Using Tasks Involving Models, Tools and Representations: Insights from a Middle Years Mathematics Project

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As part of the Task Types for Mathematics Learning Project (TTML), teachers developed and used a range of tasks which focused on tools, models or representations. Data were collected on the ways in which middle years teachers described these tasks, their preferences among particular tasks, the opportunities they saw them as providing, and the constraints they observed during their use. There was general agreement that these tasks form an important part of a balanced mathematics curriculum.

A Rationale for Explicit Tasks

There have been a number of researchers who have attempted to classify mathematical tasks. As part of the Quantitative Understanding: Amplifying Student Achievement and Reasoning Project (QUASAR), Stein, Smith, Henningsen and Silver (2000) classified tasks into four categories, which can be summarised as: Memorization involving reproducing previously learned rules or facts; Procedures without connections, Procedures with connections; and Doing mathematics. The task type which which is the basis of this paper comes under the latter two categories.

The effective use of models, tools and representations is a key component of effective mathematics teaching (Clarke & Clarke, 2003). Appropriate models and representations, in the hands of capable teachers, support children's conceptual development and can build skills. What tasks do teachers use to introduce and implement these tools, models and representations? The Task Types for Mathematics Learning (TTML) project included such *explicit* tasks (referred to as *Type 1* by project participants). A key focus in this research is to examine the opportunities and constraints experienced by teachers when using the various task types. It would appear that there are powerful constraints operating which discourage their use, because it appears that in many Australian classes the tasks used are generally routine and unlikely to lead to successful learning (see Hollingsworth, Lokan, & McCrae, 2003). Such *explicit* tasks are associated with good traditional mathematics teaching (see Watson & Mason, 1998), but their use is not always evident in the regular classroom. The term explicit is used here to emphasise that the mathematics was made explicit in the use of the task, not to imply that the teacher was "telling" the student what to do, without any student decision making.

The Explicit Tasks that are the Focus of our Project

In describing these tasks, we are not referring to exercises but to explicitly focused experiences that engage children in developing and consolidating mathematical understanding. An example is a teacher who uses a fraction wall to provide a linear model of fractions, and poses tasks that require students to compare fractions, to determine equivalences, and to solve fractional equations. The fraction wall simplifies the mathematical complexity of the concept by providing a tangible and clear model of fractions that can be otherwise abstract.

In the initial TTML professional development, teachers were provided with the following definition of these types of tasks:

The teacher commences with an important mathematical idea, and proposes tasks which involve

models or representations or tools, which help students to understand the mathematics. There is no attempt to link mathematics to its practical applications. For example, the use of a fraction wall in a chance game can assist in developing an understanding of equivalence, improper fractions, and simple operations with fractions.

Following student work on the task, the teacher leads a discussion on the mathematics which has emerged from the task, and seeks to draw out commonalities and generalisations.

After developing, adapting and using explicit tasks, the teachers filled out a survey with a series of open prompts (*n* = 31) focusing on the specific task type. There were three groups of teachers with varying experience with other types of tasks in the context of the project when the survey was completed. However, there was little difference between these groups in the patterns of responses, except that they seemed to provide more extensive responses and included more comparison references when they had experienced the use of other tasks types. This is not surprising, as they were then in a position to note appropriate contrasts. The results from this survey are presented in the following sections.

The Teachers' Definitions of the Tasks

The first survey prompt was "*If you were explaining to a group of teachers about how to use tasks of this type, how would you describe this type of task?*"

The importance of the model and the explicit focus on the mathematics were the most common components of teachers' responses. The linking of the model or tool explicitly and directly to the mathematical concept was highlighted. Sample responses included:

Using Models/tools representations to explicitly focus on a particular mathematical idea or concept. Often takes the form of teachers introducing a mathematics idea and students play a game or complete an activity. Follow up discussion on understanding/learning with students.

The use of a model or representation to assist student understanding of a particular mathematical concept to be used as a reference for further student work on the concept.

These teachers had been provided with some exemplars of tasks as well as worked in school- and cluster-based teams to develop and trial tasks. There responses were consistent with the intentions of the researchers.

Some Examples of the Tasks that Teachers Valued

In the survey, teachers were asked "*of the tasks of this type that you have tried in your class this year, which worked best?*" They were then asked to list two more of the best tasks.

