Relative Values of Curriculum Topics in Undergraduate Mathematics in an Integrated Technology Environment

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Changes to the relative value of curriculum topics, when using computer algebra systems in secondary school mathematics, have been previously considered in several studies (e.g. Artigue, 2002; Stacey, 2003). This paper extends these findings to an examination of particular topics in undergraduate mathematics, as part of a wider study investigating issues of integrated technology at the tertiary level. This study suggests that issues of curricular value are a critical factor in the successful implementation of integrated technology, and that a re-examination of the relative values of fundamental topics remains a significant challenge for undergraduate mathematics in a rapidly evolving technological environment.

A study by Oates (2004) identified the variety of ways in which integrated technology is interpreted in the literature. Responses from an exploratory survey of undergraduate mathematics colleagues conducted for this pilot study were used to propose an initial framework for describing integrated technology. Following this earlier study, a wider international survey of undergraduate mathematics educators was conducted, that received responses from forty-one tertiary institutions representing eight countries (see Oates, 2009). In addition to examining further the elements identified in the pilot study, this latter survey aimed to investigate a number of factors subsequently identified in the literature, particularly those associated with the use of technology at the tertiary level. These issues include student instrumentation (Artigue, 2002; Stewart et al., 2005); the effect of research mathematicians' beliefs about mathematical knowledge, technology and pedagogy on their use of technology (Anguelov et al., 2001; Keynes & Olsen, 2001; Kersaint et al., 2003); and the relationship between mathematicians' experience with different technologies within their own research domains and their pedagogical technical knowledge (PTK). PTK is characterised as the necessary knowledge of the principles and techniques required to teach mathematics using a given technology (Hong & Thomas, 2006).

A taxonomy of integrated technology was then developed, refined from the model proposed in Oates (2004), and informed by responses to the international survey. The complete taxonomy is not reproduced here, but a summary of the major components is provided in Table 1, along with some exemplars from the survey to illustrate the focus for each of the components. The full taxonomy describes a complex range of factors that should be considered for each of the six main components depicted in Table 1, and some examples of the factors from the *Mathematical Factors* and *Staff Factors* are provided later, in a closer examination of content issues (see Oates, 2009, pp. 202-203)

An observational study of technology implementation in undergraduate courses at The University of Auckland over the period 2001 to 2008 was then used to examine the effectiveness of the taxonomy, in describing the outcomes of technology integration initiatives. Evidence was sought to measure the effects and significance of each of the factors identified in the taxonomy. Several elements of the taxonomy were found to have played a critical role in the comparative success of the technology implementation. More importantly, the findings emphasise interdependency between the elements of the taxonomy. The results highlight that it is essential to recognise the inter-related structure of the taxonomy. Oates (2009) concludes that addressing the factors in a comprehensive

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2). Palmerston North, NZ: MERGA.

fashion leads to higher and more sustainable levels of technology integration. “While attendance to some elements in isolation may obviously stimulate changes, it is difficult to achieve effective, sustainable technology integration through such a limited approach.”

Table 1.
A Taxonomy for Integrated Technology

Taxonomy Component	Characteristic Survey Response for Taxonomy Component
Access	“It has many benefits if all the students can reach almost the same technology; otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities.” (Uruguay)
Assessment	“Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results”. (Australia)
Organisational Factors	“Bureaucracy slow to change. Use often isolated to single course.” (South Africa)
Mathematical Factors	“Less emphasis on techniques, more powerful visualisation.” (New Zealand)
Staff Factors	“Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved.” (Australia)
Student Factors	“It’s difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology.” (Canada)

Content Issues

Mathematical content issues associated with the use of technology have been widely considered in the literature at all levels of mathematics for many years, so it is thus not surprising that they were identified as a significant factor in the taxonomy and the subsequent technology implementation. The order and value of curriculum topics is included in the taxonomy under *Mathematical Factors*, together with factors such as the imperatives of individual subjects (e.g. specific computational software in applied courses); the nature of mathematics and mathematical knowledge; and the development of mathematical reasoning and skills. Given the interdependency between elements in the taxonomy, arguments about the worth of particular topics are also closely associated with elements under *Staff Factors,* such as the beliefs of staff about the nature of mathematics, technology and learning; the effects of these beliefs on their teaching and students learning; and the relationship between their specialist research domains and these beliefs (Norton & Cooper, 2001; Kendal & Stacey, 2001; Keynes & Olsen, 2001).

The focus of this paper is on changes to the value of specific topics in an integrated technology mathematics curriculum. However, there are many other factors associated with content issues that are also considered in the taxonomy. For example, Schwartz (1999) observes that the relative importance of any particular mathematics depends upon who is doing the deciding, the priority they assign to the different societal aims of education, and their beliefs in the nature and role of mathematics in addressing those aims.

