Growth of Pre-service Teachers' Knowledge and Teaching Ideas About Decimals and Fractions: The Case of Vivi

Wanty Widjaja
Sanata Dharma University
<wanty_widjaja@yahoo.com>

Kaye Stacey
The University of Melbourne
<k.stacey@unimelb.edu.au>

This paper discussed and analysed the growth of one pre-service teachers' knowledge about decimals and fractions during a teaching experiment. Evidence of her progress is based on responses to written test and interview questions. This case shows with probing questions and appropriate teaching ideas, it is possible for a pre-service teacher with initially weak and fragmented knowledge about decimals and fractions to develop a meaningful knowledge about decimals and fractions. The stronger conceptual base provided by use of a concrete representation of decimals enabled Vivi to move away from reliance on memorised facts and rules and towards conceptually based explanations of ideas.

The fact that knowledge for teaching mathematics must go well beyond knowledge about mathematical content is now well established. The seminal notion of Pedagogical Content Knowledge (PCK) coined by Shulman (1986) has been accepted as the common currency in teacher education to refer to knowledge which blends content knowledge and pedagogical knowledge for teaching. According to Shulman, PCK comprises knowledge of appropriate representations for specific content areas and procedures, and knowledge about typical students' conceptions, preconceptions and misconceptions.

A number of studies have expanded Shulman's definition of PCK to make it more operational in various content areas. Having a wide repertoire of representations enables teachers to flexibly expand on mathematical notions and properties in order to make them more comprehensive for students. Ball and Bass (2003) relate this knowledge to the importance of the teacher's role in "unpacking" mathematical content knowledge to fit a learner's perspective and in identifying central ideas for the teaching mathematics.

This paper presents the case of Vivi, one pre-service teacher whose understanding about relations between decimals and fraction and corresponding pedagogical ideas evolved during a design-research teaching experiment. A key characteristic of her initial understanding is fragmentation of content knowledge, with its symptom of reliance on rules, and reliance on memorised facts. Vivi shows how she begins to use a concrete representation as a thinking tool. The data reported in this paper is a small part of the larger study which investigates growth of Indonesian pre-service teachers' content and pedagogical content knowledge about decimals.

Misconceptions about Decimals among Pre-service Teachers

Investigating pre-service teachers' conceptions and misconceptions of decimals entails assessment of PCK. A number of studies examining pre-service teachers' knowledge on decimals (Chick, Baker, Pham, & Cheng, 2006; Stacey et al., 2001; Tsao, 2005) revealed that pre-service teachers shared some of the misconceptions apparent in children but with different prevalence of misconceptions.

Stacey *et al*. (2001) reported evidence of misconceptions among pre-service teachers that indicated their overgeneralising of some aspects of fraction knowledge to decimals (labelled as S-thinking). People holding S-thinking believe that fractions and decimals are similar. Stacey *et al* (2001) found that pre-service teachers showed little awareness of this misconception in students, even when they themselves held this misconception.

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2). Palmerston North, NZ: MERGA.

In contrast, Tsao (2005) reported a different kind of misconceptions held by pre-service teachers wherein fractions and decimals were perceived as "different entities and they did not necessarily make any connection between them" (p. 661). Earlier study by Resnick *et al.* (1989) attributed curriculum sequence differences as a factor that affected different pattern of misconceptions. In U.S.A. and Israel, where the teaching of fractions precedes the teaching of decimals, misconceptions resulting from overgeneralising fraction knowledge to decimals (S-thinking) were more common than in the sample of students from France where decimal teaching preceded fraction teaching.

Tirosh (2000) investigated 30 Israeli pre-service teachers by interviewing their knowledge about mathematical algorithms, theorems, and operations on division of fractions. She found that pre-service teachers tended to perceive mathematical properties and operations as given, well established, and unquestionable. Along with many other studies, Tirosh's findings showed discrepancies between 'knowing that' and 'knowing why' among pre-service teachers. In similar line, Stacey *et al*. (2001) reported evidence symptom of reliance on rules (e.g., rounding and truncating rules), and reliance on memorised facts without understanding. This misconception was apparent in pre-service teachers' difficulties to distinguish decimals with repeating digits or decimals with the same initial digits (labelled as A2-thinking).

