Scaffolding for Learning Equation Solving

Daphne Robson
Christchurch Polytechnic Institute of Technology
<robsond@cpit.ac.nz>

Walt Abell
Lincoln University
<abell@lincoln.ac.nz>

Therese Boustead
University of Canterbury
<therese.boustead@canterbury.ac.nz>

Students can have difficulty learning to solve equations because different types of thinking are needed. This study investigated scaffolding which allowed students to concentrate on decision making while software carried out other parts of the task. Prototype software developed for this study was trialled by adult students. Many students could solve more equations with the software than in the pre-test and half of these also transferred their learning to pen and paper.

Introduction

Students need to be able to solve simple equations if they study technical subjects which require them to work with formulae. However, they can have difficulty with learning to solve equations because different types of thinking are needed: making decisions, remembering processes accurately, and applying them correctly (VanLehn, 1989).

This paper investigates the design and development of computer software for providing assistance or scaffolding to adult students who are learning to solve equations. Prototype software was developed for this study. It provided scaffolding so that students could concentrate on the decisions at each step of their equation solving strategies, as the software carried out the rest of each step for them.

In the following sections, learning theories relevant to equation solving and scaffolding are considered. Software design and software trials are described and quantitative and qualitative results reported. Analysis and discussion focus on students' equation solving strategies and their attitudes to the scaffolding provided by the software.

Background

We shall consider two descriptions of the nature of equation solving and the type of mathematical thinking required. We then focus on the type of assistance that could help students achieve this type of thinking, and suggest a role for computer software. Terms are also defined.

Mayer (1982) described a strategy for solving an equation as being a sequence of actions. He described two types of actions. A "move" is an operation applied to both sides of an equation that results in a number, variable or term being moved from one side of the equation to the other. A "compute" involves rewriting terms on one side of an equation by carrying out a computation. Different strategies consist of different combinations of moves and computes. Mayer characterises an equation by the minimum number of actions required to solve it and points out that there may be different strategies that consist of this minimum number of actions. Thus, students learning to solve equations need to learn to how to create an equation solving strategy.

VanLehn (1989) described equation solving as an example of a "multistep problem" as it requires a sequence of steps, many of which are important to reaching the solution. A multistep problem is distinct from an "insight" problem in which only one of the steps is the key to reaching the solution. This description means that it must be more complicated

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to solve multistep problems than insight problems. VanLehn attributes the difficulty of solving equations to having to both choose an appropriate algebraic transformation at each step, and then remember and apply the transformation correctly. Lewis (1981) also described the need to know both which operator to apply at each step and how to apply the chosen operators. VanLehn described each step as consisting of two parts. First, a decision must be made about what action or algebraic transformation to do. Second, this must be carried out correctly.

In this paper, the first part of a step is called a "strategic decision" and the second part is called a "procedure". The two parts of each step are shown in the example in Table 1.

Table 1.
Strategic decisions and procedures for solving $4x-5=6$

	Strategic decision	Procedure
Step 1	Isolate $4x$ by adding 5 to both sides.	$4x-5+5=6+5$
		$4x=11$
Step 2	Isolate x by dividing both sides by 4.	$\frac{4x}{4}=\frac{11}{4}$
		$x=\frac{11}{4}$ or 2.75

A "strategy" for solving an equation consists of a sequence of strategic decisions. In the example in Table 1, the strategy for solving $4x-5=6$ consists of the two strategic decisions in the middle column. VanLehn (1996) reported that experts appear to have the ability to plan a strategy mentally, whereas novices tend to consider just one step at a time. Therefore, students learning to solve equations with expertise need to learn to mentally plan a sequence of strategic decisions to form a strategy.

Students learning to plan a strategy may receive assistance from tutors. In 1976, Wood, Bruner and Ross first used the term "scaffolding" (Azevedo & Jacobson, 2008) to refer to the type of assistance that allows a learner to solve a problem that they can not solve on their own. The term scaffolding soon became associated with Vygotsky's (1978) Zone of Proximal Development (Wood & Wood, 1996) because in this zone, guidance allows learners to solve problems which they cannot solve on their own. A feature of scaffolding is that it can provide structure to a problem and help students keep their overall goal in mind while they are focussing on an individual step (Rogoff, 1986, 1990 in Wood & Wood, 1996). Scaffolding can be used to modify a task so that students can focus on one component, for example, by replacing a difficult task with an easier task (Rittle-Johnson & Koedinger, 2005) or by performing difficult parts of a task (Quintana et al, 2004).

