Highlighting the Similarities and Differences of the Mathematical Knowledge and Strategies of Year 4 Students

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Research has shown that successful mathematics students use different types of strategies from those struggling with mathematics. Year 4 students were tested using the *One Minute Tests of Basic Number Facts* (Westwood, 2000) and a paper and pencil Number Screening Test developed by the author and colleagues. Observation of the students during the assessment procedures highlighted the vast difference in the students' speed and accuracy when recalling basic facts and the types of strategies they used when solving mathematical tasks.

In a review of research about mathematics education Bell, Costello and Kuchemann (1983) identified four components of mathematical competence: facts and skills, conceptual structures, general strategies and attitudes. Considerable research has been conducted about students' understanding of mathematical concepts and the strategies they use to solve mathematical tasks. For example, research studies by Gray and Tall (1994) have shown that young students who are successful with mathematics use different types of strategies to those who are struggling with mathematics. Students struggling with mathematics are usually procedural thinkers dependent on the procedure of counting and limited to the "count-all" and "count-back" procedures. Gray and Tall (1994) defined procedural thinking as being demonstrated when:

> Numbers are used only as concrete entities to be manipulated through a counting process. The emphasis on the procedure reduces the focus on the relationship between input and output, often leading to idiosyncratic extensions of the counting procedure that may not generalize. (p. 132)

When asked to give the number before a certain number, students have been heard to count up to the number before responding with the number required. For example, when asked to give the number before 13 a student will count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, hesitate briefly, before saying 12.

While some students are dependent on rules and procedures other students give instantaneous answers. When these students were asked to explain how they solved the task they elaborate several different strategies they could have used and checked that their solutions were correct. According to Gray and Tall (1994), the use of known facts and procedures to solve problems, along with the demonstration of a combination of conceptual thinking and procedural thinking, indicate that these students are proceptual thinkers. Gray and Tall (1994) defined proceptual thinking as:

> the flexible facility to ... enable(s) a symbol to be maintained in short-term memory in a compact form for mental manipulation or to trigger a sequence of actions in time to carry out a mental process. It includes both concepts to know and processes to do. (pp. 124-125)

Procedural thinkers usually take much longer to solve a mathematical task than proceptual thinkers. For example, when asked: "What is 17 take away 16?" proceptual thinkers will respond instantly as they aware that 17 is one more than 16. However if students attempt to solve the task using learnt procedures it will take a lot longer as they try to count back from 17 sixteen times or draw 17 tally marks and cross out sixteen of those marks.

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2). Palmerston North, NZ: MERGA.

Hiebert and Lefevre (1986), commenting on the reliance of Year 3 and 4 students on rules and procedures, noted that:

> … by the time students are in third and fourth grade, they have acquired a large array of symbol manipulation rules. In general, the rules are more sensitive to syntactic constraints than to conceptual underpinnings. (pp. 20-21)

In March 1997, Australian state and territory education ministers agreed to a national goal that "every child leaving primary school should be numerate, and be able to read, write and spell at an appropriate level" (Masters & Forster, 1997, p.1). To support the national goal a national plan was developed. The national plan requires education authorities to provide support for teachers in their task of identifying children who are not achieving adequate literacy and numeracy skills and in providing early intervention strategies for these students. To determine which students might not achieve adequate numeracy skills Numeracy benchmarks were established for students at Years 3, 5 and 7 (MCEETYA, 2005). For example, the Year 3 Numeracy Benchmarks include:

- remember, or work out, basic addition facts to 10 + 10, the matching subtraction facts (e.g. 9 + 4 = 13, 13 – 9 = 4) and extensions of those facts (e.g. 23 – 9 = 14)
- add and subtract whole numbers (to 99) by using mental and written methods or by using a calculator

The Year 5 Numeracy Benchmarks include:

- know or work out multiplication facts to 10 x 10 and use these to work out extensions of those facts (e.g. 6 x 8 = 48, so 60 x 8 = 480)
- perform simple multiplications and divisions with whole numbers such as 34 x 6 and 36 ÷ 3 by using mental or written methods

Many students mathematically ‘at risk’ have difficulties remembering basic facts and use immature problem-solving procedures to solve simple arithmetic problems As Garnett (1998) states:

> Many learning disabled students have persistent trouble “memorizing" basic number facts in all four operations, despite adequate understanding and great effort expended trying to do so. Instead of readily knowing that 5 + 7 = 12, or that 4 x 6 = 24, these students continue laboriously over years to count fingers, pencil marks or scribbled circles and seem unable to develop efficient memory strategies on their own”.