Method

This paper will report data from one pre-service teacher's responses (Vivi) to a test designed to uncover students' ways of interpreting decimals (DCT v3.1), and two interviews tasks. Each of these assessments was carried out before and after the short teaching experiment, which lasted 4 meetings. During this teaching experiment, pre-service teachers work in small group (4-6 people). The teaching aimed to engage pre-service teachers in constructing meaningful understanding of decimals and pedagogical ideas of various representations in this area. Incorporating concrete models such as linear arithmetic blocks (LAB), and emphasising place value of decimals were the main features of the teaching experiment. LAB is a linear model consists of long pipes that represent a unit and shorter pieces that represent tenths, hundredths, and thousandths in proportion. Relationships among LAB pieces and their verbal names (one, tenths, hundredths, and thousandths) was explored to strengthen decimal place value understanding.

Figure 1 presents samples of DCTv3.1, a reliable and easy to use diagnostic instrument to observe decimal misconception. The DCT (**d**ecimal **c**omparison **t**est) items included several items with repeating digits, because it is known (Stacey & Steinle, 1998) that these decimals are likely to reveal the misconceptions that are most prevalent among pre-service teachers.

Figure 2 presents the 'generic' interview questions (translated from the original Indonesian), which aimed to uncover pre-service teachers' basic ideas of decimals and teaching ideas for decimals. Prior to each interview, pre-service teacher completed a written test which aimed to explore their content and pedagogical content knowledge about decimals. The interviews were clinical in nature to elicit understanding (and misunderstanding) about decimals as well as to get insights on pre-service teachers' way of thinking behind their written test responses. A "think aloud" procedure (audio- or video-recorded) was employed during the interviews as pre-service teachers work on the problems in the presence of the researcher, who observed and asked further probing questions and to supply their explanations in writing (interview notes).

For each pair of decimals, circle the larger one or write = in between!

3.72	3.07	8.052	8.514	0.0004	0.4
17.35	17.353	4.4502	4.45	0	0.6
4.666	4.66	3.7	3.77777	0.7	0.00

Figure 1. Some items from DCT version 3.1 (pre-test version)

Vivi was one of 20 pre-service teachers participated in pre-course interview and was also involved in the post-course interview with 17 other pre-service teachers. Her case was selected because her responses during tests and interviews indicated a growth of understanding from a weak and fragmented starting point and provided insights into the complexity of thinking involved in understanding relations between fractions and decimals.

Interview questions

- Could you expand on what do you know about decimals?
- What models can be used to teach decimals to primary school children?

Figure 2. Interview question examining teaching ideas on decimals (translated)

Vivi's Understanding about Decimals and Fractions

Vivi's Initial Understanding about Decimals

Vivi is a pre-service primary teacher with weak content knowledge and indicated reliance on rounding rules as evident in her incorrect responses in comparing pairs of decimals, e.g., noting that 17.35 = 17.353, 4.4502 = 4.45, and 4.666 = 4.66. These comparison items were the only three items that Vivi marked incorrectly. When she was probed about her thinking during the pre-course interview, Vivi said that in general she solved the comparison of decimals problem "by looking at the digits behind the comma and compare which digit is larger". Note that in Indonesia, the decimal separator is a comma. This is an expert strategy, when carried out correctly, but if often fails on items with repeating digits when used without understanding.

In comparing decimals such as 17.35 and 17.353, Vivi referred to digit 3 thousandths of 17.353 "because 3 is smaller than 5 then we can ignore it becomes 17.35 = 17.35." However, in comparing 3.7 and 3.77777, she not only considered 2 decimal digits but also add an extra zero to 3.7. She explained "in comparing 3.7 and 3.77777, because 7 is larger than 5 then 3.77777 becomes 3.78 so we compare 3.78 and 3.70 and the larger one is 3.78". Clearly Vivi had knowledge of various rules that she employed in comparing pair of decimals but the significant fact is that in all this discussion, she made no reference to place value in relation to the rules that she employed.