Although the term scaffolding originally referred to the assistance provided by a human tutor, scaffolding can also be delivered with technology. Technology can be used to modify a task and is therefore suited to providing this type of scaffolding (Quintana et al, 2004). For example, calculators and computer algebra systems (CAS) can be used to perform routine calculations and algebraic manipulations, thus allowing students to focus on setting up mathematical models for solving problems (Leinbach, Pountney & Etchells, 2002). Another example of technology providing scaffolding by modifying the task is MathXpert (Beeson, 1998). Students can ask MathXpert to perform the next step in solving an equation, request a hint for the next step, or even instruct the software to perform the remaining steps. This scaffolding modifies the activity of solving an equation so that students can perform the steps they know and view worked examples of other steps. These two examples illustrate how technology can provide scaffolding by modifying the learning activity in ways that are otherwise difficult to achieve unless each student has an individual human tutor.

As described by VanLehn (1989), it can be difficult to learn to solve equations because of the need to learn and appropriately apply two styles of mathematical thinking: planning strategies, and carrying out associated procedures. Students need to learn both of these but this paper focuses on helping students learn to plan strategies. There appears to be potential to provide scaffolding with a software design that modifies the task of solving an equation so that students can focus on strategies without being hindered by errors in procedures.

This principle was the basis of the design of software developed for this study. To investigate its impact on learning, trials were conducted with adult students. In an earlier paper (Robson, Abell & Boustead, 2007), one type of equation which included fractional terms was analysed. Results showed that most students who were unable to solve this type of equation in the pre-test were able to solve a similar equation with the software. Half of these students were also able to transfer this learning to solving similar equations with pen and paper in the post-test. In this paper, the impact of the scaffolding on students learning is investigated.

Software Design

The prototype software, Equations2go (Robson, 2004), provides scaffolding that carries out procedures so that students can focus on their strategy and its strategic decisions. This scaffolding also prevents students being hindered by errors in procedures. In this paper, this type of scaffolding will be described as allowing students to focus on "strategies only".

The following example illustrates the scaffolding in the software that allows students to do strategies only. Students make a strategic decision at each step by clicking the mouse on "hot spots" on the equation and choosing options from "visual menus". The interface includes a stepping stones metaphor in which each step occurs on a stone and a successful step causes the next stone to appear. Students can request feedback, and a visual record of strategic decisions is provided by "trails" between stones. In Figure 1, a partially solved equation is shown.

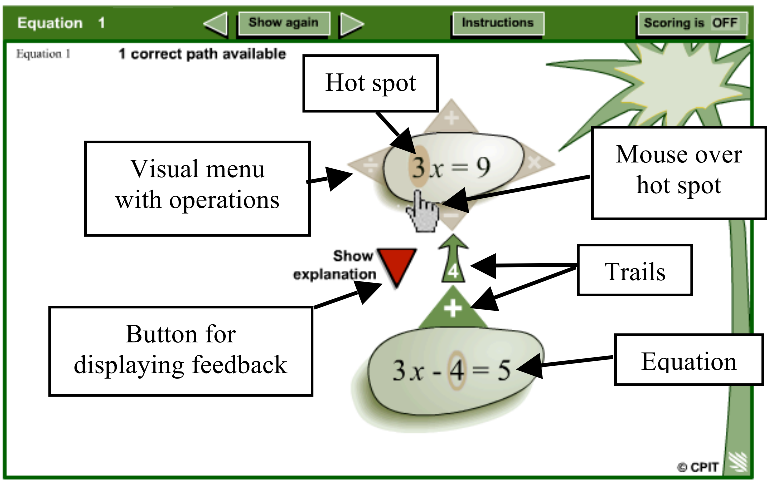

Figure 1. Partially solved equation in Equations2go.

