher leads the class in a count, teaching different concepts and skills by deciding what number to start with, what to count by (e.g., by 10s, by 19s, by ¾), whether to count forwards or backwards and when to stop. As children engage in the count, the teacher publicly records the count on the board, having thought ahead of time how to record it, stopping at particular times to elicit children's ideas for figuring out the next number and to discuss what patterns are occurring in the count and why. This activity is targeted to help children learn how to apply computational strategies, notice and use patterns to make predictions, and reason through why patterns are occurring. This activity is not simply about rote counting. Instead, the purposeful recording and choice of the counting task coupled with discussions about patterns that emerge as the count proceeds engages students in mathematical sensemaking.

Strategy sharing. The teacher poses a computational problem and elicits multiple ways of solving the problem and orchestrates a discussion for students to make sense of the strategies and relations between them. Careful use of representations and targeted questioning of students is designed to help the class learn the general logic underlying the strategies, identify mathematical connections, and evaluate strategies in terms of efficiency and generalizability.

Posing a sequence of related computational problems. The teacher poses several related computational problems, one at a time, in order to scaffold students' ability to make connections across problems and use what they know to solve a more difficult computational problem. This activity is typically used to target a particular strategy (as compared to eliciting a range of strategies). For example, posing 4 x 4, then 4 x 40 and then 4 x 39 is designed to help students consider how to use 4 x 40 to solve 4 x 39, developing their knowledge of compensating strategies in multiplication. Whereas posing 2 x 10, 10 x 10 and 12 x 10 is designed to help students identify partial products of 2 x 10 and 10 x 10 in 12 x 10, using the distributive property of multiplication to find the answer to 12 x 10 (Fosnot & Dolk, 2001).

Solving word problems. The teacher first launches a word problem to support students in making sense of the problem situation, then monitors while students are working to determine how students are solving the problem, gauges which student strategies are best suited for meeting the instructional goal of an upcoming mathematical discussion, and makes judgments about how to orchestrate the discussion to meet those goals.

The first three IAs can be used as warm-ups in the classroom and appear as such in many existing curricula. Typically, however, these activities are not instructionally specified in teachers' guides to the extent that we envision being necessary for novices. We have found that using activities that take up a short amount of instructional time, 10 to 20 minutes, is particularly useful when working with novices because they can more easily find time in field placement classrooms to practice these activities with small groups of students and then expand to working with the whole class as their competence develops. By choosing warm-ups that can be routinely used, we have also built into the IA design the opportunity for novices to use them more than once, supporting a cycle of preparation, enactment, analysis and reenactment. The fourth IA, solving word problems, is ubiquitous in elementary mathematics curricula and rarely done in ways that teach important mathematics (Hiebert et. al., 2005; NRC, 2002).

The instructional protocols we have developed for working with these IAs will guide novice teachers' planning and enactment, helping them learn how to introduce an activity, manage materials and student participation, manage discussion towards an instructional goal, work with mathematical representations, and respond to student error (see Appendix 1 for an example). One of the goals of our collaborative research is to better understand what features of ambitious instruction and key mathematical ideas in the elementary grades these particular IAs help novices learn.

Instructional activities, as we have used them, are intended to enable novices to engage in *learning the work of teaching* and teacher educators to engage in *teaching the work of teaching* along three dimensions simultaneously: principles of high quality teaching, essential teaching practices which embody those principles, and mathematical concepts and processes to be learned that provide direction for teaching practices. By doing this work, we avoid common problems in teachers' efforts to enact a kind of mathematics teaching that is not familiar to them, mechanical (non-principled) enactment, and interaction with students that is not appropriately goal-oriented.

Pedagogy of Practice

In order to enable novices to learn in, from, and for doing the work of high quality mathematics teaching on all three of these fronts, we have begun to design a pedagogy around a small set of Instructional Activities that have the characteristics we have outlined here (Grossman et al., 2009; Grossman & McDonald, 2008). Within this pedagogy, there are several places where we are able to work simultaneously on novices' skill in performing essential teaching practices, their principled enactment of those practices, and the mathematical concepts that they are learning and learning to teach.