Holton (2005, p. 306) believes that such questions must consider and balance the needs of our students: “There is clearly no one right curriculum…not all students are a subset of those going on to PhDs in mathematics.” The order of topics is also commonly debated, for example when teaching calculus. The traditional approach begins with anti-differentiation. However, the advanced numerical methods and graphing packages that are now readily accessible to many students mean that we can address concrete *summative* problems of integral calculus first, following on with the more abstract notions of rates of change and the fundamental theorem later (Harman, 2003, p. 93). Smith (1998) describes how the traditional sequence of topics in teaching differential equations (the *raison d’être* of calculus) can be turned on its head using technology. The power of CAS calculators to “draw direction fields and model population growth using real-world data allows students to explore what a differential equation object is from day one” (Smith, 1998, p. 784). Thomas and Holton (2003) similarly describe many studies that demonstrate particular uses and benefits of technology for a range of topics in undergraduate mathematics, such as the teaching of limits (p. 370), and group theory and abstract algebra (p. 371). Another series of studies found that technology has the ability to improve students’ limited conceptualisation of Riemann integration (see e.g. Hong & Thomas, 1998).

It is also argued that it is not the content itself, but the way in which it is taught that matters most (Noss, 1998; Holton, 2005). For example, it is asserted that the advanced graphical and algebraic features of CAS have the ability to transform the teaching of differential equations. King, Hillel and Artigue (2001) observe that technologies that graph slope fields and direction fields enable students to engage in qualitative analyses of previously inaccessible differential equations, rather than just the traditional analytic techniques. “Thus, the focus of a ‘differential equations’ course could shift from just finding the solution functions, to graphically organising the space of solution functions….and examining the nature of the solution functions” (King et al., 2001, p. 351). Schwartz (1999, p. 115) concludes that “for those who particularly value the importance of...personal growth and development of students, it is likely that the specific mathematical content of the curriculum will be less important than the ways in which that content is engaged.” While these aspects are equally accommodated within the taxonomy (*Staff* and *Mathematical Factors,* Oates, 2009), they will not be considered further here.

With respect to the effect of technology on content value, Oates and Thomas (2001, p. 83) provide a list of aspects they suggest should be considered. In addition to potential changes in the order of topics, they suggest that:

- (Some) content areas may be trivialised or made redundant by the use of the graphics calculator. This could include for example some routine algebraic skills.
- New content areas or richer conceptual understanding becomes accessible using the graphics calculator.
- The possibility (exists) that some of the new content areas opened up by the graphics calculator may be trivial in nature and of limited educational value.

Artigue (2002) and Stacey (2003) provide a means of examining such aspects, developing a framework that considers the *pragmatic* and *epistemic* values of individual topics, with Stacey adding a third *pedagogical* value. The *pragmatic* value recognises the usefulness of a topic, the *epistemic* value measures the importance of a topic’s place in the structure and development of mathematical knowledge, and *pedagogical* value considers whether a topic serves a purpose not related to the content itself, such as providing an opportunity to practice skills (Stacey, 2003, p. 6). The use of CAS may considerably change the relative values of a topic, often reducing the pragmatic value, and sometimes questioning the epistemic value. For example, using calculus to make a linear approximation to a function with the formula $f(x+h) \approx f(x) + h.f'(x)$ once had a pragmatic value in permitting ready approximation to the values of complicated functions. One could, for example, readily approximate the square root of 26.1 as 5.11 (with correct value 5.1088 to four decimal places) based on x = 25, whereas a by-hand calculation is very long. Whilst even a simple scientific calculator removes the pragmatic value of this formula for the student, Stacey (2003, p. 7) suggests that it still has immense epistemic value, because it is central to the principles underlying calculus. Another example is seen in differentiation of complex functions, where Stacey (2003) believes, with a few small provisos, that given the reliability of CAS in performing complex differentiation, the product-rule now has little pragmatic value. There is however a strong case for inclusion on epistemic grounds, because it demonstrates the connections to other concepts. Stacey concludes that:

> The curriculum value of topics is markedly changed by the introduction of CAS. Old justifications for teaching topics, especially pragmatic justifications, will not necessarily apply...The educational community needs to build up sophisticated rationales for curriculum areas that were not debated in the past. Justifications may be on pragmatic, epistemic, or pedagogical grounds. (Stacey, 2003, p. 7)

These examples demonstrate that assessing the relative values of individual topics is a complex task, even for a researcher of Stacey's considerable experience, and suggest that extending this across all the topics in a curriculum, and arriving at a consensus between the often diverse members of a mathematics department is a challenging task. However, even given this complexity, the contrast of responses to questions about the value of specific topics found in surveys and pilot interviews was surprising, and prompted a closer examination of this issue (Oates, 2009). The observational phase of this study included purposeful interviews with seven respondents drawn from staff involved in the Auckland implementation, and the survey of international undergraduate mathematic educators. Investigating views towards the relative value of curriculum topics when students have access to technology became a specific focus of these interviews.

