

Teacher Preparation for a Problem Solving Curriculum

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The role of the teacher is central to the success of any curriculum innovation. Thus, teachers' professional development has become an increasingly important subject of discussion in recent education literature. In the design and implementation of the project reported here, teachers' preparation for the problem-solving curriculum featured prominently. This paper discusses the challenges of selecting a suitable problem and ways of using it productively within a professional development programme that the authors carried out for the teachers involved in the project.

A survey of the curricula of a number of jurisdictions such as Australia, UK, and USA (Stacey, 2005) indicates that problem-solving appears either as a key strand in curriculum documents or as a parallel process alongside content strands. This reflects the importance that the international mathematics community places on problem solving as a part of the overall development of students' mathematical competency.

One well-known factor in the success of implementing problem solving in the classroom is the teacher (Ho & Hedberg, 2005). Teacher preparation is thus an important component in the whole enterprise of elevating problem solving to higher priority in the classrooms. In this paper, we examine the challenges involved in the carrying out of a programme that prepares teachers to do problem solving with their students in the classroom. In particular, we focus on the process of selecting suitable mathematics problems and discovering how the problems can be utilised in the classroom.

Research Setting

The study reported here is part of a project known as M-ProSE - Mathematical Problem Solving for Everyone. The fundamental goal of the project is to render the curriculum ideal of placing problem solving as central in mathematics teaching workable in actual classrooms. The school we worked with is an independent Secondary school in Singapore which is granted relative freedom by the Ministry of Education to chart the curriculum that suits the niche specialisation of the school. In this case, the school is known for its strength in mathematics and the sciences. These twin features of curricular freedom and slant towards mathematics fit well into the team's overall research design.

One of the main components of the project with the school is teacher preparation. We have just completed two stages in the teacher preparation programme: (1) Sessions with teachers that were focused on teaching problem solving processes to teachers, followed by (2) sessions with students that were focused on demonstrating to teachers ways to teach students problem solving processes. The later stages will involve working alongside the teachers as they teach problem solving to their students. As these subsequent stages will only take place in the future, this paper focuses only on the first two stages.

In the first stage, the mathematics teachers in the school attended five professional development sessions, each lasting ninety minutes. The primary goal of those sessions which were conducted by one of the authors—hereafter known as the trainer—was to help teachers, using problems as examples, to develop problem solving habits within Polya’s (1954) stages—Understanding the problem; Devise a plan; Carry out the plan; Check and Extend—while being aware of the influence of Schoenfeld’s (1985) components of problem solving, which includes Resources, Heuristics, Control, and Beliefs.

In the second stage, we used the Lesson Study method to re-direct the focus of teachers from the experience of problem solving itself to the work of teaching problem solving to students in an actual classroom. The same trainer taught an elective Year 9 student module over ten lessons, each lasting one hour. Twenty one students attended the module. The essential contents of the student module—the problems solved and the processes highlighted—were similar to the teacher module, but the pace, tone, and issues raised for discussion were adjusted to suit the needs of the students. After Lessons 1, 3, 5, 6, 8, and 10, we held post-lesson meetings with the teachers to discuss about the lessons. We focused, in particular, on the suitability of the problems introduced, the responses of students to those problems, and the adjustments needed for Cycle 2—when the teachers carry out the module in their resident classes later. These meetings helped to gather ideas for improvement as well as provided opportunities to clarify the instructional practices that were demonstrated.

Data and Analysis

Throughout the two stages of teacher preparation, we were as much focused on gathering feedback from the teachers as we were on developing their knowledge about problem solving. We saw our initial plans for the problem solving module as a provisional scheme of work; we revised the contents of the module on an ongoing basis according to the responses of the teachers. During stage one, the team members observed the teachers-at-work during the sessions and spoke to teachers regularly both at individual level and at whole-group level on their thoughts about the module as well as the feasibility of its implementation at the next stage. The team members took fieldnotes of each of the training session. Immediately after each session, the team members met to exchange notes regarding our observations about and conversations with teachers. Changes proposed during these meetings were often implemented for the next session. During stage two, teachers’ views were mainly gathered through the Lesson Study meetings. The team members continued to hold post-session meetings to tweak aspects of the module in response to points raised in the Lesson Study meetings.

The analysis for this report is based on the fieldnotes and the Lesson Study meeting accounts. The order of analysis follows the trace of how problems developed chronologically through the teacher preparation stages. The focus of analysis is on how each problem used in the sessions were crafted by the team, worked at by the teachers, and thought about by the teachers in terms of its feasibility for use in their classrooms later. For this paper, we have space only to report the analysis of one particular problem—known by us as the “sum of digits problem”—and its use that gave rise to rich discussions among teachers and the team members.