Westwood (2000) states: “Without easy recall of basic number facts, students have difficulty with even simple mental addition and subtraction problems” (p.45). There appear to be two main reasons for failure to recall basic number facts (see for example, Geary & Brown, 1991; Siegel, & Linder, 1984). Some researchers attribute difficulties to limitations of short-term memory. That is, students do not retain several pieces of information long enough in working memory to make use of the information and become confused. For example, students say: “Is it …?” Some students, however, have difficulty with the basic number facts because they simply have not had enough practice and the responses have not become automatic. This could well be the consequence of using the ‘count-all’ or count by ones strategy where students in fact, may do a triple count. To add 6 + 3 they initially count the six objects, then count the three, then attempt to count the six and three added together. In many cases this third count may be incorrect as they no longer have one-to-one correspondence.

This paper focuses on the individual differences of Year 4 students’ mathematical knowledge and skills as demonstrated by results to two assessment protocols: *The One*

Minute Basic Number Facts Tests (Westwood, 2000) and the word problems from a paper and pencil Number Screening Test (Pearn , Doig & Hunting, in press).

Previous Research

In previous work the author found that Year 3 and 4 students struggling with mathematics relied on rules and procedures even when these were inefficient and unreliable (see for example, Pearn, 1994, 1999; Pearn & Hunting, 1995; Pearn & Merrifield, 1996). Several researchers have focused on students' systematic errors in addition and subtraction, and one hypothesis is that:

> … systematic errors or buggy algorithms, as they are frequently called, are a result of students relying on rote manipulation of symbols, and that developing understanding of multi-digit procedures would eliminate most buggy algorithms (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998, p. 6).

The responses in Table 1 highlight the diverse range of strategies used by Year 3 and 4 students to the following subtraction word problem (see for example Pearn, 1999).

> Richard is 131 cm tall. Mary is 17 cm shorter than Richard. How tall is Mary?

Although most students were able to identify the word problem as subtraction, 53% of the students were unable to complete the computation successfully.

Table 1:
Examples of students' solution strategies for subtraction task

Student	Response	Strategy Type
James	Drew, or attempted to draw, 131 tally marks with constant checking. Crossed off 17 marks, then attempted to count the number of tally marks left.	Count all – start at one count by ones
Lynda	Successfully counted back by ones from 131 keeping track on her fingers. Gave correct answer.	Count back
Mike	Immediate response of 114. When asked how he worked it out he explained: "I took 10 away from 31 then 7 away from 21 and that gives 114."	Intuitive strategy – not taught at school
Barry	131 - 17 126 Written algorithm (incorrect).	Buggy algorithm or faulty procedure
Megan	131 - 17 36 (Incorrect)	Faulty procedure
Mary	131 - 17 114 written algorithm (correct response)	Traditional algorithm

Lynda successfully used the procedural strategy of counting back by ones on her fingers. Some students drew 131 tally marks but because these were randomly placed over their page these students then had difficulty keeping track of how many tally marks they had drawn. They then had difficulty counting the number of tally marks left after crossing off 17 of them. Students using this strategy were usually unsuccessful and took a long time to

complete the task. Mike used his own informal strategy to solve the task correctly. There were several different invented strategies successfully used by students who were also able to explain their strategies in detail. Barry was just one student trying to remember "the rule". Megan had her own misconception of the rule she had been taught. When asked to explain her method she said she "just worked it out".