The Relative Value of Topics with Integrated Technology

Although the restructuring of courses in 2006 was principally in response to University directives, Oates (2009, pp. 229-236) notes several examples, drawn from the observational component of the study, where the value of topics was considered during the restructuring process. The following extract from the revised syllabus for the principal first-year course lectures on "Solving Systems of Linear Equations" demonstrates the curricular value attached to this topic, and also reinforces the value attached to technology.

> **Comments on Delivery:** It is extremely important that students firmly know which matrix is in the reduced echelon form and which is not. They have to see lots of them.
>
> **Technology Exercises:** Students have to learn how input matrices of systems of linear equations…They will have to learn how to interpret the output of the command *linsolve* (or similar) to find out if a system is consistent, if it has a unique solution or not and find that unique solution if it exists.

The statement also reflects one of the major benefits of technology identified in the literature, in presenting students with the opportunity for repeated exposure to multiple dynamic representations, (see e.g. Hong & Thomas, 1998; Thomas & Holton, 2003), and the consideration given to the appropriate use of technology (conducting routine algorithmic processes).

Another proposal for the revised differentiation lectures in the same course prescribes the use of technology to assist in the necessary reduction in lectures. It notes that:

> Curve sketching will be relegated to tutorials and exercises, supported with CAS. In using CAS for curve sketching, it is particularly important to emphasise the need to consider the critical points to get an idea of what interval of the domain to graph the function over.

A further example is found in changes to the follow on second-level general mathematics course, where the teaching of methods of integration was cut from the syllabus. The restructuring proposal notes that this reduction was to be partly compensated for by the use of technology to perform these integrations. This latter example suggests that the pragmatic value of such techniques has diminished with the use of technology, although there is no specific evidence from the discussion documents that this concept was behind the decisions at the time. However, an interview response from an applied mathematics colleague suggests that the similar removal of techniques of solving differential equations from the syllabus for one course was indeed a direct result of technology changing the pragmatic value of a topic:

> Before computers, there used to be a big emphasis on special techniques for special differential equations, …students had to recognise some 15 different types of differential equation, you had no options, you had to solve it explicitly, there was no numerical option. You had to know the technique, all that's gone, if you don't recognise a differential equation, you whack it on a computer.

The value of technology is seen here not just in its computational capabilities, but also in the opportunity it allows to investigate problems and develop flexible solving strategies, as opposed to learning a catalogue of instrumental techniques. While the transition from learning techniques to using technology to investigate and calculate solutions was a comparatively natural and uncontroversial progression at Auckland, this colleague noted that such an approach is by no means universal. Indeed, many textbooks still include specific techniques as an important part of a Differential Equations course, and these courses are still taught traditionally in many places. Another colleague considered integration by substitution as a similarly specific technique with little pragmatic or epistemic value in a technological environment:

> Most students can barely see how it fits, but they get used to a standard technique of putting the things in, … the question mathematicians need to answer now is, do such mechanistic techniques generalise to more general problem-solving type situations later which are going to be useful?, and I think the answer is no.

The effect of technology on calculating eigenvalues and eigenvectors is seen as less clear by another respondent. While she concedes that some pragmatic value has definitely been lost with the ability of technology to compute these directly, she still perceives some pragmatic and epistemic value in teaching these procedures. "Matlab does not easily find families of eigenvectors, and often presents the results of complex eigenvectors in an unusual structure that requires considerable understanding from students to become recognisable." Hence while students are encouraged to use technology for such calculations in tutorials and assignments, manual procedures are still taught and examined.

The complexity of assessing any given topic's value is demonstrated in the following four responses, drawn from a series of email exchanges leading up to the 2006 course changes. This exchange, which began as a response to a query from one colleague about students working together on assignments, sparked a passionate debate about the value of a particular technique and beliefs about the nature of mathematics, teaching and technology. The topic was an exercise on *LU* factoring (a process expressing a matrix A as the product of two essentially triangular matrices in order to solve $Ax=b$):