The pre-course interview revealed Vivi held misconceptions about decimals as evident in her association of decimals with reciprocals. Vivi contends that "decimals are identical to fractions". Her strong association of decimals and fractions is based on her school experience as she pointed out "decimals are taught after fractions". Vivi's responses indicated that she held 'memorized knowledge' of familiar fractions and decimals relations such as ½, , $^{1}/_{100}$ but her false decimal=fraction links were evident when dealing with 'unfamiliar fractions' such as , , and $^{1}/_{125}$ (see Figure 3). As expressed in her explanation, Vivi focussed on the denominator of unit fractions and also, with some linking to place value, took into account the 'size' of the denominator (in the tens, in the hundreds etc) in determining the associated decimals. For example, in the interviews below she notes that 6 is between 1 and 10 so is "in the tenths", and later that 125 has 3 digits and so has an additional zero in the decimal representation.

Researcher : Can you give examples about the relations between decimals and fractions?
Vivi : For example ½ is 0.5.
Researcher : How do you know that?
Vivi : For ½, because… I am already familiar and know the answer.
Researcher : How about unfamiliar fraction, for example ?
Vivi : … because 6 *is between* 1 *and* 10 *so it is still in tenths so it is just* 0.6.
Researcher : How about ?
Vivi : It is the same for , but for hundred, we add another digit so it becomes two decimal digits.
Researcher : So for instance ?
Vivi : is … (writing) 0.15.
Researcher : How about ?
Vivi : is 0.1.

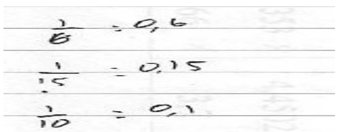

Figure 3. Vivi's pre-course interview scripts and notes

The following interview script showed a process of unrevealing Vivi's strategies and thinking in associating decimals and fractions. First, the researcher asked Vivi to find decimal representation of $^{1}/_{100}$ and Vivi could give a correct answer 0.01. Her 'alternative' strategy was evident in working with $^{1}/_{125}$ which rooted in her difficulty with basic notion of fractions and decimals. Her reference to 'size' of the denominator suggests her attempts to employ her incomplete knowledge of place value. Vivi also made remarks about long division algorithm being difficult for primary school children. We contend that this remark reflected Vivi's own struggle in remembering, executing and understanding the algorithm.

Researcher : What if $^{1}/_{125}$ *?*
Vivi : $^{1}/_{125}$ *… (long pause, writing)* 0.0125.
Researcher : How do you know that?
Vivi : Perhaps for $^{1}/_{125}$ *use a long division (she did not carry it out) but this is difficult for primary school children*
Researcher: So do you know a better way to help these children besides using long division? How do you solve all these?

Vivi : Basically I focus on the denominator, for example because 125 *consists of three digits then we need to add an extra 0. But this way is not always right.*
Researcher : Why is that?
Vivi : For instance if the fraction is $^{113}/_{125}$ *then this way is difficult so we use long division.*

In response to teaching ideas for decimals, Vivi offered fraction models first as attempts to explained relations between fractions and decimals (Figure 4). Her comments and choice of models for teaching fractions suggested that Vivi was aware of the use of such representations for teaching and learning at the primary level. Vivi's interview reveals no experience of learning decimals with representations was in her own schooling experience. This explained her lack of success in linking fraction models for learning decimals.

Vivi : ¾ is the same as 0.75.
Researcher : How do you know that?
Vivi : Because ¾ multiplied by 100 *is* 0.75. *If we don't use this way, because for primary students this is difficult, we can use cakes as example but the cake has to be round and we cut the cake into 4 parts with each represents ¼. Then ¾ is* 3 *of these cakes. Perhaps for primary school children this will be easier to understand.*
Researcher : Could you explain more how this example helps children to understand 0.75?
Vivi : It will be difficult using this cake model, perhaps we can use rope and ask children to divide the rope into 4 parts. Perhaps children will fold the rope into two and then cut it or depends on their preference – we get ½. Then to get ¼, children can fold it into 2 parts again.
Researcher : And how do you link this with decimals?
Vivi : Based on school experience, for decimals… it is difficult