The Current Study

Every year Victorian students from Years 3, 5 and 7 undertake state-wide tests to test their literacy and numeracy skills. In 2007 this was the Achievement Improvement Monitor (AIM) (VCAA). The results from these tests provide information to enable state and Federal governments to plan new programs and are meant to be a useful source of feedback and guidance to students, parents and teachers. Results from state-wide testing in Victoria revealed that Year 3 and Year 5 students from a large metropolitan primary school in the outer northern suburbs of Melbourne were not achieving at the level that the Principal expected. To provide additional information about the mathematical skills and understandings of the current Year 4 students, two additional pieces of assessment were used: The *One Minute Basic Number Facts Tests* (Westwood, 1995 in Westwood, 2000) and two versions of a paper and pencil Number Screening Test developed by the author and colleagues (Pearn, Doig & Hunting, in press).

The Sample

There were 122 students from five Year 4 classes that ranged in size from 19 to 27 students in each. The students' ages ranged from 8.92 (8 years and 11 months) to 10.25 years (10 years 3 months). The average age of the students was approximately 9 years 7 months.

The Assessment Protocols

The two assessment protocols: *The One Minute Basic Number Facts Tests* (Westwood, 2000) and a paper and pencil Number Screening Test (Pearn, Doig & Hunting, in press) were administered by the author to ensure consistency with the administration.

There are four *One Minute Basic Number Facts Tests*. Each test has 33 items that focus on one of the four processes: addition, subtraction, multiplication and division. These items are ordered randomly and not by difficulty. The addition test items include one-digit addends with either one- or two-digit sums. The subtraction test items include one-digit minuends and subtrahends with a positive one-digit difference and some two-digit minuends and one-digit subtrahends with a one-digit difference. The multiplication test contains items with one-digit multipliers and one-digit multiplicands while the division test has six one-digit dividends and 27 two-digit dividends divided by a one-digit divisor with one-digit quotients. Table 2 shows the first three items from each of the *One Minute Basic Facts Tests*.

Table 2:
The first three items from the Basic Number Facts Tests (Westwood, 2000)

Addition	Subtraction	Multiplication	Division
2 + 1 =	2 - 1 =	1 x 2 =	2 ÷ 1 =
1 + 4 =	5 - 1 =	2 x 3 =	4 ÷ 2 =

2 + 2 =	3 - 2 =	2 x 5 =	3 ÷ 1 =

The Number Screening Tests 2A and 2B (Pearn, Doig, & Hunting, in press) were designed to identify students mathematically 'at risk' in Years 3 and 4 (see for example, Pearn, 1999). Both versions of the test were deemed to be of similar difficulty which allowed students to be seated side by side for the test but unable to copy from the student beside them.

The Number Screening Tests contains 34 items that focus on number. Eight items focus on counting, six on place value, six on addition, six on subtraction and one item had multiplication as the focus. There are seven word problems. The counting tasks include items that required students to complete the sequence of counting forwards by ones from a two-digit number (including bridging across 100), counting backwards by ones from two- and three-digit numbers, counting forwards by tens from multiples and non-multiples of ten, counting forwards by fives from a multiple of five and counting forwards by twos from a non-multiple of two. The place value tasks include items that require students to write the number that is one more or less than a given number, or ten more or less than a number and ordering two and three digit numbers. The addition and subtraction tasks include one-digit and two-digit addends and subtrahends and one task requires students to find the missing addend.

Administration of Tests

Students from all five Year 4 classes were administered the *One Minute Basic Number Facts Tests* as given in the instructions for administration (Westwood, 2000, p.107). There were breaks between all the tests and as each test was administered the author ensured that students were aware of the process being used. For example, the author said something like: "Don't forget this is subtraction. You are taking the number away this time (p.107)." All students also completed the Number Screening Test (, Doig & Hunting, in press).

As the author administered the *One Minute Basic Number Facts Tests* several students from each class made the comment: "I can't do division!" Careful observation of the students during the assessment procedure revealed that students not only had difficulties with division but with all four processes. Many students completed the *One Minute Basic Number Facts Tests* using a 'counting by ones' strategy that was evidenced by the tapping of fingers, nodding of heads and the drawing of tally marks on the paper. As students completed the Number Screening Test many struggled with the word problems. They could read the problems but appeared to have difficulty deciding which process to use.