Not only did no particular task emerge as the most popular, but the most striking feature of the responses was the diversity of tasks that were valued, with 17 different tasks identified as best across the 31 teachers. The Chocolate Fraction task (Clarke, 2006) where the sharing of chocolate represents both an engaging context and a model for the development of the concept of fractions as division was identified by a number of teachers in their best three.

The reasons that the teachers gave for selecting the Chocolate Fraction task included:

Gave students something they could see. They were interested in the chocolate so it remains in most of their minds.

…was so effective in engaging the children and representing fractions as division.

The teachers were teaching in the middle years (Years 5 – 8, with student ages from around 10 to 14), with the vast majority in upper primary. An important curriculum focus

in these years is fractions and decimals, and the majority of the best explicit tasks focused on children's development in these areas. This would seem to be due in part to curriculum importance but also to the nature of the content and the availability of effective models and tools. Of the 81 nominations by the teachers as their top 3, 51 were focused directly on learning in Number. This was despite the fact that in later professional development an attempt was made to present and encourage teachers to try explicit tasks in other content areas. Teachers tended to trial and identify tasks that they had experienced during project meetings or developed as a team within the school.

There was limited justification for the teachers' preferences but the key themes appeared to be the engagement of the children followed by the importance of the model/tool/ representation in enabling mathematics learning.

The following quote was from a teacher responding to why a specific task was successful:

> Concept that hasn't been introduced was made explicit through the use of this model. Students could see clearly what maths happens when you divide/multiply by a number larger/smaller than one.

The Advantages of Explicit Tasks as Seen by the Teachers

To gain insights into the opportunities and advantages of the specific task type, the prompt was "*What do you see as the advantages of using this task type in your teaching?*"

The most common feature in the teachers' responses was the value of these tasks for developing student understanding. There were also many who commented on the engagement of students both in the sense of participation but also in the way the model (sometimes referred to as "visual" or "hands-on") allows engagement with the mathematics.

Increasingly during the phase of the project where the different task types were trialled, the discussion of the teachers involved the role of different types of tasks and the value of a range in their planning including for the range of students. The following quotes about the advantages of explicit tasks were from teachers who had trialled open-ended (see Sullivan in this volume), context-based (see Clarke and Roche in this volume) and explicit tasks:

> Yes as a starter to teach new maths that then can progress to Task type 3 [open-ended].

> In particular areas of maths eg- using operations-using this task allows students to learn the maths skills required before moving into applying it in a variety of contexts.

> It can support the concrete understanding of a maths concept for students for whom more abstract mathematical understanding may not develop as readily.

The Constraints on the Use of Explicit Tasks as Seen by the Teachers

The following prompt was included to provide insights into the constraints that teachers identified: "*What makes teaching using this task type difficult? What are the challenges in using this type of task?*"

The difficulties that appear to be related directly to the tasks types include the difficulty in identification of explicit tasks within particular content area (e.g., chance and data) and the time required to prepare the materials.

The following are representative of the range of responses and issues:

> Sometimes finding the task. For me sometimes deciding which task is actually a type 1 task. Finding the resource and preparing it for use with a grade can be difficult ie time needed to copy,

laminate, cut, etc.

Ensuring each student has sufficient background knowledge and skills. At times I found it difficult to make the task relevant to the maths program.

Understanding the model/representation is most effective when there is a purpose, ie our opportunity to apply their understanding of the model in a meaningful context. Also, particular maths concepts are easier to find models for.

Sometimes modifying for lower students.

Conclusion

In the trialling phase of this project, there was a number of issues that arose in the teaching of explicit tasks. The teacher quotes and summary comments above provide some insights into some those issues. They can be summarised as follows:

- While these tasks are not contextualised, there is sometimes a "hook" that helps to engage the students. The chocolate fraction task is an example of this.
- Some content areas, particularly Number, seem to provide more opportunity for successful explicit tasks.
- Extensive exposition is not necessarily required. The provision of the model or representation can enable the students to generate the mathematical ideas and justification.
- The model, representation or tool needs to be linked closely to the mathematical concept begin developed.
- The mathematical focus is pivotal and it seems that teachers might be less willing to deviate from the intent than with contextualized tasks.

The teachers in this project were able to articulate the purpose, opportunities and constraints of explicit tasks. Such tasks are an important component of curriculum, but teaching them well is not simple. However, as one of the teachers pointed out, *it is important to be reminded why* we need to include them.

References

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