In a pedagogy of practice for teaching, the teacher educator becomes responsible for:

- exhibiting, demonstrating, and naming the elements of an instructional activity;
- situating the activity in theoretical and empirical evidence that it is likely to result in student learning;
- giving novices the opportunity to deliberately practice the elements of the activity that are "routine" with coaching from teacher educators;
- structuring collaborative work on problems of teaching practice so as to attend to the development of novices' knowledge of important mathematics and their knowledge about how students make sense of that mathematics in ways that are connected with that work;
- scaffolding novices' preparation for doing the activity with particular elementary level learners in ways that call attention to important mathematics and students' ways of making sense;
- rehearsing the enactment of the plans for doing the activity so as to provide deliberate practice of its routine elements as well as opportunities to respond in a principled way to the kind of non-routine information that comes from students;
- organizing opportunities for novices to teach using the activity and to record their practice and their students' work
- analyzing with novices how an Instructional Activity can maintain its integrity while playing out differently in different classroom contexts;
- assessing the learning of novices around the key practices that are embedded in the activity;
- refining the design of the Instructional Activity in consideration of what elementary mathematics students are able to learn with it.

The design of our pedagogies of practice are based on research on the learning of complex interactive activities (Ericcson, et al., 1993; Ericsson, 2002; Patel, Kaufman, & Magder, 1996). They enable the kind of coherent back-and-forth between course work and teaching in classrooms that has been identified as essential for enabling novices to become competent beginners at ambitious teaching (Bransford, Brown, & Cocking, 1999; Feiman-Nemser, 2001; Grossman, et al., 2009; Mewborn & Stinson, in press). Learning theorists refer to this as the "generative dance" between knowledge and knowing, essential to being able to use knowledge in the unpredictable interactions required by this kind of work (Barab & Duffy, 1998; Cook & Brown, 1999; Gasson, 2005; Latour, 1991). Using skill and knowledge involves being skilled at the routine elements of participation structures so that it is possible to interpret and respond to the non-routine information generated by students engaged in mathematical work (Dreyfus, 2004; Feldman & Pentland, 2003; Ghousseini, 2008; Leinhardt & Steele, 2005).

Teacher educator rehearsal. During planning meetings, course instructors rehearse a provisional version of an IA that is to be used in an upcoming class. This version consists of a plan including the mathematical instructional purpose of the lesson in which the activity is used, goals for K-5 student participation, and a rough draft of a protocol that guides the enactment of the activity by specifying routine participation structures. Attention to access and equity are planned into the participation structures that support key practices such as posing questions to address the mathematical instructional purpose, eliciting student reasoning, using representations to support student thinking, and managing the participation of all students in the activity. Also integral to rehearsing an IA is the instructor's preparation to provide novices with opportunities to engage as learners of particular mathematical content. The rehearsal and coaching process for the teacher educator results in the articulation and refinement of the mathematical learning goals that can be accomplished with the IA in a particular lesson. Such a process is generative of teacher educator learning because it involves the articulation and sharing of knowledge of content and teaching among instructors.

Refinement of IA plan and production of protocol. Following the planning/rehearsal of the lesson using an IA, the instructor leads the design team in producing a highly-specified written protocol to guide the enactment of the activity with novices. To construct the protocol, the team draws on knowledge generated and articulated during the rehearsal. The knowledge is codified into two categories: routine practices and non-routine practices. The routine practices consist of specified moves in the protocol that require little exercise of judgment on the part of the novice, thus freeing cognitive capacity for the novice to attend to non-routine practices. Non-routine practices are in general underspecified due to their dependency on unpredictable elements of the context, such as what students would say or do. The non-routine parts of the protocol are strategically designed to develop the professional judgment of the novice in relation to important aspects of the context. Possible responses to students are included to represent choices and examples.

Enactment of IA with novices. The enactment of the IA with novices goes through a number of phases. First the novices participate in the IA as learners of mathematics by doing the mathematics themselves while the teacher educator teaches the lesson. The teacher educator leads them through the activity, modeling the key practices of ambitious teaching in the way they were outlined in the instructional protocol. Following this, the teacher educator and novices analyze the work of teaching that was modeled by the teacher educator. Using the protocol as a scaffold, they parse the work of teaching and the decisions that guided its design and enactment. The teacher educator deliberately engages novices in considering alternative solutions to problems of practice that fall at the intersection of content and student thinking. The protocol, during these conversations, becomes a tool for both enactment and investigation. Following the analysis, the novices prepare for rehearsing the IA by writing a plan with the protocol as a guide.

Rehearsal of IA by novices. Using the same or a modified protocol, the novices next rehearse the IA with feedback from the teacher educator. The rehearsal allows the novices to begin to master routine aspects of the IA and to begin developing their judgment in adjusting instruction appropriately to the performances of various learners. During this rehearsal, one of the roles of the teacher educator is to coach the novices by observing their efforts to enact the practices outlined in the protocol as guided by specified principles of high quality teaching. Rehearsals for one instructional activity can last from 20 to 40 minutes. during which novice teachers or the teacher educator may stop action to ask a question, suggest an alternative course of action, or note how teachers' decisions were appropriate. The teacher educator also deliberately participates in the rehearsal by making assertions and errors or asking questions that elementary students are likely to give in such activity. These interjections force the novice to reason about and productively respond to student thinking in the moment, trying on the actions that might be used in the actual classroom setting. The rehearsal/coaching process works dialectically with the protocol of the IA, taking up and articulating in action aspects of the protocol while at the same time expanding its nature as a tool for action.

