The 'Number Proficiency Index': Establishing the Starting Point for Mathematical Instruction in High School

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This paper summarises part of a longitudinal study to investigate the possibility of establishing an Index that would indicate the appropriate starting point for instruction for students entering high school and reports on the first group of students to sit the Index tests in 2005, comparing the predictive nature of the tests with other state and national tests through correlations. The Index tests focus on the number construct and it is envisaged that by identifying the student's deficiencies or strengths in this construct it may be possible to devise interventionist strategies or extension activities that would increase the probability of success in later years.

Premature teaching of mathematical concepts prior to educational and conceptual readiness can have a significant negative impact on subsequent conceptual understanding and this alone should be sufficient reason to ensure that teachers are aware of the preparedness of their students (Clarke, 2005; Stein and Lane, 1996). The main purpose of this paper is to examine the predictive reliability of a new bank of tests designed to measure the level of understanding of the number strand. Numerous studies have shown that there is often a decline in achievement after transition (Carvel, 2000; Collins & Harrison, 1998; Galton, Gray, & Ruddrick, 1999) so it is imperative that this decline does not continue to impact on performance if the situation can be remedied by interventionist strategies and pedagogical adjustments (Doig, McCrae, & Rowe, 2003). Transition does not only affect students who struggle but can also impede the progress of more able students who find that the first year of high school presents few new challenges, with an increased volume of work but little increase in difficulty over the work completed in primary school, often leading to disillusionment and lack of control (Green, 1997; Kirkpatrick, 1992). As many mathematics classes are not *streamed* in the first year of high school lessons are often targeted to the middle ability students, which can have a detrimental effect on both the lower ability student and those more able.

In 2005, tests were administered to 172 new students in Year 8 (the first year of High School in Western Australia) in four different schools. Each student received a final score, which was referred to as their Number Proficiency Index (NPI). Data for these same students were gathered from the Western Australian Literacy and Numeracy Assessment (WALNA) for Year 7 and the Monitoring Standards in Education (MSE9) test at Year 9. From 2008, the National Assessment Program Literacy and Numeracy (NAPLAN) replaced the previous full-cohort literacy and numeracy assessment programs of all Australian States and Territories for students in Years 3, 5, 7 and 9. Further data were collected from the students exit point from high school by weighting their final scores according to the difficulty of the course undertaken. Multiple correlations were calculated to investigate the predictive nature of the WALNA test, MSE9 test and the NPI. The results reported in this paper are for the first group of students to have passed through the five years of high school from 2005 to 2009. The tests have also been given to students entering high school in each of the years from 2006 to the present.

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Whilst acknowledging that the measure of mathematical ability is not solely determined by the number construct, it is the intent of this report to show that it the most reliable predictor of success in later years. If it can be shown that deficiencies in the understanding of number contribute to unsatisfactory mathematical performance in later years, then it may be possible to alert teachers to these deficiencies so that they may provide alternative strategies to assist understanding. Also, very able students may be exposed to more challenging work, either in the same strand or in a related strand involving applications, rather than repeating what they have already mastered. Indeed, one of the main objectives in establishing the Index is to provide teachers with a *simple numerical value* that may be used as a guide to the most appropriate starting point for mathematical instruction in the first year of high school. This does not imply *streaming* in the true sense of the term but more of an adaptive approach to teaching pedagogies to suit different abilities.

Theoretical Framework

As attitudes to learning are influenced greatly by a student's perception of their potential to be successful and this is profoundly influenced by their self-belief in their ability or lack thereof (Dweck, 2000), the determination of the NPI may increase confidence in students who have a negative distorted perception of their own ability and even encourage a more adventurous approach to mathematics for those students with above average understanding of number.

Teachers have a profound influence on the achievement of their students in mathematics (Askew et al 1997). Further, teachers' beliefs influence their practices and preferred pedagogies (Beswick, 2007) and teachers will tend to choose the middle ground when faced with a large class of students with different abilities. The challenge is to get teachers to acknowledge that it is wrong to believe that students are all at the same stage of development when entering high school and will thus make the necessary changes to their teaching style to accommodate these differences.

Given that the correlation between a student's NPI and their exit results in Year 12 is sufficiently high to be used as a predictive model then it seems reasonable to assume that an improvement in a student's NPI would indicate better *number sense* and result in improved performance in later years. Number sense is defined as not only a person's general understanding of number and operations but also their ability to use this understanding and flexibility to develop strategies for dealing with numbers and operations usefully (McIntosh, Reys, & Reys, 1993).

