Reconceptualising Problem Solving in the School Curriculum

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In this paper, we discuss the development of a very specific problem solving curriculum in an independent school in Singapore as part of the first phase of our research project. We are using a design research methodology to fine-tune the problem solving curriculum in which we are introducing the mathematics practical, an idea borrowed from science education.

While we acknowledge the importance of problem solving in the Singapore school mathematics curriculum, we are also concerned about the routine aspects in which problem solving has been implemented in the curriculum. There are several other reasons why we wish to propose a new approach to problem solving in the school mathematics curriculum. First, problem solving research has been ambivalent in its findings (see Schoenfeld, 1992; Lester, 1994; Schoenfeld, 2007; English, Lesh & Fennewald, 2008). There is still much to be done. To attain the goal of problem solving requires a close connection to research. Stacey (2005) has suggested that:

> To get closer to the goal requires research directed to understanding the problem solving process for mathematics (in all its aspects), developing effective classroom processes, and designing excellent tasks. Moreover, the research needs to be closely intertwined with curriculum development and teacher development projects so that it can make an impact on practice (p. 341).

Second, one of the major issues has been in the fourth stage of the Polya model, namely "looking back". Silver, Ghousseini, Gosen, Charalambous, and Strawhun (2005) have strongly pointed out that: "…instructional interventions intended to develop in students an inclination to "look back" at their solution to a problem in order to generate alternate solutions have been largely unsuccessful" (p. 288). Students do not generalise and extend a problem and do not think that it is important to do so when solving problems.

Third, the assessment of problem solving in the classroom has focused on assessing the products rather than the processes of the problem solving process. Consequently, the focus has been on finding the correct answer. A radical change in the assessment procedure is needed. Fourth, curriculum documents state that problem solving heuristics are to be taught in our primary and secondary school mathematics classes. Resources used by teachers tended to emphasise the learning of heuristics but did not focus on mathematics content and processes at a deep level or the kind of mathematical thinking used by mathematicians. For these reasons, our team has devised a problem solving curriculum with a kind of *mathematics practical*, that focuses not only on the products of the problem solving process but also on the important processes.

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1). Palmerston North, NZ: MERGA.

The Mathematics Practical

Our interest in using the mathematics practical for developing the problem solving curriculum emerges from science. It is a well-known fact that practical work is an important aspect of studying science. Science as inquiry is basic to science education and a controlling principle in the ultimate organisation and selection of students' activities (National Council of Research [NRC], 1996). In doing practical work students are hands-on and involved in activities such as asking questions, planning and conducting simple investigations, using simple instruments to measure and gather data, and using data to make inferences. Millar (1991) mentioned two rationales for practical work: (1) to facilitate the learning and understanding of science concepts, and (2) to develop competence in the skills and procedures of scientific inquiry.

The science practical lesson is very much accepted by students as part of science education and many have an understanding that it is to teach them how to 'do' science. Practical work to achieve the learning of the scientific processes has a long history of at least a hundred years. Despite much debate of how exactly it is to be carried out (Woolnough & Allsop, 1985), practical work is accepted as a mainstay in science education. In an attempt to direct the students to follow the Polya model, especially when they are struggling with a problem, we decided to construct a worksheet like that used in science practical lessons and told the students to treat the problem solving class as a mathematics 'practical' lesson. In this way, we hoped to achieve a paradigm shift in the way students looked at these 'difficult, unrelated' problems which had to be done in this 'special' class.

The Approach

We are using "design experiments" (Brown, 1992; Collins, 1999; Wood & Berry, 2003) as the methodological backbone of our project entitled Mathematical Problem Solving for Everyone (M-ProSE). Design experiments arose from the attempts of the education research community to address the demands of research in real-life school settings in all its complexity. It argues for the application of multiple techniques to study a complex phenomenon such as mathematical problem solving. This approach permits the use of several methods such as participant observation, interview, video-taping, and paper-and-pencil testing to provide corroborative evidence for findings. The envisaged outcome of M-ProSE is to produce a workable "design" (an initiative, artefact or intervention, for instance) that can be adapted to other settings. In Gorard's (2004) words, "The emphasis [in design experiments], therefore, is on a general solution that can be 'transported' to any working environment where others might determine the final product *within their particular context"* (emphasis added).