Enactment of IAs in K-5 classrooms. Once novices have planned and rehearsed the IA at the university, they have the opportunity to make use of the IAs in field placements and full time student teaching. Outside of the university setting, they must manage the IA with a group of elementary students, balancing instructional practices and principles they have learned with the particular mathematical goal and the needs of a group of elementary students. The novices enact the IAs and bring back records of practice (including video and audio clips, student work, and personal field reflections) to the methods class for additional analyses and rehearsals. This process supports the development of the IAs as generative vehicles for learning ambitious teaching as novices teach and then use the methods course as a setting to hone their practices. The connection between course experiences and teaching in the field becomes intertwined in ways that allow for authentic engagement in and analysis of practice. The process continues as novices take IAs into their first year of teaching, continuing to experiment with and adapt the use of the IAs to meet their instructional goals and the needs of each student. Participating novices are thus also engaged in a kind of "design research" as they learn through several cycles of using and refining the IAs.

Rehearsing Instructional Activities

To provide an image of what aspects of ambitious instruction we rehearse using instructional activities as the unit of practice, we present a brief teaching scenario (see Figure 1). In the left-hand column, we describe a teacher's skilled use of the instructional activity. It represents the kind of teaching we are aiming for. Our research and development process will help us understand what it will take to help novice teachers reach this level of competence. In the right hand column is commentary about what novices might do with teacher educator guidance during public rehearsals to help them enact features of ambitious instruction. As you read, imagine what features of ambitious teaching are demonstrated in the scenario. Focus your attention on three features of classroom talk teachers need to attend to as they work on their role in fostering productive mathematical discussions are: (1) supporting students to know what to share and how to share; (2) supporting students to be positioned competently; (3) achieving a mathematical goal. Our use of instructional activities becomes a way through which we can work specifically on these core practices.

Sequencing a set of related problems. Sequencing a set of related problems is a short ten to twenty minute instructional activity.[3] The teacher uses a carefully selected sequence of problems to develop and highlight particular computational approaches or ideas about operations. Each problem is posed one at a time, and students discuss their strategies before the subsequent problem is presented. Teachers may direct students to consider whether earlier problems in the string can help them think about later ones. Teachers choose to accompany the discussion with particular representations such as number lines, arrays, or other models. This way, children can consider the strategies from the prior problem as well as the numbers, and they are prompted to think about the relationship of the problems in the string as they go along. This sequencing activity can be used by teachers at a wide range of grade levels by choosing problems that scaffold key computational strategies across any operation.

In preparation for this rehearsal, novice teachers have had the opportunity to experience the teacher educator leading the activity, have watched videos of classroom instruction with teachers leading the activity, examined local school curricula in which strategic sequences are posed, and planned to rehearse the particular sequence that is the focus of the rehearsal. Coursework has also involved working through possible representational tools to use (e.g., the number line).

Once planned, one or more teachers co-lead the lesson with their peers in the role of their students. The teacher educator stops the "action" at opportunistic times to provide direction for what novices might do or re-do and why.

Figure 1: Connection between skilled enactment of instructional activity and guided public rehearsals

Skilled enactment of instructional activity by practicing teacher	Feature of ambitious instruction and how they are rehearsed during guided rehearsals
Instructional goal Mr. K wants to prompt his first and second grade students to think about strategies for subtraction. In particular, he wants them to think about when it might be more strategic to count up to find the answer to a subtraction problem (i.e., when the numbers are close together) and when it might be more strategic to take away or count back (when the numbers are far apart). His students are also in the process of articulating why you get the same answer to a subtraction problem when you count up or count back. For this discussion, he wanted to note if that issue came up but not pursue it because of his intent to keep this as a short warm up. Sequence chosen for today 11-3 16-12	*Feature of instruction*: Posing a string of related computational problems allows for convergent discussion around a particular idea. To do this, teachers have to identify the goal they are working on and think of number choices and discussion prompts that help them achieve this goal. *Rehearse*: Novices identify their mathematical purpose as they plan for rehearsal. They may state the purpose to their peers before rehearsal begins.

[3] Professional development materials developed by Catherine Fosnot and Maarten Dolk informed the specification of this instructional activity. (Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work*. Portsmouth, NH: Heinemann.)