Differences in curriculum delivery in high school have been widely recognised as one of the major factors in students failing to sustain their academic progress after transition (Kruse, 1996). Other factors include a change in the students' belief in themselves as learners, expectations of performance on the part of teachers and the inappropriateness of the content with respect to the ability of the students (Kirkpatrick, 1992). These factors are compounded when a student becomes bored or frustrated with the lack of challenge (Green, 1997) because they have had to spend a large part of their first year in high school repeating what they have already mastered (Yates, 1999).

Students being promoted chronologically are assumed to be at the same level of intellectual development, but a simple observation of the *physical* differences observable in any class in the first year of high school would show this to be a naive assumption. As success in the first year of high school can have a profound influence on the choice of subjects and vocational pathways (Siemon, Virgona, & Cornielle, 2001), it is vital that the students are provided the best opportunities. To do this, teachers need to be acutely aware of the base level of each of the students. The Index may help to indicate which students would benefit from teaching strategies that emphasise mastery, understanding and improvement (Midgley & Maehr, 1998).

Research that has shown transition difficulties do not last long for most students (Davison, 1996) did not focus on mathematics and it appears from the results of this study that the difficulties can last for many years although transition is, most likely, only one contributing factor to a complex problem. The lack of basic mathematical skills can have a compounding effect on success over many years and there is even a possibility that these same students are at an even greater disadvantage in being less skilful in adjusting to change in teaching style and pedagogy after transition (Eccles, Lord, & Midgley, 1991).

Despite the number of changes occurring in the educational landscape in Australia it should be noted that the decline in achievement following transition (Collins & Harrison, 1998; Galton, Gray, & Ruddrick, 1999) is not dependent on the age of the child when transition occurs as the same decline is evident in countries which have different transition periods (Anderman, Maehr, & Midgley, 1999).

Students entering high school are often not equipped to handle the emphasis on measuring performance (American Psychological Association, 1996) and exposure to a set curriculum that is based on the assumption that students are all at the same level of understanding (and skills), particularly in the number construct. As understanding and skills are intertwined (Wu, 1999) and not a dichotomy of one or the other it would be advantageous to students if intervention programs could be devised and implemented in the initial year of high school to challenge or improve the skills, and therefore the prospects of success of these students. Of course, this would require teachers to accept and adopt a range of pedagogies and the prospects of this happening are not great given current research findings (Cohen & Hill, 2001; Fullan, 1993). The linkage and continuity problems between schools may not be solved by the introduction of a national curriculum (Gorwood, 1991) so the importance of this research is not diminished by the changes occurring in Australia at present.

Methodology

Feeder Primary Schools. Before administering the tests the number of contributing primary schools was determined. Table 1 shows the data for the 2005 intake.

Table 1
Number of Feeder Primary Schools (Minimum 2 students) - 2005.

School Type	Number of Primary Schools
Government School	7
Catholic School (co-educational)	15
Catholic School (boys only- Boarding)	11
Anglican School (co-educational)	13

It seems that all of the schools were dealing with a large number of feeder schools and these numbers became much larger when the minimum number of contributing students was not used as a parameter. This makes the creation of linkages between primary and secondary schools very difficult and this requires further consideration as there is evidence that improved linkages can have a positive effect on transition (McGee, 1987). In the competitive environment of the private school system it may be that it is more important to *sell* the school rather than ensure that there is a sustained and effective linking arrangement between the schools that concentrate on the learning progression of the child (Abrams, 2000; Herrington & Doyle, 1998). This lack of linkage can exacerbate the problems faced by teachers in trying to establish the proficiency of students in their care often resulting in repetitive curriculum delivery and testing.

The Anglican school has its own primary school on site that contributes over half of the high school entry year so the number of other feeder schools was somewhat surprising.

The NPI Tests

The NPI tests contributing to this research were administered during Year 8 of high school in 2005. Many schools in Western Australia are now starting high school at Year 7 and other schools are adopting a middle school approach by grouping Years 7, 8 and 9 as a distinctive identity in terms of administration. The tests will continue to be given either at the end of Year 7 or at the beginning of Year 8, between the national NAPLAN tests in Year 7 and Year 9.