Analysis of the Data

In Table 3 the total students' scores for *The One Minute Basic Number Facts Tests* (Westwood, 2000, p.107) are compared for the four processes: addition, subtraction, multiplication and division. Results have been rounded to two decimal places. The author has used the Westwood term "normal range" to indicate the range of scores for 50% of the students in the particular age group (i.e. +/- 0.68 standard deviation). The "critically low score" is one standard deviation below the mean for the age group. According to Westwood that means a student designated as having "a critically low score" is in the bottom 16% of the age group.

Table 3:
Analysis of the One Minute Basic Number Facts Tests (Westwood, 2000)

	Addition	Subtraction	Multiplication	Division	Word problems
Range of scores	2 - 33	0 - 33	0 - 33	0 - 33	0 – 6
Median	23	14	12	6	3
Mean	22.34	15.93	13.21	8.56	2.66
Standard Deviation	6.49	7.48	7.73	7.72	1.743
Normal range	17.92 – 26.75	10.84 – 21.01	7.96 – 18.47	3.27 – 13.77	1.47 – 3.84
Critical low score	16	8	5	1	1

Students achieved higher scores for addition than subtraction. Subtraction scores are generally higher than multiplication scores, which are generally better than scores for division. While some students were unable to correctly answer any items there were some students who completed the 33 items for each process in less than one minute.

Ten students answered all 33 addition basic facts correctly within one minute. However 12 students correctly answered 14 or less addition facts. The mean for the addition facts for this group of students was 22.34, the median was 23, and the critically low score for addition was 16. Five students answered 33 subtraction basic facts correctly in less than one minute, 13 answered 8 facts or less. The mean for subtraction was 16 with the median of 14. The critically low score for subtraction was 8. Four students answered all 33 multiplication basic number facts correctly in less than one minute. Twelve students correctly answered 4 or less multiplication facts. The mean was 13 and the median was 12 correct multiplication facts and the critically low score for multiplication was 5 multiplication facts correct. Three students correctly answered 33 division number facts in less than 1 minute while 12 students correctly answered one or less number facts in one minute. The mean was 9, the median was 6 and the critically low score was 1 for division.

All students completed one version of the Number Screening Test 2. Many students from all classes struggled with the word problems from both versions. There were 12 students (8%) who did not attempt any word problems. There were 30 students (25%) who were only successful with one task but nine students (6%) correctly answered all six whole number word problems. Table 4 shows the percentage of students successful with each of the whole number word problems. While 78% of students successfully answered the addition word problem only 34% were successful with one subtraction problem (Task 2) while only 25% were successful with the other subtraction task. Nearly half the students were successful with the multiplication problem and more than half succeeded with one of the division problems with only 23% successful with the second division problem.

Table 4:
Success with word problems from the Number Screening Test (in percentages).

Addition	Subtraction		Multiplication	Division	
Task 1	Task 2	Task 5	Task 3	Task 4	Task 6
78	34	25	49	57	23

In Figure 1 the results from the *One Minute Basic Number Facts Tests* are compared to the results from the word problems from the Number Screening tests.