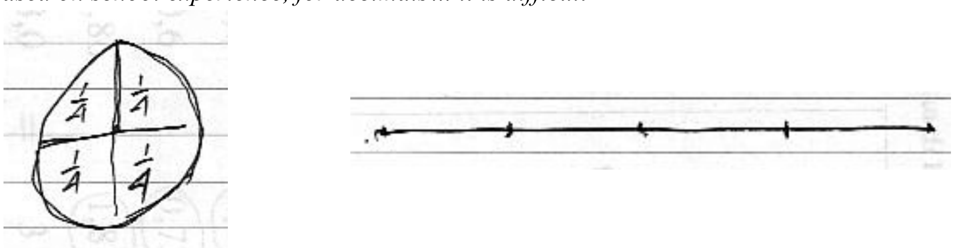

Figure 4. Vivi's representations for teaching fractions during pre-course interview

Vivi's Evolved Understanding about Decimals and Fractions in the Post-course Interview

Vivi showed improved knowledge in the post-test and post-course interview. When asked to explain her answers to the newly completed DCTv3.1 in the subsequent interview, she did not use rounding or truncating rules as in the pre-test. Moreover, the post-course interview documented Vivi's improved understanding about the decimal and fraction relations. She showed an 'aha' moment during the post-course interview. More importantly, Vivi was able to make a meaningful understanding of fractions and the division process by utilizing the concrete model LAB as a thinking tool.

Researcher : Could you explain how do you find the decimal for using the LAB pieces?
Vivi : First means 1 piece divided into 6 parts. We need to divide the whole piece into 10 pieces and distribute them among 6 people.
Researcher : Then?

Vivi : Because there were 10 parts and we use six parts so there were 4 parts remaining. We can divide each of them into 10 shorter parts.
Researcher : So first, shall we go back and work out the names of each piece again?
Vivi : First we start with one and then divide it into 6 parts, so each has 1 tenth and we have a remainder of 6 tenths… no that is wrong.
Researcher : Each part has how many of what?
Vivi : Each has one tenth and there are 4 remaining tenths and we divide them again into 10, so we have 40 hundredths and divide it again into 6 parts, etc… six six six.
Researcher : So how do you find the decimal notation for
Vivi : is the same as zero point six, six, six
Researcher : Will it stop?
Vivi : No it still continue
Researcher : What is repeating?
Vivi :The six so it is zero point one six six six six repeating

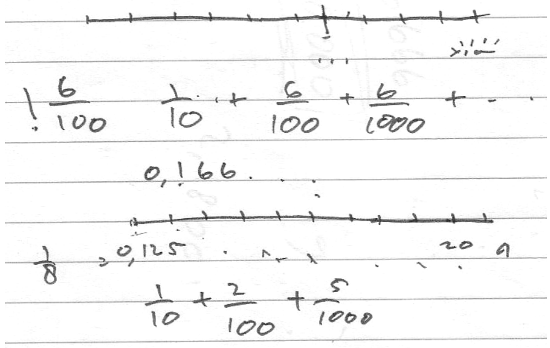

Figure 5. Vivi's post-course interview notes

Clearly the post-course interview script above documented the fact that initially Vivi still had tendency to confuse decimals and fractions. Probing questions that highlight the differences between fractions and decimals were helpful in assisting Vivi to reach her 'aha' moment. She worked this out by drawing the model of LAB (see Figure 5) and articulating decimal place value relations during process of division. The most important step for her was a realization that a starting point in finding the decimal notation of is noticing the relations between 1 and 10 tenths before carrying out division by 8. As articulated in her thinking aloud, Vivi kept made reference to relations between 2 tenths and 20 hundredths, followed by division of 20 hundredths into 8, getting 2 tenths a remainder of 4 hundredths and this led her to 0.125 with no difficulty.

Researcher : Could you now explain to me how do you find the decimal for ?
Vivi : Similar to previous process, I divide 1 pipe into 10 tenths… (writing) … we have used up these 8 pieces so there are 2 remaining, divide each of these into 10 to get 20… there are four remainder, multiply them by 10 and dividing by 8 we get 5.
Researcher : So what is the decimal notation then?
Vivi : 0.125.