Participants and the Structure of the Tests

A total of 172 students sat the original test in 2005. The students came from four different schools and eight teachers were involved in the administration of the tests. The tests are a combination of 20 multiple-choice questions and 60 short-answer questions on basic number skills with many of the questions being modified from an original 60 multiple-choice questions, first used in 2003. The questions do not necessarily increase in difficulty and are not grouped. Responses are either awarded one mark for a correct answer or no marks for an incorrect answer. Instructions are prescriptive and teachers are not allowed to offer any assistance. No examples are given on the test papers and students are not permitted to use calculators or other mathematical aids.

The tests are prepared in two booklets, both containing 40 questions and can be administered across two 40-minute periods or in one 80-minute session. Both tests contain 10 multiple-choice and 30 short-answer questions and are constructed in such a way that topics alternate throughout the test and do not necessarily increase in difficulty as it was not assumed here that individual difficulties could be pre-determined. Estimation and number line questions are used extensively in all topics as can be seen in Figure 1.

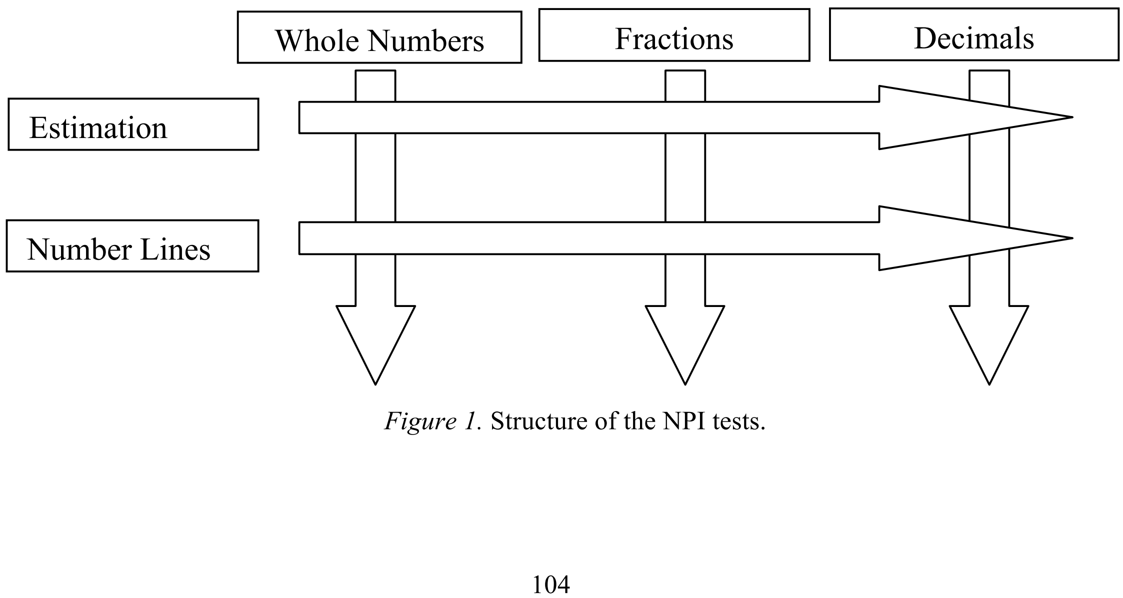

Figure 1. Structure of the NPI tests.

Data Collection and Reporting

The NPI is reported to teachers in raw score form, ranking lists, dot frequency diagrams, and separate results by gender and topic. The most important report for teachers simply indicates which of the three distinct categories *low*, *medium* and *high* the student falls into based on their raw score result from the NPI tests. This then indicates to the teacher those students who may need additional support and those students who do not need to repeat what they have already mastered. The categories are defined below:

- *Low* identifies the group of students who may benefit from interventionist strategies that may enable them to proceed successfully with work in number and measurement.
- *Middle* indicates the students who would be expected to work through the existing curriculum with very little change in content or delivery method.
- *High* identifies the group of students who would be provided with extension activities either in the same context as existing content or in a different context related to other areas of mathematics.

Discussion and Findings

The numbers in each category identified by their results in the NPI tests of 2005 are shown below in Table 2. The strategy to be adopted is also indicated in the table. It is interesting to note that 43 of the students had a very good understanding of number and could therefore benefit from becoming actively engaged in investigative learning and hands-on activities. The same could be said of the 31 students categorised as *low* but with a different emphasis concentrating on motivation and engagement. Whilst the recommended strategy for the *middle* group implies that no change is necessary to current practices it may be necessary to research the practices adopted by teachers in the transition year to ensure that they are encouraging students to think mathematically (Ruthven, 2002) and are not simply using repetitive processes that do not lead to sustained learning (Walshaw & Anthony, 2008).