Sum of Digits Problem

The problem is stated as such:

Find the sum of all the digits of the numbers in the sequence $1, 2, 3, \dots, 10n - 1$.

This problem is chosen because it lends itself appropriately to our agenda of highlighting the importance of Polya's stages in problem solving. The suitability of this problem in relation to Polya's stages is elaborated below.

Understand the problem: Students are expected to be familiar with a similar-looking problem of finding the sum of the numbers in a sequence. We anticipate that many students (and teachers) would apply the results of this more familiar problem straightaway to the problem given. This provides a good opportunity to highlight the importance of "understanding the problem" in the work of problem solving. In this case, to successfully solve the problem, one needs to understand the difference between "sum of all digits of the numbers" and "sum of numbers" as well as its implications to the solution structure.

Devise/Carry out a plan: When students realize that the problem they are given is not what it seems but actually a novel problem, they are then led to the importance of consciously "devising a plan" instead of merely taking ad hoc shots at it by plucking one known formula after another without a coherent strategy. The problem provides a good opportunity to introduce to students a number of heuristics such as "solve a simpler problem", "observe a pattern", "use systematic listing", and "construct alternative representations" when devising and/or carrying out the plan.

Check and Extend: The problem also lends itself easily to discussion of other alternative solutions as a way to "look back". The problem can also be easily adapted to related problems such as "find the sum of all the digits in the odd numbers of the sequence".

Discussion

The trainer's way of conducting the sessions was to focus on the processes embedded in the Polya stages and influenced by the Schoenfeld components, using problems to illustrate the processes. As an example, in the first session, he focused on Polya's "understanding the problem", and supported that goal by choosing the problems that were likely to bring out the importance of "understanding the problem". Subsequently, in the following sessions, he focused on different stages of the Polya model, often revisiting familiar problems introduced earlier, such as the sum of digits problem. In other words, the trainer brought to the *foreground* the processes of problem solving, using the problems themselves as *background* support to illustrate the processes.

This approach was met with resistance by some teachers. First, it was observed that teachers behaved more like students during the sessions—they were far more interested in solving problems than learning about the processes. In other words, their preference was the reverse—problem solving at the *foreground* of their mental activity, understanding the processes served as a *background* support to aid in solving the problem. Second, the teachers raised concerns about revisiting the problems in different lessons - to them, it resulted in a "lack of closure" to a problem. They were uneasy that problems were not dealt with in class and only left as homework later. They preferred to complete the problem within the lesson it was introduced. Third, teachers expressed concern that the problems covered were too difficult for the students. One teacher said that he "felt pressured himself" and questioned if they were appropriate for students.

The project team met over several times to re-consider the module for students. We tried to take into account the teachers' concerns. We decided to switch the prominence of problems and processes—by having a “problem of the day” at the foreground and talking about processes as supports to aid in the solving of problems. By having only one problem of the day, we have more time to discuss the problem and thus address the “closure” problem. In addition, we made attempts to simplify the problems so that students will not be overwhelmed by them, as feared by the teachers.

During stage two, the sum of digits problem was presented to the students as “problem of the day” during Lesson 1. Instead of the more general problem, the problem was stated as such:

Find the sum of all the digits of the numbers in the sequence 1, 2, 3, ..., 9999.

The students' homework showed that most students were able to take productive steps to the solution. The most common solution was that of using pattern observation to obtain the answer $4 \times 45 \times 1000$.

We learnt from this project that the choice of suitable problems hinges on a number of considerations. We need to consider the teacher's response to the problem. For example, Paper 2 of this symposium series has already highlighted the criticality of the teacher's mathematical knowledge in successfully using the scoring rubric. For the sum of digits problem, some of the teachers found the problem challenging even to themselves, but given time, they were far more confident and were able to obtain the required solution. Nevertheless, we adjusted the level of difficulty of the problem and presented it to students as the sum of digits up to 9999 instead of $10^n - 1$. As it turned out, the students were largely able to offer productive attempts at the problem.

The nature of the problem is also critical in the selection of a problem. In the case of the sum of digits problem, the teachers found it suitable in two ways: there are different ways to solve the problem so the problem is rich in the sense that students were free to approach using alternative strategies; the problem allows even the weakest student to make some successful attempts (for example, in calculating the sum of digits from 1 to 99), thus boosting students' confidence at mathematical problem solving.

References

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