25-6 32-28 *Commentary:* Mr. K recognizes that any instructional activity could raise multiple goals. He makes a plan by identifying what his mathematical priority will be, knowing full well that he could always change course based on what he learns about his students' thinking.	
Launching the activity He begins by telling his students that they are going to work on a set of problems as a warm up to their lesson. He is interested in knowing how they solved the problem. He tells them that he is going to start by writing the problem on the board and he wants his students to figure out the answer and indicate they are ready to share by placing their thumbs on their chests, so that everyone in the room has some good thinking time. *Commentary*: Mr. K used strategies to position his students competently. He used a system of monitoring whether students were ready to share (putting thumbs on their chest), which helped mitigate the pressure of hands shooting up and waving in the air as other students figured out the answer.	*Feature of instruction*: The choice of a management device should minimize the public display of speed to get an answer Begin with a problem in the sequence that you know your students can do. *Rehearse*: What to say at the beginning of the lesson to focus students' attention, cultivate interest in students' ideas, show enthusiasm for the task and get started right away.
Posing the first problem: 11 - 3 He begins by writing 11 – 3 on the board. This problem is easy for the students in his class. When he sees nearly all the thumbs up, he calls on Jaden, "What do you think the answer is?" Jaden confidently states, "8." Accepting his response, Mr. K asks, "Are there any other answers students want to share?" When no one indicates a different answer, he asks Jaden how he knew the answer was 8. Jaden replies, "I have a slide show in my mind, and I just knew the answer was 8." Mr. K acknowledges that and asks if anyone counted to figure out the answer. Chloe raises her hand to share, "I counted back on my fingers 11, 10, 9 and since I took 3 away, I knew the answer would be 8." Mr. K asks for a show of hands for how many people solved it the same way. Many students raised their hands. Mr. K notes Chloe's counting on the board. *Commentary:* He began the string using numbers (11	*Feature of instruction:* Eliciting students' answers should be coupled with questions that help the teacher understand strategies that students use. The teachers' questions help build norm that making reasoning not just answers is important for mathematical work. *Rehearse:* Use follow up questions to understand the details in students' strategies, how and what they counted. Ask questions in a way that enables students to put their ideas forth. Practice representing students' strategies on the board.

– 3) that would not be intimidating to any of his students but that would be a good entry point for the goal he had selected for the day.	
Pose second problem: 16 - 12 He tells his students to be ready for the next problem and writes 16-12 on the board. When the students have shown with their thumbs that they are ready, Mrs. H asks, "Who will share the answer to this problem? Sadie?" Sadie answers 4 and no students offer a different answer. Sadie explained that she counted up from 12 to figure out her answer, "I counted 13, 14, 15, 16 and kept track on my fingers like this" (she holds up her fingers and demonstrates). Jack says he got the same answer as Sadie but he solved it in a different way, "I knew that 6 minus 2 equals 4 so 16 minus 12 has to be four too. You know, you can just ignore the 10 since they both have 10." Mr. K wants his students to notice that in the first problem one person counted down to solve the problem and in the second problem, Sadie counted up. He probes, "Hmm. I see how you did that Sadie. This is a subtraction problem and I'm wondering how come you didn't count back?" Sadie explained, "Well I thought I would run out of fingers if I counted back." Mr. K follows up, "Yes I can see that you would. Did anyone count back to figure this out?" Cole says, "I did. I just used the hundreds chart and counted back 12 until I landed on 4." Mr. K says , "Oh I see how you kept track" and continues with, "Okay, let's all be thinking about whether you are counting back or counting up on the next problem and let's talk about why you might choose one or the other." *Commentary*: Mr. K asked students to share their answers and how they got their answers, directing students to repeat their count and asking others to listen and follow along on their own fingers. Mr. K also made decisions about how to represent his students' strategies and ideas on the board to provide models for how to record one's thinking and to provide visual models for relating, for example, distance between numbers and the decision to count up from one number to the next or take away the	*Feature of instruction:* Elicit students' ideas and with the goal of the activity in mind, orient students' ideas to one another. *Rehearse*: Use questions to make the instructional goal salient. Learn how to respond to different strategies that are shared, developing ways to focus students' attention on mathematical ideas. Use representations to show students' thinking.