In the first step of Figure 1, a student chose to add four to both sides of the equation. The software carried out this calculation and displayed the result on a new stone. The mouse is shown hovering over a hot spot on this new stone causing a menu to be displayed with operations for the second step. The prototype version of Equations2go developed for this study contains the following four equations.

$$3x - 4 = 5 \qquad 3F - 2 = 5F + 1 \qquad 3\,(4a - 5) = 6 \qquad \frac{d}{3} + \frac{d}{5} = 1$$

The software trials are described in the following section, along with data analysis, the sample and limitations of the study.

Trials

The trials were conducted during class sessions and with ethics approval. A pre- and post-test format was used with the intervention being use of the software. This format was chosen so that changes in students' equation solving strategies could be attributed to the scaffolding in the software. During the trials, students carried out the following six tasks:

1. Answered a short pre-questionnaire.
2. Answered a pre-test by solving 7 equations, four of which were similar to the equations in the software.
3. Used the software for 20 minutes and their actions were logged by the software.
4. Answered a short post-questionnaire which included a question with a Likert Scale about their attitude to doing strategies only.
5. Answered a post-test which had similar equations to the pre-test.
6. Discussed several follow-up questions in pairs or small groups and recorded their answers. One of the questions asked about doing strategies only.

The equation solving strategies that students used before, during and after using the software could be seen in the pre- and post-tests and in the logged data. The attitudes of students to doing strategies only could be seen from the post-questionnaires and the records of the discussion groups. Although 75 students took part in the trials, the data for 13 students were discarded as invalid because of being incomplete.

Limitations to this study were caused by other types of scaffolding in the software provided by feedback, the interface design, and the limited time that students used the software because the prototype software had only four equations. There were 29 discussion groups: most groups had two students but some groups had three or four students. The discussion groups included some of the students whose data were invalid, but the comments of these students were included because it was not possible to identify and exclude their contribution to the discussion groups.

Results and Discussion

The results are presented in two parts: first, those that relate to the impact Equations2go had on learning and second, those that describe student attitudes to doing strategies-only. The data for one student is examined in detail.

Impact on Learning

All students were able to make correct strategic decisions with Equations2go, 98% of students solved one or more equations in the software and 84% solved all four equations in the software.

Most students (95%) who could not solve equations in the pre-test were able to solve similar equations with Equations2go. The logged data were not available for five students and 20 students were already able to solve the equations in the pre-test that were similar to those in Equations2go. Of the remaining 37 students, 35 solved one or more types of equations with the software that they were unable to solve in the pre-test. Of the other two students, one appeared to run out of time to do the last question in the software and it was only this type that she got wrong in the pre-test and the other was unable to complete any equations during the 20 minutes of software trial but was able to make more correct strategic decisions with Equations2go than with pen and paper.

When the numbers of correct strategic decisions made by students in the pre- and post-tests were compared, it was found that more than half of the students improved after using Equations2go. Of the 62 students, 36 made more correct strategic decisions in the post-test than in the pre-test, seventeen did not show any improvement and the remaining 9 could already solve the equations in the pre-test. Furthermore, 30 students solved an equation in the post-test after not being able to solve a similar equation in the pre-test, and the number of students who correctly made all strategic decisions for all equations increased from 9 in the pre-test to 14 in the post-test.

The feature which made it easier for students to solve equations with Equations2go than with pen and paper was the scaffolding which allowed students to do the strategies only. This appears to have helped students solve equations with the software and many students have transferred this learning to pen and paper by making more correct strategic decisions in the post-test.

Student Attitudes

Students' attitudes to doing the strategies only are summarised in this section, and the attitudes and equation solving strategies of one student are examined.

The majority of students reported in the post-questionnaire that doing strategies only was helpful to their learning: 76% rated it very helpful, helpful or fine while the remaining students rated it unhelpful or sometimes unhelpful.

Students' reactions to doing the strategies only were reported by 19 of the 29 discussion groups. The other 10 discussion groups did not comment specifically on this. The comments are summarised in Table 2.

Table 2.
Discussion group comments about strategies being separated.