1. (It's a) bad idea to teach this obsolete, tedious *LU* factorisation, which no one needs anymore…(while) it still has some applicability, … currently no one client department needs it…we should give preference to teaching ideas, not techniques.
2. The issues divide neatly between the importance of the technique and the extent to which it should be laboured as a teaching item. The topic is intrinsically important because it is at the centre of all practical scientific computation…Is it important as a teaching item? Sometimes a mathematical concept has to be introduced without a directly practical application…I personally think that the opportunity to introduce *LU* with pivoting is like finding a flower in the desert.
3. This flower unfortunately has to be uprooted, together with integration by trigonometric substitution and other techniques which have lost relevance for the wider audience. A regular person with regular needs will be much better off using Maple. Such techniques should be taught in specialised courses…It takes too much precious time which can be better spent on building understanding.
4. I believe that a good portion of (any) honest technique is useful for students learning mathematics, as a training of ability for prolonged logical concentration. Separating learning of ideas from learning of adequate technical support looks similar to learning by heart a French song without learning French language.

Questions about the *pragmatic* and *epistemic* value of *LU* factorisation are clearly evident in this debate. Some see little pragmatic value remaining in this topic, and are happy to see the responsibility assumed by technology, others believe strongly in the epistemic and pedagogical value. Several studies consider ways in which technology may possibly be used to resolve such conflicting arguments. Hillel (2001) illustrates this with an activity using CAS to perform elementary row reductions on the coefficient matrix of a relatively large homogeneous system with non-trivial solutions. The purpose of the activity is clarification of theory covered in class:

> Grasping the relation between elementary row operations and equivalent systems is the key notion, not the actual procedure for row-reducing matrices. Once understood, I see no reason why students should not be given free rein to use CAS and go directly to row reduced echelon form of a matrix without actually performing the row operations (and later, to go directly from a system of equations to a CAS-generated solution). (Hillel, 2001b, p. 374)

LaTorre (1993, p. 306) describes a similar process using CAS to perform all the individual steps of solving a system of linear equations, observing that in this manner, students "control the whole process from start to finish by deciding when and where to pivot, which row interchanges to make, and then effect back substitution interactively. The calculators carry only the computational burden". Both these examples suggest a reduction in the pragmatic value of Gaussian elimination with technology, but both clearly believe that the procedure retains its epistemic value in aiding students' understanding. However, as witnessed in the email debate about *LU* factorisation, and the examples presented earlier by Stacey (2003), reaching consensus on exactly which elements of a particular topic may retain their pragmatic or epistemic value and which may be assigned to technology is a difficult task, which often depends on who is making the decision (cf. Schwartz, 1999; Holton, 2005). This was confirmed in interviews, with respondents' beliefs about the value of Gaussian elimination differing considerably in some respects. All agreed that there is significant reduction in the pragmatic value, although one respondent believes that some pragmatic value is retained when the system has none or infinite solutions, as many CAS platforms do not distinguish these. There was also some agreement that technology was not always of benefit, especially for weaker students for whom learning is often reduced to a mechanical process. For such students, technology may exacerbate the problem as they become overly reliant on it, or be of little use if they do not know when its use is appropriate. One subject observes with respect to his marking of examination questions on Gaussian elimination that:

> The main problem doesn't seem to be that they can't do the operations (for which the calculator can help them); it's that they don't know what operations to do. They'll do three pages of working and still won't have any zeros in their matrix…the students don't understand what the goal is. I'm not sure how technology can help with this.

While one interview candidate was completely comfortable with students' using technology to directly calculate row-reduced matrices, another saw little educational value in such use. The former used the analogy of driving a car without knowing how it works to illustrate his point. The technological "black-box that provides a solution to a set of linear equations is probably all most people need". The latter sees little value in technology for the more important role of developing critical thinking and effective problem-solving strategies:

> Depends what one wants, I can't see how a student can understand the process by pushing a button, it may be OK for an engineer who just needs a seriously good program to provide the numbers at the end, they don't need to know anything about Gaussian elimination, …but most students haven't got a clue what their answer means, they know nothing more about their solutions than that they are a result of what they do.

He sees little pragmatic value for Gaussian elimination in such an environment, regardless of whether technology is used. Students who have learnt to row-reduce flawlessly without technology are frequently no better off than those who perform the operations using a calculator, so technology is seen as having no significant learning advantage other than to save time, check on working or avoid arithmetic errors.

Conclusion

These discussions demonstrate that, notwithstanding the holistic consideration of the taxonomy as advocated by Oates (2009), content issues were a significant factor in the technology implementation at The University of Auckland. Reaching consensus on the relative values of topics in the undergraduate mathematics curriculum was especially problematic. The findings of this study support the complexity of assessing the values of topics as described by Stacey (2003), and the conclusion by Oates (2009, p. 242), that "a re-examination of the changing pragmatic and epistemic values of specific topics, and the goals of mathematics education, within a rapidly evolving technological environment, remains a pressing challenge for undergraduate mathematics educators."

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