second number from the first. Finally, Mr. K focused his students' attention by asking them to track whether they chose to count up or back as they solved the problems.	
Pose third problem: 25 - 6 Mr. K wrote the next problem on the board, 25 - 6. This time when he asked for answers, three different answers were shared, 18, 19 and 21. Mr. K asked his class, "Now when you look at these numbers, is there one that you think we could cross off because it doesn't seem reasonable? Take a second to talk to a partner about which one you might cross off." His students turn to their neighbours to consider this question. Mr. K listens in on pairs and chooses to call on a pair with a student who thought the answer was 21 to see how she revised her thinking. Bea says, "Well, I thought it was 21 because I just thought, 'Easy, 6 minus 5 is 1 and you just keep 2. So it's 21. But now I see that if you take 5 away from 25, you'd already get to 20, so taking 6 away can't be 21.'" Mr. K notices a lot of nods in the room and asks what happened when the pairs checked the answers. Did they arrive at 19 or 18? Isaac says, "If you count back you take away 25, then 24, 23, 22, 21, 20 and you're left with 19." Mr. K shows Isaac's thinking on the board by crossing out the numbers 25 through 20 to show 19 left. ~~25~~ ~~24~~ ~~23~~ ~~22~~ ~~21~~ ~~20~~ 19 Mr. K once again noted that Isaac counted back when he showed how he figured out the answer. *Commentary*: When several answers were given for this third problem, he asked students to consider whether there was an answer they could eliminate by thinking about the reasonableness of the answer and allowed students to think through this with a partner. Mr. K uses partner sharing in strategic ways to monitor for students ideas, give them a chance to process what they heard in the whole group, and provide opportunities to think about the reasonableness of the answer. He also decided who he might call on and asked permission from the student when he thought it would help support that student to participate.	*Feature of instruction*: Monitor for students' understanding and thinking to make decisions about which ideas to select for public discussion and how those choices connect to instructional goal. When students make errors, use them as opportunities to do more mathematical thinking. Cultivate an environment where students can revise their thinking. *Rehearse*: Use representations that might help model how students could record thinking and that might help students follow the strategies that are being shared and begin to build connections. Use different participation structures successfully to engage each student in thinking about critical mathematical ideas. Practice ways of responding to student error that build norms that revisions to ideas are opportunities for further mathematical thinking.

Pose last problem: 32 - 28	
Mr. K tells the students he will pose one last problem and this time ask why they counted backwards or forwards, 32 – 28. He hears audible gasps from the students as many turn to their fingers. Once they are ready to share, Mr. K asks for answers and students offer the answer, 4.	*Feature of instruction*: Realizing mathematical goal through the way discussion is orchestrated.
Mr. K asked, “Now did you count up or back to get the answer? Turn to your neighbour and tell them which strategy you chose and why?”	*Rehearse:* How to close activity and assess where students are in aiming instructional goal.
Mr. K again uses this opportunity to listen to what students are saying and he has his eye on a few students who sometimes miscount to hear how they solved the problem. He sees an opportunity to call on Ali and leans over quietly to see if he’d be willing to share his thinking. Ali gives him a tentative yes and Mr. K smiles at him and says he’ll help if he needs it. Mr. K calls the group together and asks Ali to tell the group what method he chose.	
Ali replies, “Well, I just went up from 28.” “Can we hear you count?” Mr. K asks. Ali begins quietly and Mr. K tells him, “Let’s see your hands up high. Do you want to stand up and show us? And maybe the class can follow along with you with their hands.”	
Ali begins counting numbers, holding up a finger with each count, “29, 30, 31, 32.”	
Mr. K asks, “Now why didn’t you count back 28?”	
Ali perks up a bit and says, “Well, I know that 32 is really close to 28.”	
Mr. K, “And why would that make a difference?”	
“Well, I don’t have as many numbers to count. I know that if I count back, I would have to count 28 times, but if I count up, it will not be that many,” Ali explains.	
Maddie volunteers, “Yeah and me and Jason noticed that um, that’s what we did with the other numbers!”	
Mr. K asks, “What do you mean.”	
Maddie says, “See on 11 minus 3, the numbers are kind of far apart and so are 25 – 6. You don’t have to count back that much.”	
Mr. K responds, “Does someone have a different way of talking about this?” David shares, “Well, you can think about where the numbers are on the number line. There are not a lot of numbers between 28 and 32. But there are a lot of numbers between 11 and 3. So if you want to start counting up, you don’t have to count as many spaces.”	
Mr. K states, “I think this is an important idea for us to think about. When we need to subtract numbers,	