However, finding other forms of representation for teaching decimals is not that simple. In the post-course interview Vivi opted to use of LAB model as her thinking tool and not the division algorithm because she contended that "it was easier to imagine". She recommended LAB and bamboo sticks, a similar linear model to LAB, as teaching tools for decimals. This suggested that Vivi has not extended her knowledge of representations to other forms beyond those offered in the teaching experiment.

Discussion and Conclusions

The interview data revealed Vivi's lack of experience with concrete models in learning decimals and weak knowledge of the links between fractions and decimals led to inappropriate extension of fraction models for teaching decimals at the beginning. In Vivi's case, inappropriate extension was related to and further confirmed her misconception of decimals and reciprocals.

Vivi's strong association of decimals to fractions reflects the sequence of Indonesian curriculum in teaching fractions before decimals which emphasize on computational skills in working with fractions. In general, one advantage of this approach is that pre-service teachers acquire knowledge on the relations between decimals and fractions and certain degree of fluency in converting between fractions and decimals. This is observed in general for many pre-service teachers in the larger study. However, Vivi represents a case of pre-service teachers with no fluency in converting between fractions and decimals (except for the ones considered as 'common' or 'familiar') and a lack of meaningful understanding about the hidden place-value related fractions that underlie decimal notation. A heavy emphasis on a computational approach combined with deficient knowledge of fractions does not assist pre-service teachers in making meaningful links between fractions and decimals.

Vivi is a showcase of pre-service teacher who attempted to juggle incomplete knowledge of fractions, decimals and rules to give 'an alternative' interpretation for decimals and fractions relations. Her reliance on rounding rules documented in the pre-test and pre-course interview suggested a way to simplify problems of working with decimals of longer decimal digits by utilizing rules without understanding. Comparing decimals with some initial decimal digits in common is a problematic task for adult students as reported by Steinle and Pierce (2006) and Stacey and Steinle (2006) and often reveals their misconceptions. In this case, Vivi is no exception. One of the main reasons for this difficulty is reliance on incomplete algorithms without understanding.

It should be noted here that Vivi's ability to extend the use of LAB as a thinking tool is not a common finding in this study. In general, the LAB model is utilized as a representational tool to model decimal numbers. One of the explanations for Vivi's success in using LAB as a thinking tool was partly due to the nature of individual clinical interview. During this process, with probing questions, Vivi was compelled to resolve her previous incomplete knowledge of decimals and fractions.

It was encouraging to learn that a pre-service teacher with weak content knowledge such as Vivi was able to progress to develop a meaningful interpretation of links between fractions and decimals. Vivi's case suggests that pre-service teachers with weak initial content knowledge gained advantage from their active participation in the teaching experiment. Vivi still had more to learn and her new knowledge did not seem well integrated, but she had made progress. The incorporation of concrete models for learning decimals was a new experience for almost all pre-service teachers involved in the teaching experiment. This experience has strengthened her understanding and expanded Vivi's knowledge of alternative ways of teaching decimals by incorporating concrete models.

References

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). Edmonton, AB: CMESG/GCEDM.

Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 297-304). Prague: PME.

Resnick, L. B., Nesher, P., Leonard, F., Magone, S., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20*(1), 8-27.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education, 4*(3), 205-225.

Stacey, K., & Steinle, V. (1998). Refining the classification of students' interpretation of decimal notation. *Hiroshima Journal of Mathematics Education, 6*, 49-69.

Stacey, K., & Steinle, V. (2006). Contrasting Decimal Conceptions of Adult and School Students. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 424). Prague: PME.

Steinle, V. (2004). *Changes with age in students' misconceptions of decimal numbers.* Unpublished PhD thesis, University of Melbourne, Melbourne.

Steinle, V., & Pierce, R. (2006). Incomplete or incorrect understanding of decimals: An important deficit for student nurses. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 161-168). Prague: PME.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5-25.

Tsao, Y.-L. (2005). The number sense of pre-service elementary school teachers *College Student Journal 39*(4), 647-679.