Table 2
Results in Categories -2005 Test Year.

Category	Number	Strategy
Low (score less than 40)	31*	Intervention
Middle ($40 < x < 70$)	98	No change
High (score greater than or equal to 70)	43	Extension

Note. * Four students were special needs students with recognised learning difficulties.

The correlations shown in Table 3 below refer only to the 2005 Year 8 intake. The exit scores refer to the final mark obtained by the student when leaving high school and are taken from the scaled Western Australian Tertiary Entrance Examination (TEE) marks as provided by the Curriculum Council of Western Australia except for those students that did not sit the final examinations in which case the students scaled school mark was used. The scaling factors used were the same as those applied to the courses in Western Australia for the determination of the Tertiary Entrance Score (TES) that then provided each student with a Tertiary Entrance Rank (TER). Students who completed the lowest of the available senior mathematics courses (Modelling with Mathematics) had their marks adjusted to reflect the relative simplicity of the course.

Table 3

Correlations -2009 Graduation Year.

Comparison	Correlations	n*
WALNA7 v MSE9	0.93	164
WALNA7 v NPI	0.84	170
MSE9 v NPI	0.91	169
WALNA7 v Exit Score**	0.66	143
MSE9 v Exit Score	0.71	155
NPI v Exit Score	0.79	163

Note. * Student numbers varied due to the length of time between comparative data.

** The Exit Score is the weighted score obtained by the student in their final year of high school.

Tracking the Pathway of the Students through High School

Once the students had completed their final year examinations it was possible to compare their results to their positioning from the NPI tests nearly five years earlier. No student categorised as *high* in the NPI tests completed the Modelling with Mathematics course in 2009, which was not a TEE course and did not require the students to sit an examination, and only four of these students in this category attempted the easiest of the three TEE courses with examinations available in Western Australia. No student in the *low* category attempted any course higher than Modelling with Mathematics and seven students did not do any mathematics courses in 2009. Students who were originally categorised as *middle* in 2005 were represented in all three TEE courses and their final results were spread across the entire state distribution.

Implications

By showing that the NPI is the most reliable predictor of success teachers may become more cognisant of the importance of establishing a measure of a student's number facility and making changes to existing practices. It is not surprising that the correlations between the WALNA in Year 7 and the MSE in Year 9 were so high considering that the content covered in both tests was very similar and there was less than two years between the tests. Correlations decreased with the passing of time so whilst the correlation between the NPI results and the exit scores is only moderately strong it is the strongest of the three exit score comparisons.

High scores on the NPI tests do not necessarily mean that a child is proficient in the context of number but merely that they have some procedural skills in the strand and are thus ready to begin their next stage of learning on the way to becoming competent. It is imperative that teachers know the ability of the students in their care *before* they start teaching the curriculum. If we are to accept that teachers and their practices have a profound influence on their student's achievement then it is clearly evident that every effort must be made to assist teachers in understanding the potential of their students. Unfortunately, there is little confidence in believing that teachers will change from what they have been comfortable doing (Fullan, 1993) or any guarantee that teachers who identify problems with their student's understanding of basic facts will change their pedagogy (Walshaw & Anthony, 2008).

It is hoped that this research will not only identify problems in the assumptions of teachers about the most suitable starting point for instruction, but will also be instrumental in offering realistic, potential solutions to the problem. At the very least it should help teachers to understand the futility of trying to teach high order concepts to students who have not mastered a basic understanding of the number construct. It may also provide the opportunity for more able students to avoid the repetitive nature of some of the content delivered in the first year of high school.

Further research will continue and will include a broader cross-section of school types and factors such as cultural and socio-economic considerations, pupil perceptions, organisational issues and gender differences.

Conclusion

If we accept that teachers are reluctant to change their ingrained conventional practices (Cohen & Hill, 2001) then we need to make it as easy as possible for them to identify the standing of their students in relation to their peers and against minimum benchmarks. The NPI has a sufficiently high correlation to the students exit score to be used as a predictor for success throughout high school. Given that teachers acknowledge the results of the NPI tests and consequently make changes to cater for both strengths and deficiencies this research may help to alleviate some of the concerns regarding the number of students excluded from effective mathematics learning because of their lack of challenge or their lack of proficiency with basic number skills.

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