we might want to think about whether it would be faster to count up or count back. If the numbers are close together, it might be faster to count up. In your math journals, write down an example of a problem where you think counting up would be a good idea. I'll take a look at those as you're working." Mr. K gives the students a minute to think of their example and then launches into the problem of the day. *Commentary*: Sequencing a set of related problems is an instructional activity that typically has convergent goals for discussion. In this example, Mr. K used his time for this warm up to focus students' attention on a particular idea related to subtraction. For his purposes on this day, he did not aim to elicit all of the ways students could think about subtracting one number from another. Instead, he wanted to raise issues for the students about when it might be more strategic to take away the second number (the subtrahend) and when it might be more strategic to count from one another to another. He chose the numbers in the string so they would bring out this issue, choosing numbers that were only 3 or 4 apart versus numbers that were over 10 apart. He posed the problems one at a time, and as the string progressed, he explicitly asked students to think about whether they would count up or count back to get the answer and why. And finally, he ended the warm-up by asking every student to give an example of numbers where it would be strategic to count up so that he could examine each student's journal for evidence of where they are with respect to this idea.	

Teacher Educator Learning

We have engaged together over the last three years—first imagining, planning for and then carrying out major innovations in our university methods courses. We have engaged in continuous cycles of designing, trying out our designs in different contexts, observing and/or recording what happens, and analysing our observations and records as a way of refining both instructional activities and pedagogies of practice. All of these activities, as well as all of the teaching of novices that has occurred as we have worked together, have been recorded in field notes and on video, capturing the early stages of the "daily operations" involved in this work.

This professional education pedagogy has been an important site for our own learning because it specifies unfamiliar roles for both teacher educators and novice teacher-learners. It also assumes a different kind of connection between what happens in university courses and what happens in classrooms than the one that now exists in most programs. Below, we point to some of the aspects of our professional learning through our views of teacher learning and our emerging theoretical ideas about practice.

Getting Inside Practice

To claim that we could guide novice teachers to use specific mathematical activities meant that we ourselves had to be able to teach with them successfully, keeping in view the instructional goal and our ability to engage a range of learners across grade levels in worthwhile mathematical work. We spent time with K-5 children in classrooms, experimenting with the activities ourselves and learning from practicing teachers' experimentations. We have learned how pedagogical supports within the instructional activities provide access to worthwhile mathematics and develop K-5 student learning. Experimenting with the instructional activities ourselves has meant, in varying degrees, opening ourselves up to certain vulnerabilities – while we may have felt confident with mathematical ideas themselves, we had to work on how to use mathematical representations, successfully manage a group of children, and take up and respond to a range of student ideas, both expected and unexpected. Moreover, we had to make our own practices of teaching public to one another at group meetings and cope with the uncertainty such risks always bring. These public exchanges, however, were critical in our understanding of the instructional activities and our ability to get inside the teaching practices that the activities required.

Learning How to Lead Public Guided Rehearsals

Learning what it means to construct meaningful rehearsals of these instructional activities during methods courses has been an enormous challenge. We wanted to use rehearsals to both prepare and propel novices to experiment with and learn from the instructional activities. We are learning from each other what coaching in the context of public rehearsals entails, why we might make particular coaching moves, how to make decisions about what kind of feedback to give and when, and how that relates to what novices are actually learning. We have learned what moves on our part help teachers learn – how we break the activity apart, what aspects of the practice we focus on first, and what artefacts and structures support learning.

Understanding the Experiences of Novices Across Contexts

Aiming to support novice teacher learning also means becoming more attuned to what novices tell us and what they do with the learning experiences we have tried to design and engineer for them. Following our students into their field experiences, student teaching, and their first year of teaching has made us confront quite directly whether our students are learning to do what we are trying to teach them to do. Conventionally, university professors judge their success by whether students write good papers or do well on exams. By crossing boundaries that are rarely crossed, we are developing a different sense of who we are and what we are able to do as teacher educators.

Managing organizational features of working with teacher education programs

While our designs for teacher education are clearly centred on what happens inside our courses, we have learned about how different features of the organizational structures of our teacher education programs and our "partnerships" with local schools both support and constrain the pedagogies we are developing. For example, for our work to grow and be successful we recognize the need to find ways to disrupt the prevailing ways of separating coursework from fieldwork, bring teaching partners and university supervisors into our collaboration. We need to understand, draw on and at times counter the prevailing views of mathematics teaching in our preservice placements. We need to face the daunting diversity in how teaching is enacted in different classrooms by experienced teachers. We need to figure out how to align instruction with curriculum, often in situations where the curriculum is not the same across the classrooms in which our novices do their fieldwork. We need to be able to experiment and innovate within well-established teacher education programs in ways that allow for change and learning.