Type of comment	Number of discussion groups	Example of comment
Positive	11	*"Great for learning order of ops etc, without getting bogged down by numbers (ie doing the calculations). Would love a copy."*
Qualified positive	4	*"Good for people just learning rules. Further on we may need a bit of practice of actual calculation."*
Negative	4	*"It made it too easy."*

Some students felt Equations2go made it too easy and four discussion groups noted this. One student explained to the researcher that she made mistakes in procedures rather than strategies, and so she didn't like the algebraic procedures being done for her. Another student wrote: *"The program was very helpful but I don't know if I will be able to do them on paper because the program made it easy to do them on the computer"*. However, she completed the post-test and then went back to the post-questionnaire and added: *"Now that I've just done the post-test I think the program helped a lot"*.

Data from the pre- and post-tests for this student show that her marks, which were allocated to strategic decisions, improved from 4 out of 18 in the pre-test to 9 out of 18 in the post-test. This was a result of her improved strategies in Questions 3 and 4 which she solved correctly in the post-test. Her solutions to Question 3 and 4 in the pre- and post-tests are shown in Table 3.

Table 3.
Student's solutions to Questions 3 and 4 in pre-test and post-test

	Pre-test	Post-test
Question 3	$\frac{4x}{4}-7=3\div 4$ $\frac{x-7}{-7}=3\div 4=\frac{0.75}{-7}$ $x=\frac{0.75}{-7}$	$\frac{2D-9}{+9}=3+9$ $\frac{2D}{2}=\frac{12}{2}$ $D=6$
Question 4	$3-\frac{2a}{a}=3a+4a-4\ \div a$ $3-2=3a+4a-4\div a$ $3-2=\frac{7a}{a}-\frac{4}{-4}$ $\frac{3-2-4}{6}=\frac{7a}{a}\ \frac{6a}{6}$ $a=\frac{-3}{6}$	$\frac{4-3y}{+3y}=2+7y-2y\ +3y$ $2-4=2+8y$ -2 $\frac{2}{8}=\frac{8y}{8}$ $y=\frac{1}{4}$

In the pre-test, this student attempted to use an inefficient strategy for both these equations by isolating the unknown before isolating the unknown term. This inefficient strategy required procedures involving fractions and she carried these out incorrectly. In the post-test, however, she chose an efficient strategy with simple procedures which she carried out correctly.

In the Equations2go equation that is similar to Question 3, this student's first attempt was similar to her pre-test attempt but it was not accepted. She went on to solve the equation successfully three times in Equations2go and it was this new strategy that she used successfully in the post-test.

In the Equations2go equation that is similar to Question 4, the student attempted to use her pre-test strategy three times, solved it correctly five times, and successfully used her new strategy in the post-test. For both types of equation, she successfully applied strategies she learned with Equations2go to the post-test.

Although this student felt that Equations2go made it too easy, she was pleasantly surprised to find that she was able to successfully transfer her learning from Equations2go to the post-test. Equations2go appears to have helped this student learn to solve equations, and this is supported by evidence from the logged data, pre- and post-test performance, and post-questionnaire comments.

Conclusion

The software simplified the task of equation solving as 95% of students who could not solve equations in the pre-test were able to solve similar equations with Equations2go. By allowing students to do strategies only, the software appeared to provide scaffolding in Vygotsky (1978)'s Zone of Proximal Development, as it helped students solve equations which they could not solve on their own. Furthermore, when the scaffolding was removed, many students were able to transfer their learning to pen and paper. More than half of the students who had incorrect or missing strategic decisions in the pre-test improved in the post-test.

The majority of students reported that doing the strategies only helped them learn. Two discussion groups recognised that the scaffolding was suited to an early stage of learning and a few students felt that doing the strategies only made it too easy. However, analysis of the data for one student showed how she learned more than she realised from an activity she found easy. Although this information is only for one student, there is potential to further investigate the learning of students when they think scaffolding makes equation solving too easy.

The principle behind the design of the scaffolding was based on VanLehn's (1989) assertion that equation solving is difficult to learn because several types of thinking are required. It also depended on the suitability of computers for providing scaffolding by modifying the task (Quintana et al, 2004) in a way that is difficult to achieve in a standard classroom environment. The results of this study may be relevant to other fields where analysis of the learning theories would suggest that students need to concentrate on a particular part of the task. As in this study, it may be possible to help students do this by designing scaffolding in which a computer carries out other parts of the task.

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