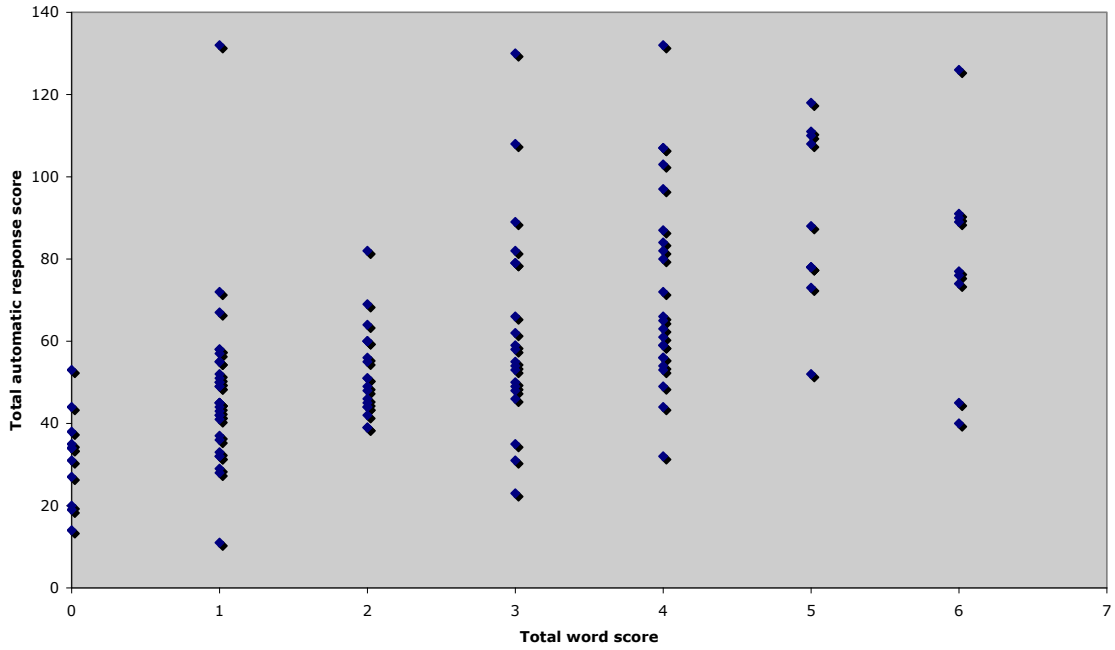

Figure 1. Comparison of *The Basic Number Facts Tests* and Word Problem scores.

There were large differences in scores for the one minute Basic Number Fact Tests and the Number Screening Tests. These results could be classified into four categories:

- high score for Basic Number Facts and for the word problems from the Number Screening tests
- high score for basic Number facts but low score for the word problems from the Number Screening tests
- low score for basic Number facts but high score for the word problems from the Number Screening tests
- low score for basic Number facts and low score for the word problems from the Number Screening tests

Success with the Basic Number Facts did not appear to guarantee success on the Number Screening Test or vice versa. Students in the top left hand quadrant had good recall of basic facts but were not successful with the word problems. Students in the bottom right hand quadrant were successful with the written word problems but were not as successful with automatic recall of basic facts.

One student from the bottom left hand quadrant scored a total of 44 for the *One Minute Basic Number Facts Tests* and solved one word problem. He systematically answered correctly all the addition and subtraction facts he attempted (22 for addition and 11 for subtraction). He chose specific multiplication & division number facts and correctly answered eight of the 11 multiplication facts he attempted and 3 of the 11 division facts he

attempted. He made several errors with multiplication facts but his difficulties were not obvious. However when responding to the division facts he answered as though they were subtraction facts. That is, he confused the division sign with the subtraction sign. Although he attempted all the word problems on the Number Screening Test he was only successful with one. Marcus added all the numbers in the word problems regardless of the task.

Conclusion

The results highlighted the large range of mathematical knowledge of skills of students at Year 4. Some students deemed by their teachers to be good at mathematics had good recall of learnt facts but were unable to solve simple word problems. Some students who were unable to recall number facts instantly were able to solve the word problems. *The One Minute Basic Number Fact Tests* (Westwood, 2000) identified students struggling to recall basic number facts and those who had instant recall. The Number Screening Tests were designed to identify students mathematically 'at risk' but in this case were also able to identify students who were successful at Year 4. A large number of the Year 4 students assessed using these two assessment protocols used inefficient counting strategies for both types of assessment. These strategies were demonstrated when the students tapped their fingers, blinked or rolled their eyes, and using tally marks on both tests. Teachers need to ensure that student develop more flexible strategies that allow them to develop fluency with number facts and know when, and how, to use them.

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