Conclusion

Mathematics educators are among the leaders in teacher education in theorizing about practice-based education (Ball & Cohen, 1999; Lampert & Ball, 1998; Smith, 2001). Yet, we are not satisfied that we are preparing teachers in ways that enable them to grow as professionals who take students' disciplinary knowledge and dispositions seriously. We recognize the need to advance our work as teacher educators. This paper aimed to contribute to conversations about what teacher educators might learn to do if they are to prepare novice teachers for ambitious instruction.

References

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 3-31). San Francisco: Jossey-Bass.

Barab, S. A., & Duffy, T. M. (1998). From practice fields to communities of practice. In D. Jonassen & S. Land (Eds.), Theoretical Foundations of Learning Environments. Manwah, NJ: Erlbaum.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education, 24*, 417-436.

Bransford, J. D., Brown, A. L., & Cocking, R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school*. Washington, D.C.: National Academy Press.

Bryk, T. (October, 2007). Ruminations on re-inventing an R and D capacity for educational improvement. Paper prepared for the American Enterprise Institute Conference on the "Supply Side of School Reform." Washington, D.C.

Carpenter, Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Chapin, S. H., O'Connor, C., & Anderson, N. (2003). *Classroom discussions: Using math talk to help students learn* (Grades 1-6). Sausalito, CA: Math Solutions Publications.

Cobb, P., Dean, C., & Zhao, Q. (2006). *Conducting design experiments to support teachers' learning.* Paper presented at the Annual Conference of the American Educational Research Association, San Francisco, CA.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G., Trigatti, B., Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics project. *Journal for Research in Mathematics Education, 22*, 3-29.

Cohen, D., Raudenbusch, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25,* 119-142.

Cook, J., & Brown, J. S. (1999). Bridging epistemologies: The generative dance between organizational knowledge and organizational knowing. *Organization Science, 10,* 381-400.

Dreyfus, S. E. (2004). The five-stage model of adult skill acquisition. *Bulletin of Science, Technology & Society, 24* (3), 177-181.

Ericsson, K. A., Krampe, R. T., & Tesch-Romer, C. (1993). The role of deliberate practice in the acquisition of expert performance. *Psychological Review, 100*, 363–406.

Ericsson, K. A. (2002). Attaining excellence through deliberate practice: Insights form the study of expert performance. In M. Ferrari (Ed.), *The pursuit of excellence in education* (pp. 21-55). Hillsdale, NJ: Erlbaum.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record, 103*(6), 1013-1055.

Feldman, M. S., & Pentland, B. T. (2003). Reconceptualizing organizational routines as a source of flexibility and change. *Administrative Science Quarterly, 48*, 94-118.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal, 30,* 555–583.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work*. Portsmouth, NH: Heinemann.

Franke, M. L., Carpenter, T. P., Levi, L., & E. Fennema (2001). Capturing teachers' generative growth: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38,* 653-689.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28*, 130-162.

Gasson, S. (2005). The dynamics of sensemaking, knowledge, and expertise in collaborative, boundary spanning design. *Journal of Computer-Mediated Communication,* 10(4), article 14. Retrieved October 16, 2006 from http://jcmc.indiana.edu/vol10/issue4/gasson.html

Ghousseini, H. N. (2008). *Learning with routines: Preservice teachers learning to lead classroom mathematics discussions.* Ph.D. dissertation, the University of Michigan.

Gravemeijer, K., & van Galen, F. (2003). Facts and algorithms as products of students' own mathematical activity. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 114-122). Reston, VA: National Council of Teachers of Mathematics.

Grossman P. & McDonald M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*,184 –205.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9).

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., & Garnier, H. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis, 27*, 111-132.

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.

Kamii, C., & Joseph, L.L. (2004). *Young children reinvent arithmetic, 2nd grade: Implications of piaget's theory*. New York: Teachers College Press.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203-235.

Kazemi, E., & Hubbard, A. (2008). New directions for the design and study of professional development: Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education, 59,* 428-441.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academies Press.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Lampert, M., & Ball, D. L. (1998). *Teaching, multimedia, and mathematics: Investigations of real practice*. New York: Teachers College Press.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., Franke, M. (in preparation) *Using designed instructional activities to enable novices to manage ambitious mathematics teaching.* To appear in M.K. Stein & L. Kucan (Eds.) Instructional explanations in the discipline. New York: Springer.

Lampert, M., Boerst, T., & Graziani, F. (in press). Using organizational assets in the service of ambitious teaching practice. *Teachers College Record.*

Lampert, M. & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning in and for ambitious practice. *Elementary School Journal,109,* 491-509.

Latour, B. (1991). Technology is society made durable. In J. Law (Ed.) *A sociology of monsters* (pp. 103-131). London: Routledge.