Although, voices have been raised against a traditional approach to problem solving based on the works of Polya and Schoenfeld (see English, Lesh, & Fennewald, 2008), in our opinion, the teaching of traditional problem solving has been successful under certain circumstances such as in Schoenfeld's undergraduate classes (Schoenfeld, 1985). The processes are sound because these are the same processes *professional mathematicians* use. The methods of teaching are generally not complicated. What we see as the root of the lack of success is that problem solving is not assessed. Because it is not assessed, students are not motivated to learn it and as such, students are more interested to learn the other components of the curriculum which would be assessed. We think that the way out of this perennial quandary is by making a paradigm shift. In a pilot project at an independent school (see Tay, Quek, Toh, Dong, & Ho, 2007), we decided to construct a worksheet like that used in science practical lessons and told the students to treat the problem solving class as a mathematics 'practical' lesson. In this way, we hoped to achieve a paradigm shift in the way students looked at these 'difficult, unrelated' problems which had to be done in this 'special' class.

Procedure

Regarding the *Curriculum Design and Development,* the M-ProSE project team has commenced work on the design and development of a curriculum for problem solving in school mathematics (see Figure 1). We have already elaborated on some reasons for proposing a new problem solving curriculum; also, we understand that what we choose to teach, how we teach it and how we assess what we taught are important considerations in devising this new curriculum.

MA2110: Problem Solving in Mathematics
Modular Credits: 1

Overview

The aim of this module is to develop the students' ability to solve problems in mathematics and think mathematically. Students will learn a model of mathematics problem solving and the strategies, resources and dispositions that are crucial for successful problem solving. The main mode of learning is a series of 'mathematics practical' lessons. In these lessons, students will work on specially crafted mathematics problems on a special 'practical' worksheet that guides them systematically and metacognitively through the problem solving process.

Figure 1. Problem solving course description

We have completed the first phase whereby we designed a 10 week lesson sequence in one Year 9 class in an independent school where students have the flexibility of taking some elective courses besides their core course in mathematics. The students enrolled for this course took it as an elective. This class was taught by one of our team members. Initial work included the selection of problems for the course, designing the practical worksheets, designing an assessment rubric and teacher capacity building (the practical worksheet and assessment rubric will be discussed in the other symposium presentations). It is to be noted that in our project, the problems used are chosen judiciously to elicit the problem solving processes that we value and not necessarily tied to some specific content being taught in the students' regular classes. Some problems that we have used include:

1. Find the sum of all the digits of the numbers in the sequence 1, 2, 3, …, 9999.
2. You are given two jugs, one holds 5 gallons of water when full and the other holds 3 gallons of water when full. There are no markings on either jug and the cross-section of each jug is not uniform. Show how to measure out exactly 4 gallons of water from a fountain.
3. Two bullets are placed in two consecutive chambers of a 6-chamber pistol. The cylinder is then spun. Two persons play Russian Roulette. The first points the gun at his own head and pulls the trigger. The shot is blank. Suppose you are the second person and it is now your turn to shoot. Should you pull the trigger or spin the cylinder another time before pulling the trigger?

The M-ProSE team recognises teachers as key to any successful curricular or instructional reform. In particular, in the area of non-routine problem solving, past research (see, for example, Schoenfeld, 1992; Lester, 1994) has highlighted the need for teachers themselves to be versatile and have experience in using non-routine problem solving strategies. M-ProSE does more than just equipping teachers with the problem solving strategies and heuristics. We have introduced to the teachers, Polya's (1954) problem solving model and Schoenfeld's (1985) problem solving framework and how to work with the practical worksheet. Finally, M-ProSE will continue to work with teachers to refine the curriculum and teaching methods via a series of lesson feedback sessions, as the project unfolds. We used lesson study style professional development for the teachers to improve their own professional development with our team's help. We have collected some data, but additional data will be collected as the project progresses.

In the other papers in this symposium, we elaborate on the use of the practical worksheet by students using examples from our research. We also discuss the use of an assessment rubric that assesses more comprehensively the problem solving processes such as 'looking back' along with generalisation and extension. Finally, we look into teacher preparation for implementing this very specific problem solving curriculum.

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