Leinhardt, G. & Steele, M. D. (2005). Seeing the complexity of standing to the side: Instructional dialogues. *Cognition and Instruction, 23*, 87-163.

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35*, 3-14.

Mewborn, D. S., & Stinson, D. W. (in press). Teacher education as assisted performance. Teachers College Record.

National Research Council. (2002). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Newmann, F., & Associates (1996). *Authentic achievement: Restructuring schools for intellectual quality.* San Francisco: Jossey Bass.

Patel, V. L., Kaufman, D. R., & Magder, S. A. (1996). The acquisition of medical expertise in complex dynamic environments. In K.A. Ericsson (Ed.), *The road to excellence: The acquisition of expert performance in the arts, sciences, sports, and games* (pp. 127–166). Hillsdale, NJ: Lawrence Erlbaum.

Reckwitz, A. (2002). Toward a theory of social practice: A development in culturalist theorizing. *European Journal of Social Theory, 5* (2), 243-263.

Schifter, D., Bastable, V., & Russell, S. J. (1999). *Developing mathematical ideas: Building a system of tens, casebook and Facilitator's guide*. Parsippany, NJ: Dale Seymour.

Schoenfeld, A. H. (Ed.) (2008). A study of teaching: Multiple lenses, multiple views. Journal for research in Mathematics Education monograph series. Reston, VA: National Council of Teachers of Mathematics.

Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education, 20*, 163-183.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 314-340.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, England: Cambridge University.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & C. D. Pearson (Eds.), *Review of Research in Education* (Vol. 24, pp. 173-109). Washington, D. C.: American Educational Research Association.

Appendix 1: Example of protocol used to guide planning and enacting a sequence of relating

Posing a Set of Related Problems

Step 1: Choose a purposeful sequence of related problems

Be sure that you have thoughtfully chosen or created a string with a specific mathematical purpose. What is the string designed to highlight? What relationships or strategies do you want participants to notice?

Step 2: Introduce task to students and anticipate the flow/pacing

Keeping your purpose in mind will help you decide when to delve and when to gloss over particular problems and/or strategies. For example, you may want to tell participants you expect them to "just know" the first problem in a string (although this is not always the case!) Also, if someone shares a rather complicated strategy that does not match your goals, you may choose not to ask them a lot of probing questions. In contrast, if someone has shared the strategy you'd like people to focus on, slow the conversation down by asking someone else to restate.

Decide what management device you want to use for kids to signal that they have their answer. How does your management routine convey messages about competence, status, competition, speed, etc.?

Step 3: Pose the first problem

Start with a problem that you know the kids will find easy.

Get answer(s) from kids.

Decide if you want to link answer to a particular representation.

Decide if you want to be in charge of the representation or have the kids create or direct your representation.

Listen to response and decide if clarification, elaboration or explanation is needed. If a student shares a strategy you want to highlight, decide how much elaboration, re-voicing and rephrasing you want to do or request that other students do.

Decide if you want to request a different strategy or if you want to ask students to comment on or build upon current strategy.

Decide how to utilize other student voices to explain mathematical reasoning.

Decide how to record students' mathematical reasoning.

Step 4: Pose second problem

Think about how to keep the problems of the string visible to the students if you have also been recording their strategies.

Request answer(s) from student.

Decide how you want a student to link their answer back to representation. Request student to describe how they got an answer(s).

Decide how you want students to treat different answers and strategies shared thus far. Do you want to comment or have students comment on their similarity or differences? Do you want to make an explicit link to how the strategies used on previous problems might support solving this problem?

Step 5: Pose remaining problems.

Pose each problem one at a time and consider all ideas from steps 3 & 4.

NOTE: If the last problem in the string is an application of the ideas that the string is designed to focus attention to, explicitly tell students you are posing a new problem. "*Now I'm going to pose a new problem with different numbers. See if the work we've just done with ___ idea, helps you get the answer for this one.*"

Step 6: Highlighting the big ideas and closing the task

Discuss the specific strategy that this string was designed to address. Work with students to make connections among the problems that were posed to them within the string. Make the mathematical strategy/concept that this string highlighted explicit for students.

Decide if it is necessary to pose another similar problem where students might be able to use the strategy just discussed and highlighted in the string.

Note: this protocol was used primarily at the University of Washington. We are in the process of experimenting with different kinds of protocols and specifications of activities across our three sites. We are also doing more work to link our protocols to instantiations of the activities in the various elementary curricula encountered in the US.

Challenges that might/will occur:
Children offer incorrect responses
Many children seem to not be participating
You ask for any connections among the problems and get no response a
Children are not seeing connections among problems: they are not using previous problems in the string to solve the harder problems that come nearer the end of the string